Physics 111

Exam #1

October 2, 2020

Name_____

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}| = (5kg) \times \left(2\frac{m}{s}\right) = 10\frac{kg \cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple-choice questions are worth 3 points and each free-response part is worth 7 points.

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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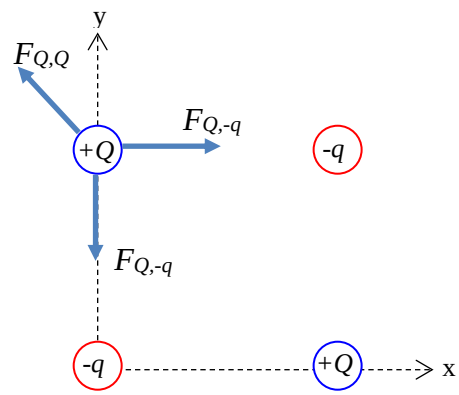
1. Point Charges

a. Consider the four point-charges shown on the right on the corners of a square with sides of length L. The charge in the upper left corner and the charge in the lower right corner have charges $+Q$. The other two charges are negative in sign but have unknown magnitude q. If the net force on the upper left charge is zero, what is the magnitude of the other charges (assumed equal) in the upper right and lower left corners (the red circles) in terms of Q?



From the horizontal forces:

$$F_{Q,-q} - F_{Q,Q} = \frac{kQq}{L^2} - \frac{kQQ}{2L^2}\cos 45 = \frac{kQ}{L^2}\left(q - \frac{\sqrt{2}Q}{4}\right) = 0$$

$$\rightarrow q - \frac{\sqrt{2}Q}{4} = 0 \rightarrow q = \frac{\sqrt{2}Q}{4}$$

Or, from the vertical forces:

$$-F_{Q,-q} + F_{Q,q} = -\frac{kQq}{L^2} + \frac{kQQ}{2L^2}\sin 45 \ = \frac{kQ}{L^2}\left(-q + \frac{\sqrt{2}Q}{4}\right) = 0$$

$$\rightarrow -q + \frac{\sqrt{2}Q}{4} = 0 \rightarrow q = \frac{\sqrt{2}Q}{4}$$

b. How much work was done to assemble the distribution of point-charges if $Q = 2\mu C$ and $L = 1cm$? Assume that each charge was brought in one at a time, from very far away and placed at the charge's final location.

Labeling the charges clockwise from the upper we have:
$W_1 = 0$ to place the first charge.

$$W_2 = -q_2 \Delta V_1 = -\left(-\frac{\sqrt{2}Q}{4}\right)\left[\frac{kQ}{L} - 0\right]$$

$W_2 = \frac{\sqrt{2}kQ^2}{4L} = \frac{\sqrt{2}\times 9\times 10^9 \frac{Nm^2}{C^2}\left(2\times 10^{-6}C\right)^2}{4\times 0.01m} = 1.27J$ to place the 2nd charge.

$$W_3 = -q_3(\Delta V_1 + \Delta V_2) = -(Q)\left[\left(\frac{kQ}{\sqrt{2}L} - 0\right) + \left(-\frac{\sqrt{2}kQ}{4L} - 0\right)\right]$$

$W_3 = \frac{kQ^2}{L}\left(-\frac{1}{\sqrt{2}} + \frac{\sqrt{2}}{4}\right) = \frac{9\times 10^9 \frac{Nm^2}{C^2}\left(2\times 10^{-6}C\right)^2}{0.01m}\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{4}\right) = -1.27J$ to place the 3rd charge.

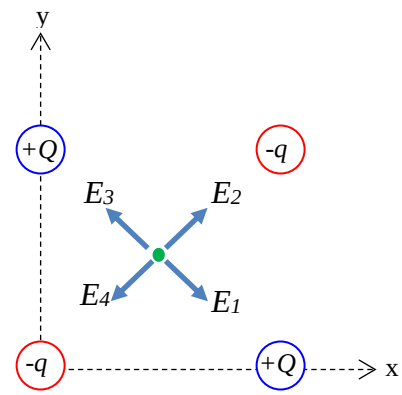
$$W_4 = -q_4(\Delta V_1 + \Delta V_2 + \Delta V_3)$$
$$= -\left(-\frac{\sqrt{2}Q}{4}\right)\left[\left(\frac{kQ}{L} - 0\right) + \left(-\frac{\sqrt{2}kQ}{4\sqrt{2}L} - 0\right) + \left(\frac{kQ}{L} - 0\right)\right]$$

$W_3 = \frac{kQ^2}{L}\left(\frac{2\sqrt{2}}{4} - \frac{\sqrt{2}}{16}\right) = \frac{9\times 10^9 \frac{Nm^2}{C^2}\left(2\times 10^{-6}C\right)^2}{0.01m}\left(\frac{2\sqrt{2}}{4} - \frac{\sqrt{2}}{16}\right) = 2.23J$ to place the 4th charge.

$$W_{net} = W_1 + W_2 + W_3 + W_4 = 0J + 1.27J - 1.27J + 2.23J = 2.23J$$

c. What is the net electric field at the center of the square?

Labeling the charges clockwise from the upper left charge, we have that due to the symmetry in the problem, the net electric field at the center of the square is zero.

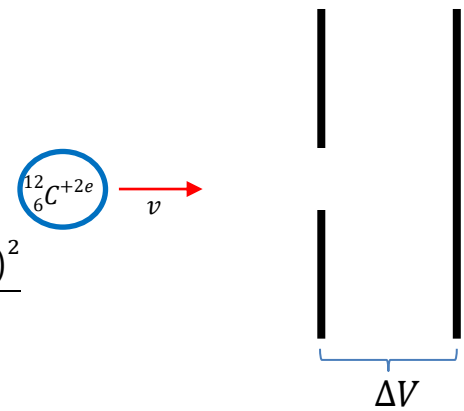


d. Which of the following statements about electric fields and electric potentials is valid?

1. If the electric potential at a particular point is zero, the electric field at that point is zero.
2. If the electric field at a particular point is zero then the electric potential at that point must be zero.
3. If the electric field throughout a particular region is constant, then the electric potential throughout that region must be zero.
4. If the electric potential throughout a particular region is constant, then the electric field in that region must be zero.
5. All of the above statements about electric fields and electric potentials are valid.
6. None of the above statements about electric fields and electric potentials are valid.

2. RC Circuits

a. A carbon ion ($^{12}_{6}C^{+2e}$) is accelerated from rest to a speed $5 \times 10^{5}\frac{m}{s}$. The carbon ion is then directed toward a hole in a set of capacitor plates as shown below. What potential difference across the plates would be needed to bring the carbon ion to rest just at the rightmost plate?



$$W = -q\Delta V = \Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$W = -q\Delta V = -\tfrac{1}{2}mv_i^2$$

$$\Delta V = \frac{mv_i^2}{2q} = \frac{(12 \times 1.67 \times 10^{-27}kg)\left(5 \times 10^{5}\frac{m}{s}\right)^2}{2 \times (2 \times 1.6 \times 10^{-19}C)}$$

$$\Delta V = 7828V = 7.83kV$$

b. If the plates of the capacitor are square with a side of length $L = 85cm$, and are separated by a distance of $d = 3cm$, what is the capacitance of the system and how much charge was stored on the capacitor when fully charged?

$$C = \frac{\kappa\epsilon_0 A}{d} = \frac{8.85\times10^{-12}\frac{C^2}{Nm^2}\times(0.85m)^2}{0.03m} = 2.1 \times 10^{-10}F$$

$$Q = CV = 2.1 \times 10^{-10}F \times 7828V = 1.7 \times 10^{-6}C = 1.7\mu C$$

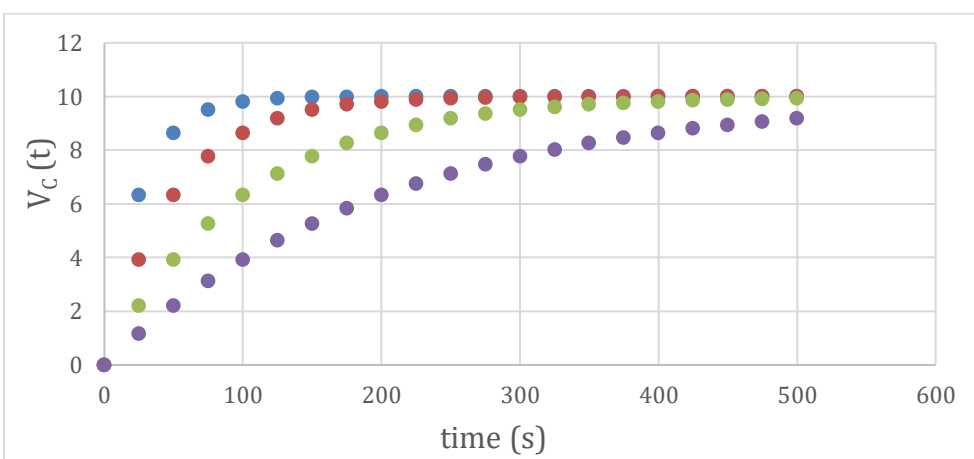
c. How much energy is stored in the fully charged capacitor and what is the electric field between the plates?

$W = \frac{1}{2}CV^2 = \frac{1}{2} \times 2.1 \times 10^{-10}F \times (7828V)^2 = 0.0064J = 6.4mJ$

$E = -\frac{\Delta V}{\Delta x} = -\frac{7828V}{0.03m} = -2.6 \times 10^5 \frac{V}{m}$, or $2.6 \times 10^5 \frac{V}{m}$ pointing to the left opposite to the velocity of the carbon ion.

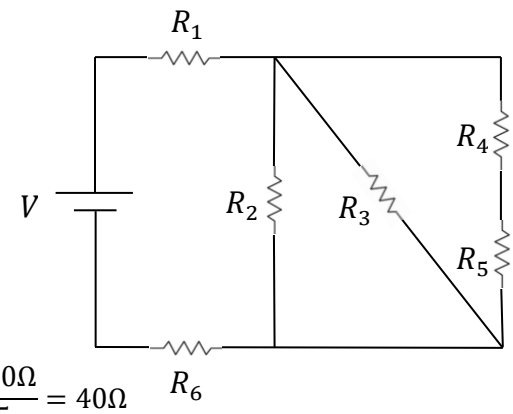
d. Suppose that you had four individual capacitors (each with a different capacitance) that were each separately fully charged using a battery with potential difference V and connected to a resistor with resistance R, forming four separate RC circuits. The capacitors charge through their individual resistor and plots of the potential difference across the charging capacitors as a function of time are shown below. From the graph below, which curve has a time constant of $\tau = 50s$?

1. The blue curve.
2. The red curve.
3. The green curve.
4. The purple curve.
5. It is not possible to determine which curve has a $\tau = 50s$ time constant without more information.



3. Electric Circuits

Consider the circuit shown below in which several identical resistors each with resistance $R = 100\Omega$ are connected to a $10V$ battery.



a. What is the current produced by the battery?

Resistors R_4 and $R_5$are in series.
$R_{45} = R_4 + R_5 = 100\Omega + 100\Omega = 200\Omega$

Resistors R_2, R_3 and $R_{45}$are in parallel.
$$\frac{1}{R_{2345}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_{45}} = \frac{5}{200\Omega} \rightarrow R_{2345} = \frac{200\Omega}{5} = 40\Omega$$

Resistors R_1, $R_{2345}$and R_6 are in series.
$R_{eq} = R_{123456} = R_1 + R_{2345} + R_6 = 100\Omega + 40\Omega + 100\Omega = 240\Omega$

The total current: $I_T = \frac{V}{R_{eq}} = \frac{10V}{240\Omega} = 0.0417A = 41.7mA$

b. What is the current through and the potential drop across the resistor labeled R_2?

The potential drop across resistors R_1 and R_6 are given by $V_1 = I_T R_1 = 0.0417A \times 100\Omega = 4.17V$and $V_6 = I_T R_6 = 0.0417A \times 100\Omega = 4.17V$ respectively.

Thus the potential drop across V_2, given by conservation of energy, is $V - V_1 - V_{2345} - V_6 = 0 \rightarrow V_{2345} = V_2 = V - V_1 - V_6 = 10V - 4.17V - 4.17V = 1.66V$.

The current through is given by Ohm's law: $V_2 = I_2 R_2 \rightarrow I_2 = \frac{V_2}{R_2} = \frac{1.66V}{100\Omega} = 0.0166A = 16.6mA$

c. Suppose that we found a resistor laying around the lab with the equivalent resistance of the circuit in part 3a. We connect this resistor to the same $10V$ battery and an initially uncharged $3000\mu F$ capacitor. As soon as the battery is connected to the resistor/capacitor combination the capacitor begins to charge through the resistor. At what time will the capacitor have a stored energy equal to 48% of its maximum amount?

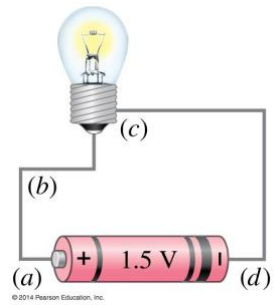
$$E = \tfrac{1}{2}CV^2 = \tfrac{1}{2}C\left(V_{max}\left(1 - e^{-\frac{t}{RC}}\right)\right)^2 = \tfrac{1}{2}CV_{max}^2\left(1 - e^{-\frac{t}{RC}}\right)^2$$

$$E = E_{max}\left(1 - e^{-\frac{t}{RC}}\right)^2 \rightarrow t = -RCln\left(1 - \sqrt{\frac{E}{E_{max}}}\right)$$

$$t = -240\Omega \times 3000 \times 10^{-6}F \times ln\left(1 - \sqrt{\frac{0.48E_{max}}{E_{max}}}\right) = 0.85s$$

d. Consider the following circuit in which a light bulb is connected to a battery. At which point in the circuit is the current the largest?

1. Point A
2. Point B.
3. Point C.
4. Point D.
5. It's the same at all points.
6. There is not enough information to answer this question.



Physics 111 Equation Sheet

Electric Forces, Fields and Potentials

$$\vec{F} = k\frac{Q_1Q_2}{r^2}\hat{r}$$

$$\vec{E} = \frac{\vec{F}}{q}$$

$$\vec{E}_Q = k\frac{Q}{r^2}\hat{r}$$

$$PE = k\frac{Q_1Q_2}{r}$$

$$V(r) = k\frac{Q}{r}$$

$$E_x = -\frac{\Delta V}{\Delta x}$$

$$W = -q\Delta V = -q\left[V_f - V_i\right]$$

Magnetic Forces and Fields

$$F = qvB\sin\theta$$

$$F = IlB\sin\theta$$

$$\tau = NIAB\sin\theta = \mu B\sin\theta$$

$$PE = -\mu B\cos\theta$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\varepsilon_{induced} = -N\frac{\Delta\phi_B}{\Delta t} = -N\frac{\Delta(BA\cos\theta)}{\Delta t}$$

Constants

$$g = 9.8\,\tfrac{m}{s^2}$$

$$1e = 1.6\times10^{-19}C$$

$$k = \frac{1}{4pe_o} = 9\times10^{9}\,\tfrac{Nm^2}{C^2}$$

$$e_o = 8.85\times10^{-12}\,\tfrac{C^2}{Nm^2}$$

$$1eV = 1.6\times10^{-19}J$$

$$m_0 = 4p\times10^{-7}\,\tfrac{Tm}{A}$$

$$c = 3\times10^{8}\,\tfrac{m}{s}$$

$$h = 6.63\times10^{-34}Js$$

$$m_e = 9.11\times10^{-31}kg = \frac{0.511MeV}{c^2}$$

$$m_p = 1.67\times10^{-27}kg = \frac{937.1MeV}{c^2}$$

$$m_n = 1.69\times10^{-27}kg = \frac{948.3MeV}{c^2}$$

$$1amu = 1.66\times10^{-27}kg = \frac{931.5MeV}{c^2}$$

$$N_A = 6.02\times10^{23}$$

$$Ax^2 + Bx + C = 0 \rightarrow x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

Electric Circuits

$$I = \frac{DQ}{Dt}$$

$$V = IR = I\left(\frac{rL}{A}\right)$$

$$R_{series} = \sum_{i=1}^{N} R_i$$

$$\frac{1}{R_{parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$$

$$P = IV = I^2R = \frac{V^2}{R}$$

$$Q = CV = \left(\frac{ke_0A}{d}\right)V = \left(kC_0\right)V$$

$$W = U = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}$$

$$Q_{\text{charge}}(t) = Q_{\max}\left(1 - e^{-\frac{t}{RC}}\right)$$

$$Q_{\text{discharge}}(t) = Q_{\max}e^{-\frac{t}{RC}}$$

$$C_{parallel} = \sum_{i=1}^{N} C_i$$

$$\frac{1}{C_{series}} = \sum_{i=1}^{N}\frac{1}{C_i}$$

Light as a Particle & Relativity

$$E = hf = \frac{hc}{\lambda} = pc$$

$$KE_{\max} = hf - \phi = eV_{stop}$$

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\phi)$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$p = \gamma mv$$

$$E_{total} = KE + E_{rest} = \gamma mc^2$$

$$E_{total}^2 = p^2c^2 + m^2c^4$$

$$E_{rest} = mc^2$$

$$KE = (\gamma - 1)mc^2$$

Geometry

Circles: $C = 2pr = pD \qquad A = pr^2$

Triangles: $A = \tfrac{1}{2}bh$

Spheres: $A = 4pr^2 \qquad V = \tfrac{4}{3}pr^3$

Light as a Wave

$$c = f\lambda = \frac{1}{\sqrt{\varepsilon_o\mu_o}}$$

$$S(t) = \frac{energy}{time\times area} = c\varepsilon_o E^2(t) = c\frac{B^2(t)}{\mu_0}$$

$$I = S_{avg} = \tfrac{1}{2}c\varepsilon_o E_{\max}^2 = c\frac{B_{\max}^2}{2\mu_0}$$

$$P = \frac{S}{c} = \frac{Force}{Area}$$

$$S = S_o\cos^2\theta$$

$$v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{n}$$

$$\theta_{inc} = \theta_{refl}$$

$$n_1\sin\theta_1 = n_2\sin\theta_2$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$M_{total} = \prod_{i=1}^{N} M_i$$

$$d\sin\theta = m\lambda \text{ or } \left(m+\tfrac{1}{2}\right)\lambda$$

$$a\sin\phi = m'\lambda$$

Nuclear Physics

$$E_{binding} = \left(Zm_p + Nm_n - m_{rest}\right)c^2$$

$$\frac{\Delta N}{\Delta t} = -\lambda N_o \rightarrow N(t) = N_o e^{-\lambda t}$$

$$A(t) = A_o e^{-\lambda t}$$

$$m(t) = m_o e^{-\lambda t}$$

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

Misc. Physics 110 Formulae

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\Delta(mv)}{\Delta t} = m\vec{a}$$

$$\vec{F} = -k\vec{y}$$

$$\vec{F}_C = m\frac{v^2}{R}\hat{r}$$

$$W = \Delta KE = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right) = -\Delta PE$$

$$PE_{gravity} = mgy$$

$$PE_{spring} = \tfrac{1}{2}ky^2$$

$$x_f = x_i + v_{ix}t + \tfrac{1}{2}a_x t^2$$

$$v_{fx} = v_{ix} + a_x t$$

$$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x$$