Physics 111

Exam #1

September 29, 2017

Name_____

Please read and follow these instructions carefully:

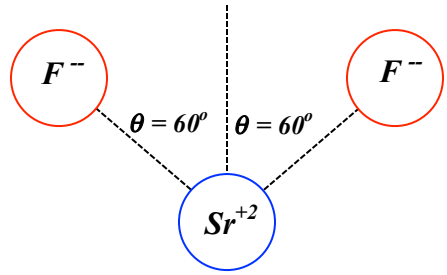
- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg) \times (2\frac{m}{s}) = 10\frac{kg \cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 3 points and each free-response part is worth 9 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

_____

1. Strontium (II) fluoride (SrF_2) is a binary ionic compound that is used as an anti-reflective coating on some lenses. Strontium (Sr^{+2}) and fluorine (F^-) ions come together to form the compound. Suppose that a Sr^{+2} ion is located at the point $(x,y)=(0,0)$ while the F^- ions are located on either side of the Sr^{+2} as shown below. Further let the spacing between a Sr^{+2} and F^- ion be $d=0.5nm$.



a. How much work (in eV) would be done to assemble this collection of charges if each charge is brought in from very far away and placed in the configuration above?

$$W_{Sr}=-q\Delta V=0$$

$$W_{F,1}=-q\Delta V=-(-e)\left[\frac{2ke}{d}-0\right]=\frac{2ke^2}{d}$$

$$W_{F,2}=-q\Delta V=-(-e)\left[\left[\frac{2ke}{d}-0\right]+\left[-\frac{ke}{2d\sin\theta}-0\right]\right]=\frac{2ke^2}{d}\left(1-\frac{1}{4\sin\theta}\right)$$

$$W_{net}=W_{Sr}+W_{F,1}+W_{F,2}=0+\frac{2ke^2}{d}+\frac{2ke^2}{d}\left(1-\frac{1}{4\sin\theta}\right)=\frac{2ke^2}{d}\left(2-\frac{1}{4\sin\theta}\right)$$

$$W_{net}=\frac{2ke^2}{d}\left(2-\frac{1}{4\sin\theta}\right)=\frac{2\times9\times10^9\,\frac{Nm^2}{C^2}\times\left(1.6\times10^{-19}C\right)^2}{0.5\times10^{-9}m}\left(2-\frac{1}{4\sin60}\right)=1.6\times10^{-18}J$$

$$w_{net}=1.6\times10^{-18}J\times\frac{1eV}{1.6\times10^{-19}J}=10eV$$

The distance between the fluorine atoms is given by $r=2x=2(d\sin\theta)$.

b. What is the magnitude and direction of the electric field at the location of the strontium ion (point $P=(0,0)$) due to the two fluorine ions?

$$E_{net,x}=E_{R,-q}\sin\theta-E_{L,-q}\sin\theta=0$$

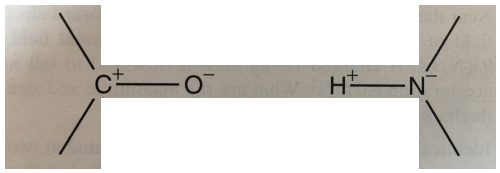
$$E_{net,y}=E_{R,-q}\cos\theta+E_{L,-q}\cos\theta=2E_{R,-q}\cos\theta=2\left(\frac{ke}{d^2}\right)\cos\theta$$

$$=2\left(\frac{9\times10^9\,\frac{Nm^2}{C^2}\times1.6\times10^{-19}C}{\left(0.5\times10^{-9}m\right)^2}\right)\cos60=5.8\times10^9\,\tfrac{N}{C}$$

$$E_{net}=\sqrt{E_{net,x}^2+E_{net,y}^2}=5.8\times10^9\,\tfrac{N}{C}$$

$$\phi=\tan^{-1}\left(\frac{E_{net,y}}{E_{net,x}}\right)=90^0$$

c. The CO and HN groups shown below are important in the study of organic chemistry. The C and O have charges $\pm 0.4e$ while the H and N have charges $\pm 0.2e$. Which of the following gives the net electrostatic force between the CO group and the HN group? Note the distances between individual atoms follow the convention: from atom 1 to atom 2 as: r_{12}. Ignore the internal forces between C and O and between H and N.



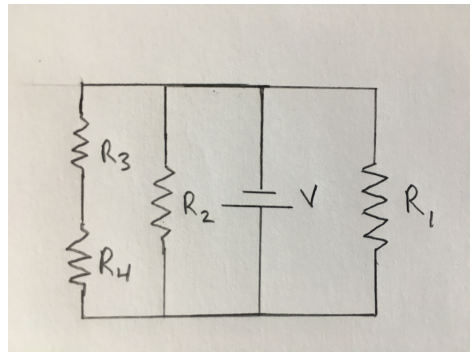
1. $F_{net} = F_{CH} + F_{CN} + F_{OH} + F_{ON} = k(0.4e)(0.2e)\left[\frac{1}{r_{CH}^2} + \frac{1}{r_{CN}^2} + \frac{1}{r_{OH}^2} + \frac{1}{r_{ON}^2}\right]$.
2. $F_{net} = F_{CH} - F_{CN} + F_{OH} - F_{ON} = k(0.4e)(0.2e)\left[\frac{1}{r_{CH}^2} - \frac{1}{r_{CN}^2} + \frac{1}{r_{OH}^2} - \frac{1}{r_{ON}^2}\right]$.
3. $F_{net} = -F_{CH} + F_{CN} + F_{OH} - F_{ON} = k(0.4e)(0.2e)\left[-\frac{1}{r_{CH}^2} + \frac{1}{r_{CN}^2} + \frac{1}{r_{OH}^2} - \frac{1}{r_{ON}^2}\right]$.
4. $F_{net} = F_{CH} + F_{CN} - F_{OH} - F_{ON} = k(0.4e)(0.2e)\left[\frac{1}{r_{CH}^2} + \frac{1}{r_{CN}^2} - \frac{1}{r_{OH}^2} - \frac{1}{r_{ON}^2}\right]$
5. $F_{net} = -F_{CH} - F_{CN} - F_{OH} - F_{ON} = -k(0.4e)(0.2e)\left[\frac{1}{r_{CH}^2} + \frac{1}{r_{CN}^2} + \frac{1}{r_{OH}^2} + \frac{1}{r_{ON}^2}\right]$.

d. Imagine that you had three $+q$ charges on the corners of an equilateral triangle. If the charges were released from rest, which of the following is(are) true? There could be more than one answer to this question.

1. The three charges will move toward each other and the work done on the charges will be negative.
2. The three charges will move apart and the work done on the charges will be negative.
3. The three charges will move toward each other and the work done on the charges will be positive.
4. The three charges will move apart and the work done on the charges will be positive.
5. The three charges will move at a constant speed when they are very far apart.
6. The charges will move and work will be done, but the direction and the sign of the work done cannot be determined from the information given.

2. Circuits

a. Consider the resistor circuit on the right in which a collection of resistors is wired in combination to a battery. The battery is rated at $V_B = 24V$ and each resistor is $R = 100k\Omega$. What is the equivalent resistance of the circuit and the total current produced by the battery?



R_3 and R_4 in series: $R_{34} = R_3 + R_4 = 100k\Omega + 100k\Omega = 200k\Omega$

R_{34} and R_2 in parallel:

$$\frac{1}{R_{234}} = \frac{1}{R_2} + \frac{1}{R_{43}} = \frac{1}{100k\Omega} + \frac{1}{200k\Omega} = \frac{3}{200k\Omega} \rightarrow R_{234} = \frac{200k\Omega}{3}$$

R_{234} in parallel with R_1:

$$\frac{1}{R_{1234}} = \frac{1}{R_{eq} = R_1} + \frac{1}{R_{234}} = \frac{1}{100k\Omega} + \frac{2}{200k\Omega} = \frac{5}{200k\Omega}$$

$$\rightarrow \mathrm{R}_{eq} = R_{1234} = \frac{200k\Omega}{5} 40k\Omega$$

The total current is: $I_{Total} = \frac{V}{R_{eq}} = \frac{24V}{40000\Omega} = 6 \times 10^{-4} A$.

b. Suppose that a resistor, with equivalent resistance of that determined in part a, were connected to a capacitor. The capacitor is constructed out of two parallel circular plates of diameter $D = 12.7cm$ separated by $2.54cm$ of paper ($\kappa = 3.7$). The resistor and capacitor are connected to the same $24V$ battery and allowed to fully charge. Then the battery is disconnected and the capacitor is connected to the resistor and is allowed to discharge through the resistor. How long would it take to dissipate 63% of the initial stored energy?

The capacitance:

$$C = \frac{\kappa \varepsilon_0 A}{d} = \frac{3.7 \times 8.85 \times 10^{-12} \frac{C^2}{Nm^2}}{0.0254m} \left[\pi \left(\frac{0.127m}{2} \right)^2 \right] = 1.63 \times 10^{-11} F.$$

The energy changes with time (since the potential across the capacitor changes with time) according to:

$$E(t) = \tfrac{1}{2} CV^2 = \tfrac{1}{2} C \left(V_{\max} e^{-\frac{t}{RC}} \right)^2 = E_{\max} e^{-\frac{2t}{RC}}$$

$$0.37 E_{\max} = E_{\max} e^{-\frac{2t}{RC}} \rightarrow t = -\frac{RC}{2} \ln(0.37) = -\frac{40000\Omega \times 1.63 \times 10^{-11} F}{2} \ln(0.37).$$

$$\therefore t = 3.24 \times 10^{-7} s$$

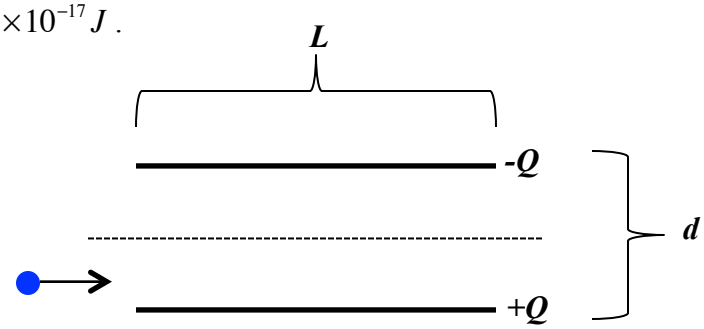
c. As the capacitor charges to its maximum potential using a battery of potential V, which of the following is true? In the following cases let V_R be the potential across the resistor at a time t and V_C be the potential across the capacitor at the same time t.

1. V_R and V_C remain constant in time since the battery is of constant potential.
2. $V_R \downarrow$ and $V_C \downarrow$.
3. $V_R \downarrow$ and $V_C \uparrow$.
4. $V_R \uparrow$ and $V_C \downarrow$.
5. $V_R \uparrow$ and $V_C \uparrow$.

d. A series combination of a resistor and a capacitor is the electronic control circuit is used in a cardiac pacemaker. The resistor-capacitor circuit can be "tuned" by changing the resistance of the pacemaker and this will control the rate at which the heart beats. Suppose that a particular pacemaker is designed to have a heart rate of $N\,\frac{beats}{\text{min}}$. If you as a physician needed to double the heart rate of your patient, so that the heart rate is $2N\,\frac{beats}{\text{min}}$, the capacitor in pacemaker the circuit

1. needs to discharge faster, so the resistance should be decreased.
2. needs to discharge faster, so the resistance should be increased.
3. needs to discharge slower, so the resistance should be decreased.
4. needs to discharge slower, so the resistance should be increased.
5. does not affect the timing, regardless of the resistance used.

3. Two parallel metal plates are shown below. The bottom plate has a charge $+Q$ while the upper plate has a charge $-Q$. The plates have a length $L = 10cm$ and are separated by $6cm$ of air. A constant electric field with magnitude $2500\frac{V}{m}$ points in the positive y-direction. A proton is incident from the left somewhere between the lower plate and the midpoint between the plates with a kinetic energy $K_i = 3.52\times10^{-17}J$.



a. From the point where the proton enters the electric field on the left to the point where it leaves the electric field on the right, what is the vertical displacement of the proton? The dashed line in the figure represents the midpoint between the two plates.

The only force that acts when the proton is between the plates is due to the electric field. This electric field points vertically up and thus the net force on the proton (and the acceleration) points vertically upward. The acceleration is given by: $F_{net,y} = eE = ma_y \rightarrow a_y = \dfrac{eE}{m}$. The vertical displacement of the proton in the field is given by

$$\Delta y = \tfrac{1}{2}a_y t^2 = \frac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{L}{v_{ix}}\right)^2 = \frac{1}{2}\left(\frac{1.6\times10^{-19}C\times2500\frac{N}{C}}{1.67\times10^{-27}kg}\right)\left(\frac{0.1m}{2.1\times10^5\frac{m}{s}}\right)^2 = 0.029m.$$

Here, the time for the proton to cross the field is determined from the horizontal motion ($\Delta x = v_{ix}t \rightarrow t = \dfrac{\Delta x}{v_{ix}} = \dfrac{L}{v_{ix}}$) and the horizontal component of the proton's velocity is determined from the initial kinetic energy (

$$K_i = \tfrac{1}{2}mv_{ix}^2 \rightarrow v_{ix} = \sqrt{\frac{2K_i}{m}} = \sqrt{\frac{2\times3.52\times10^{-17}J}{1.67\times10^{-27}kg}} = 2.1\times10^5\tfrac{m}{s}).$$

b. What is K_f, the final kinetic energy of the proton, when it leaves the electric field on the right?

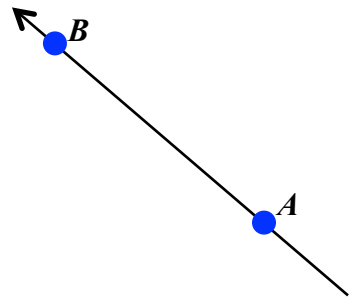
The electric field does work on the proton and the work done is given by

$$W = -q\Delta V = -e[-E\Delta y] = eE\Delta y = 1.6\times10^{-19}C\times2500\tfrac{N}{c}\times0.029m = 1.21\times10^{-17}J.$$

$$W_{net} = \Delta K = K_f - K_i \rightarrow K_f = W_{net} + K_i = 1.2\times10^{-17}J + 3.5\times10^{-17}J = 4.7\times10^{-17}J.$$

c. Suppose that an electric field points from the right to the left, diagonally up the page as shown below. An *electron* is fired from point B, at potential V_B toward point A, with potential V_A. If $V_B < V_A$, which of the following is true as an electron moves from point B to point A?

1. $\Delta K > 0$ and $\Delta EPE < 0$.
2. $\Delta K < 0$ and $\Delta EPE < 0$.
3. $\Delta K > 0$ and $\Delta EPE > 0$.
4. $\Delta K < 0$ and $\Delta EPE > 0$.
5. The kinetic and electric potential energies both change but in a way that cannot be calculated from the information given.



d. After your experiment with the proton in part a, you need to remove the $\pm Q$ charges from the plates. To do this you connect the $-Q$ plate to the $+Q$ plate together with a resistor R. The system then discharges through the resistor. Which of the following gives the energy dissipated per unit time across the resistor as heat and light?

1. $P = I_{\max} V_{\max} e^{-\frac{t}{RC}}$.
2. $P = \frac{V_{\max}^2}{R} e^{-\frac{2t}{RC}}$.
3. $P = I_{\max}^2 R e^{-\frac{t}{RC}}$.
4. $P = \frac{I_{\max}}{V_{\max}} e^{-\frac{t}{RC}}$.
5. $P = \frac{V_{\max}}{I_{\max}} e^{-\frac{t}{RC}}$.

Physics 111 Equation Sheet

Electric Forces, Fields and Potentials

$$\vec{F} = k\frac{Q_1Q_2}{r^2}\hat{r}$$

$$\vec{E} = \frac{\vec{F}}{q}$$

$$\vec{E}_Q = k\frac{Q}{r^2}\hat{r}$$

$$PE = k\frac{Q_1Q_2}{r}$$

$$V(r) = k\frac{Q}{r}$$

$$E_x = -\frac{\Delta V}{\Delta x}$$

$$W = -q\Delta V = -q\left[V_f - V_i\right]$$

Magnetic Forces and Fields

$$F = qvB\sin\theta$$

$$F = IlB\sin\theta$$

$$\tau = NIAB\sin\theta = \mu B\sin\theta$$

$$PE = -\mu B\cos\theta$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\varepsilon_{induced} = -N\frac{\Delta\phi_B}{\Delta t} = -N\frac{\Delta(BA\cos\theta)}{\Delta t}$$

Constants

$$g = 9.8\,\tfrac{m}{s^2}$$

$$1e = 1.6\times10^{-19}C$$

$$k = \frac{1}{4\pi\varepsilon_o} = 9\times10^{9}\,\tfrac{Nm^2}{C^2}$$

$$\varepsilon_o = 8.85\times10^{-12}\,\tfrac{C^2}{Nm^2}$$

$$1eV = 1.6\times10^{-19}J$$

$$\mu_o = 4\pi\times10^{-7}\,\tfrac{Tm}{A}$$

$$c = 3\times10^{8}\,\tfrac{m}{s}$$

$$h = 6.63\times10^{-34}Js$$

$$m_e = 9.11\times10^{-31}kg = \frac{0.511MeV}{c^2}$$

$$m_p = 1.67\times10^{-27}kg = \frac{937.1MeV}{c^2}$$

$$m_n = 1.69\times10^{-27}kg = \frac{948.3MeV}{c^2}$$

$$1amu = 1.66\times10^{-27}kg = \frac{931.5MeV}{c^2}$$

$$N_A = 6.02\times10^{23}$$

$$Ax^2 + Bx + C = 0 \rightarrow x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

Electric Circuits

$$I = \frac{\Delta Q}{\Delta t}$$

$$V = IR = I\left(\frac{\rho L}{A}\right)$$

$$R_{series} = \sum_{i=1}^{N} R_i$$

$$\frac{1}{R_{parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$$

$$P = IV = I^2R = \frac{V^2}{R}$$

$$Q = CV = \left(\frac{\kappa\varepsilon_0 A}{d}\right)V = (\kappa C_0)V$$

$$W = U = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}$$

$$Q_{\text{charge}}(t) = Q_{\max}\left(1 - e^{-\frac{t}{RC}}\right)$$

$$Q_{\text{discharge}}(t) = Q_{\max}e^{-\frac{t}{RC}}$$

$$C_{parallel} = \sum_{i=1}^{N} C_i$$

$$\frac{1}{C_{series}} = \sum_{i=1}^{N}\frac{1}{C_i}$$

Light as a Particle & Relativity

$$E = hf = \frac{hc}{\lambda} = pc$$

$$KE_{\max} = hf - \phi = eV_{stop}$$

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\phi)$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$p = \gamma mv$$

$$E_{total} = KE + E_{rest} = \gamma mc^2$$

$$E_{total}^2 = p^2c^2 + m^2c^4$$

$$E_{rest} = mc^2$$

$$KE = (\gamma - 1)mc^2$$

Geometry

Circles: $C = 2\pi r = \pi D \qquad A = \pi r^2$

Triangles: $A = \tfrac{1}{2}bh$

Spheres: $A = 4\pi r^2 \qquad V = \tfrac{4}{3}\pi r^3$

Light as a Wave

$$c = f\lambda = \frac{1}{\sqrt{\varepsilon_o\mu_o}}$$

$$S(t) = \frac{energy}{time\times area} = c\varepsilon_o E^2(t) = c\frac{B^2(t)}{\mu_0}$$

$$I = S_{avg} = \tfrac{1}{2}c\varepsilon_o E_{\max}^2 = c\frac{B_{\max}^2}{2\mu_0}$$

$$P = \frac{S}{c} = \frac{Force}{Area}$$

$$S = S_o\cos^2\theta$$

$$v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{n}$$

$$\theta_{inc} = \theta_{refl}$$

$$n_1\sin\theta_1 = n_2\sin\theta_2$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$M_{total} = \prod_{i=1}^{N} M_i$$

$$d\sin\theta = m\lambda \text{ or } (m+\tfrac{1}{2})\lambda$$

$$a\sin\phi = m'\lambda$$

Nuclear Physics

$$E_{binding} = (Zm_p + Nm_n - m_{rest})c^2$$

$$\frac{\Delta N}{\Delta t} = -\lambda N_o \rightarrow N(t) = N_o e^{-\lambda t}$$

$$A(t) = A_o e^{-\lambda t}$$

$$m(t) = m_o e^{-\lambda t}$$

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

Misc. Physics 110 Formulae

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\Delta(mv)}{\Delta t} = m\vec{a}$$

$$\vec{F} = -k\vec{y}$$

$$\vec{F}_C = m\frac{v^2}{R}\hat{r}$$

$$W = \Delta KE = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right) = -\Delta PE$$

$$PE_{gravity} = mgy$$

$$PE_{spring} = \tfrac{1}{2}ky^2$$

$$x_f = x_i + v_{ix}t + \tfrac{1}{2}a_x t^2$$

$$v_{fx} = v_{ix} + a_x t$$

$$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x$$