Physics 111

Exam #1

October 8, 2021

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $\left|\vec{p}\right| \approx m\left|\vec{v}\right| = \left(5kg\right)\times\left(2\,\frac{m}{s}\right) = 10\,\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple-choice questions are worth 3 points and each free-response part is worth 7 points.

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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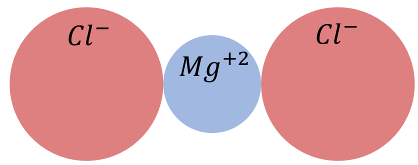
1. Point Charges

Magnesium and chlorine come together to form an ionic bond to make magnesium chloride ($MgCl_2$). An ionic bond holds the atoms together by an electrostatic attraction between the atoms. The two valence electrons of magnesium are donated, one each to each of the chlorine atoms. This effectively makes $Mg^{+2}$and two Cl^{-} and these are what are responsible for the electrostatic attraction of the ionic bond.

a. How much work was done to assemble $MgCl_2$ and how much energy (in eV) is stored in the system as a potential energy? Assume each charge is brought in separately from very far away and placed in the positions shown below, where the ionic radii of magnesium and chlorine are $r_{Mg} = 0.072nm$ and $r_{Cl} = 0.181nm$ respectively.



$$r = r_{Mg} + r_{Cl} = 0.072nm + 0.181n = 0.253nm$$

$$W_{Mg} = 0$$
$$W_{Cl} = -(-e)\left(\frac{k2e}{r}\right) = \frac{2ke^2}{r}$$
$$W_{cl} = -(-e)\left(-\frac{ke}{2r}\right) - (-e)\left(\frac{k2e}{r}\right) = \frac{3ke^2}{2r}$$

$$W_{net} = W_{Mg} + W_{Cl} + W_{Cl} = 0 + \frac{2ke^2}{r} + \frac{3ke^2}{2r} = \frac{7ke^2}{2r}$$
$$W_{net} = \frac{7ke^2}{2r} = \frac{7 \times 9 \times 10^9 \frac{Nm^2}{C^2} \times (1.6 \times 10^{-19}C)^2}{2 \times 0.253 \times 10^{-9}m} = 3.2 \times 10^{-18}J$$
$$W_{net} = -\Delta U_e \rightarrow \Delta U_e = -3.2 \times 10^{-18}J \times \frac{1eV}{1.6 \times 10^{-19}J} = -19.9eV$$

b. What is the net electric field at the location of the rightmost chlorine ion due to the magnesium and leftmost chlorine ion?

$$E_{net,x} = E_{Mg} - E_{Cl} = \frac{2ke}{r^2} - \frac{ke}{(2r)^2} = \frac{7ke}{4r^2}$$
$$E_{net,x} = \frac{7 \times 9 \times 10^9 \frac{Nm^2}{C^2} \times 1.6 \times 10^{-19}C}{4(0.253 \times 10^{-9}m)^2} = 3.9 \times 10^{10}\frac{N}{C}$$

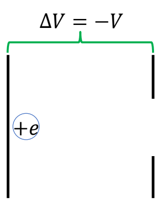
The net electric field points away from the magnesium ion (in the positive x-direction) with magnitude $3.9 \times 10^{10}\frac{N}{C}$.

c. What is the net electrostatic force on the rightmost chlorine ion due to the magnesium and leftmost chlorine ions?

$F = qE = 1.6 \times 10^{-19} C \times 3.9 \times 10^{10} \frac{N}{C} = 6.3 \times 10^{-9} N$ in magnitude.
The direction is in the negative x-direction (towards the magnesium ion) since the charge on the chlorine is negative and will feel a force opposite to the direction of the electric field.

d. Suppose that you wanted to accelerate a proton from rest to a speed v_f, so that maybe you could probe the structure of $MgCl_2$ with the proton. To do this you solicit designs from companies that build proton accelerators. Below is a collection of proton accelerator designs from four companies using different sets of capacitors. Which of the following designs will allow you to effectively accelerate the proton to right at a speed v_f?

1.



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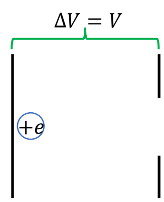

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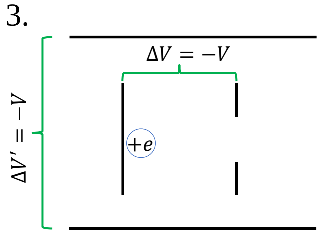

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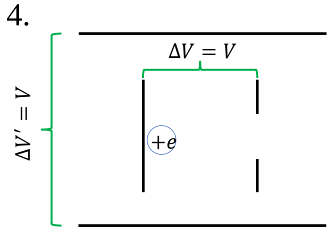
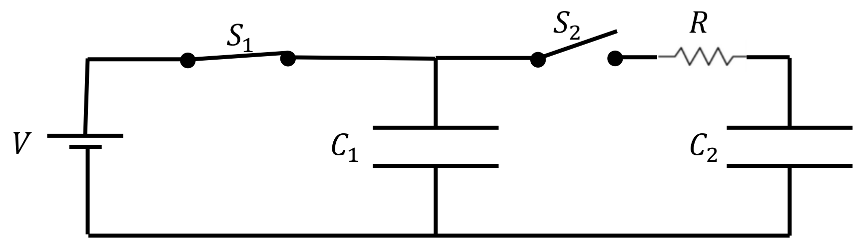


5. None of the above designs will allow me to build a successful proton accelerator.

2. Capacitors
Suppose that you have the circuit shown below in which a capacitor $C_1 = 2\mu F$ is charged by keeping switch S_1 closed and switch S_2 open. The capacitor C_1 is connected to a $V = 2000V$ battery.



a. How much charge (in magnitude) is on each plate and how much energy is stored in the $2\mu F$ capacitor when fully charged?

$$Q_1 = C_1 V = 2 \times 10^{-6} F \times 2000V = 4 \times 10^{-3} C$$
$$U_{e,1} = \frac{1}{2} C_1 V^2 = \frac{1}{2} \times 2 \times 10^{-6} F (2000V)^2 = 4J$$

b. At a later time, the switch S_1 is opened and the switch S_2 is closed. This connects capacitor C_2, which is initially uncharged and has an unknown capacitance, into the circuit. A $R = 1M\Omega$ resistor connects the two capacitors. When the switch S_2 is closed, what is the initial current that flows through the resistor and what is the capacitance of C_2? Assume that the time constant for this circuit is $1.5s$.

$$\tau = R_{eq} C_{eq} \rightarrow C_{eq} = \frac{\tau}{R_{eq}} = \frac{1.5s}{1 \times 10^6 \Omega} = 1.5 \times 10^{-6} F = 1.5\mu F$$
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \rightarrow C_2 = \left(\frac{1}{C_{eq}} - \frac{1}{C_1}\right)^{-1} = \left(\frac{1}{1.5\mu F} - \frac{1}{2\mu F}\right)^{-1} = 6\mu F$$

$$V_R = IR \rightarrow I = \frac{V_R}{R} = \frac{2000V}{1 \times 10^6 \Omega} = 0.002A = 2mA$$

c. What is the final charge on each capacitor and the energy stored in each capacitor after a long time has passed? Hint: Apply conservation of energy and charge.

$V_{C_1} - V_R - V_{C_2} = 0 \rightarrow V_{C_1} = V_{C_2}$ since after a long time, the current goes to zero and thus $V_R \rightarrow 0$.

$$V_{C_1} = V_{C_2} \rightarrow \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \rightarrow Q_2 = \left(\frac{C_2}{C_1}\right) Q_1$$

$$Q_{total} = Q_1 + Q_2 = Q_1 + \left(\frac{C_2}{C_1}\right) Q_1 = \left(1 + \frac{C_2}{C_1}\right) Q_1 = \left(1 + \frac{6\mu F}{2\mu F}\right) Q_1 = 4Q_1 = 4 \times 10^{-3} C$$

$Q_1 = 1 \times 10^{-3} C$ and $Q_2 = \left(\frac{C_2}{C_1}\right) Q_1 = \left(\frac{6\mu F}{2\mu F}\right) 4 \times 10^{-3} C = 3 \times 10^{-3} C$

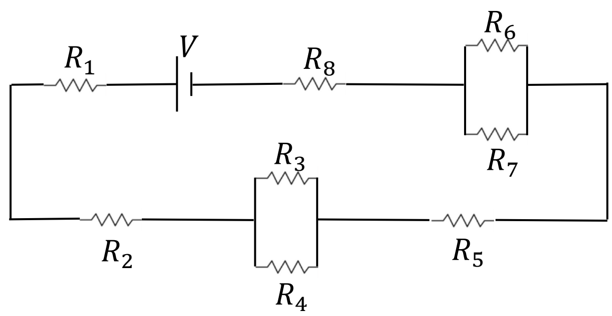
$$U_{e,1} = \frac{Q_1^2}{2C_1} = \frac{\left(1 \times 10^{-3} C\right)^2}{2 \times 2 \times 10^{-6} F} = 0.25J$$
$$U_{e,2} = \frac{Q_2^2}{2C_2} = \frac{\left(3 \times 10^{-3} C\right)^2}{2 \times 6 \times 10^{-6} F} = 0.75J$$

d. When connected as in part c, the energy stored in capacitors C_1 and C_2 after a long time, it turns out, *is less than* the value that you calculated for capacitor C_1 alone, in part a. Which of the following below could be a possible reason why the total energy stored in the capacitors in part c is less than that stored in part a?

1. There is energy lost in charging capacitor C_2.
2. Energy is lost in discharging C_1.
3. Energy is lost to heat in the resistor.
4. Energy is not conserved in this experiment.
5. None of the above adequately explain why the total energies are different.

3. Electric Circuits
Consider the circuit shown below in which eight identical resistors each with resistance $R = 100\Omega$ are connected to a $10V$ battery.

a. What is total current produced by the battery?



R_3 and R_4 in parallel, thus

$$\frac{1}{R_{34}} = \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{100\Omega} + \frac{1}{100\Omega} = \frac{2}{100\Omega}$$

$R_{34} = 50\Omega$

R_6 and R_7 in parallel, thus

$$\frac{1}{R_{67}} = \frac{1}{R_6} + \frac{1}{R_7} = \frac{1}{100\Omega} + \frac{1}{100\Omega} = \frac{2}{100\Omega}$$

$R_{67} = 50\Omega$

R_1, R_2, R_{34}, R_5, R_{67}, and R_8 in series, thus

$$R_{eq} = R_{12345678} = R_1 + R_2 + R_{34} + R_5 + R_{67} + R_8 = 4R_1 + 2R_{34}$$

$$R_{eq} = 4(100\Omega) + 2(50\Omega) = 500\Omega$$

$$I_T = \frac{V}{R_{eq}} = \frac{10V}{500\Omega} = 0.02A = 20mA$$

b. What is the potential difference across resistor R_6 and the current through resistor R_3?

$$V - V_{R1} - V_{R2} - V_{R34} - V_{R5} - V_{R67} - V_{R8} = 0$$

$$V = 4V_{R1} + 2V_{R34} \rightarrow V_{R34} = \frac{V-4R_1}{2} = \frac{10V-4(2V)}{2} = 1V = V_{R67}$$

Where, $V_{R1} = I_T R_1 = 0.02A \times 100\Omega = 2.0V$

Thus, $V_{R6} = V_{R7} = V_{R67} = 1V$

$$V_{R34} = V_{R3} = V_{R4} = 1V \rightarrow I_3 = \frac{V_{R3}}{R_3} = \frac{1V}{100\Omega} = 0.01A = 10mA$$

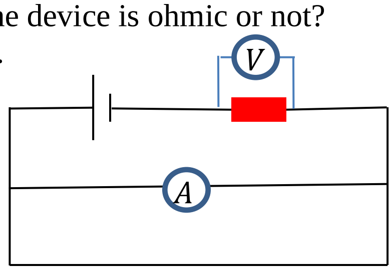
c. What is the drift velocity of the electrons through resistor R_1 if the wire in the resistor were made of nickel ($\rho_{Ni} = 8900\frac{kg}{m^3}$ and $m = 58.69\frac{g}{mol}$) with a circular cross section of diameter $d = 0.5mm$? Assume that nickel donates one free charge carrier per atom to the current.

$$n = \frac{\rho_{ni} N_A}{m} = \left(\frac{8900\frac{kg}{m^3} \times 6.02 \times 10^{23} \frac{Ni\ atoms}{mol}}{\frac{kg}{mol}} \right) \times \frac{1\ charge\ carrier}{atom} = 9.13 \times 10^{28}$$

$$I = neAv_d \rightarrow v_d = \frac{I}{neA} = \frac{0.02A}{9.13 \times 10^{28} \times 1.6 \times 10^{-19} C \times (\pi (0.25 \times 10^{-3} m)^2)} = 7 \times 10^{-6} \frac{m}{s}$$

d. Suppose that you wanted to test whether a device (shown in red in the circuits below) was ohmic or not. To do this, you connect the device to a battery and make measurements of the potential across the device as a function of the current through the device. Which of the following circuits will allow you to test whether the device is ohmic or not?

1.



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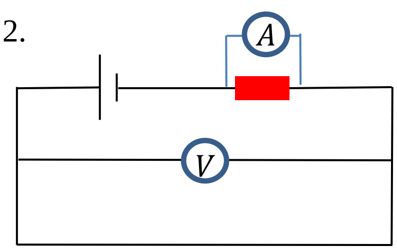

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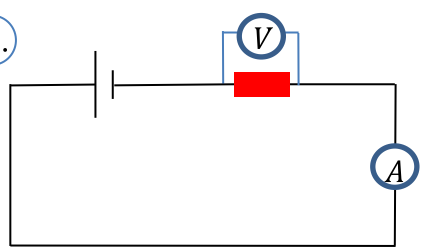

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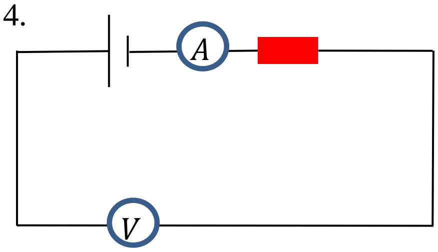


5. None of the circuits above will let me determine if the device is ohmic or not.

Physics 111 Formula Sheet

Electrostatics

$F = k\frac{q_1 q_2}{r^2}$

$\vec{F} = q\vec{E}; \quad E_{pc} = k\frac{q}{r^2}; \quad E_{plate} = \frac{q}{\epsilon_0 A}$

$E = -\frac{\Delta V}{\Delta x}$

$V = k\frac{q}{r}$

$U_e = k\frac{q_1 q_2}{r} = qV$

$W = -q\Delta V = -\Delta U_e = \Delta K$

Magnetism

$\vec{F} = q\vec{v} \times \vec{B} \rightarrow F = qvB\sin\theta$

$\vec{F} = I\vec{L} \times \vec{B} \rightarrow F = ILB\sin\theta$

$B = \frac{\mu_0 I}{2\pi r}$

$\varepsilon = \Delta V = -N\frac{\Delta\phi_B}{\Delta t}$

$\phi_B = BA\cos\theta$

Electric Circuits - Capacitors

$Q = CV; \quad C = \frac{\kappa\epsilon_0 A}{d}$

$C_{parallel} = \sum_{i=1}^{N} C_i$

$\frac{1}{C_{series}} = \sum_{i=1}^{N}\frac{1}{C_i}$

$Q_{charging}(t) = Q_{max}\left(1 - e^{-\frac{t}{\tau}}\right)$

$Q_{discharging}(t) = Q_{max}e^{-\frac{t}{\tau}}$

$I(t) = I_{max}e^{-\frac{t}{\tau}} = \frac{Q_{max}}{\tau}e^{-\frac{t}{\tau}}$

$\tau = RC$

$U_C = \frac{1}{2}qV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$

Electric Circuits - Resistors

$I = \frac{\Delta Q}{\Delta t}$

$I = neAv_d; \quad n = \frac{\rho N_A}{m}$

$V = IR$

$R = \frac{\rho L}{A}$

$R_{series} = \sum_{i=1}^{N} R_i$

$\frac{1}{R_{parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$

$P = \frac{\Delta E}{\Delta t} = IV = I^2 R = \frac{V^2}{R}$

Light as a Wave

$c = f\lambda$

$S(t) = \frac{\text{Energy}}{\text{time}\times\text{Area}} = c\epsilon_0 E^2(t) = c\frac{B^2(t)}{\mu_0}$

$I = S_{avg} = \frac{1}{2}c\epsilon_0 E_{max}^2 = c\frac{B_{max}^2}{2\mu_0}$

$P = \begin{cases} \frac{S}{c}; & \text{absorbed} \\ \frac{2S}{c}; & \text{reflected} \end{cases}$

$S = S_0\cos^2\ \theta$

$v = \frac{c}{n}$

$\theta_{\text{incident}} = \theta_{\text{reflected}}$

$n_1\sin\theta_1 = n_2\sin\theta_2$

$P = \frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i}$

$M = -\frac{d_i}{d_0}; \quad |M| = \frac{h_i}{h_0}$

Light as a Particle/Relativity

$E = hf = \frac{hc}{\lambda}$

$K_{max} = hf - \phi$

$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$

$\frac{1}{E'} = \frac{1}{E} + \frac{(1-\cos\phi)}{E_{rest}}; \quad E_{rest} = mc^2$

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$p = \gamma mv$

$E_{total} = E_{rest} + K = \gamma mc^2$

$K = (\gamma - 1)mc^2$

$E_{total}^2 = p^2c^2 + m^2c^4$

Nuclear Physics

$N = N_0 e^{-\lambda t}$

$m = m_0 e^{-\lambda t}$

$A = A_0 e^{-\lambda t}$

$A = \lambda N$

$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

Constants

$g = 9.8 \frac{m}{s^2}$

$1e = 1.6 \times 10^{-19} C$

$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$

$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$

$1eV = 1.6 \times 10^{-19} J$

$\mu_0 = 4\pi \times 10^{-7} \frac{Tm}{A}$

$c = 3 \times 10^8 \frac{m}{s}$

$h = 6.63 \times 10^{-34} J \cdot s = 4.14 \times 10^{-15} eV \cdot s$

$N_A = 6.02 \times 10^{23}$

$1u = 1.66 \times 10^{-27} kg = 931.5 \frac{MeV}{c^2}$

$m_p = 1.67 \times 10^{-27} kg = 937.1 \frac{MeV}{c^2}$

$m_n = 1.69 \times 10^{-27} kg = 948.3 \frac{MeV}{c^2}$

$m_e = 9.11 \times 10^{-31} kg = 0.511 \frac{MeV}{c^2}$

Physics 110 Formulas

$\vec{F} = m\vec{a}; \quad F_G = \frac{GM_1 m_2}{r^2}; \quad F_s = -ky; \quad a_c = \frac{v^2}{r}$

$W = -\Delta U_g - \Delta U_s = \Delta K$

$U_g = mgy$

$U_s = \frac{1}{2} k y^2$

$K = \frac{1}{2} m v^2$

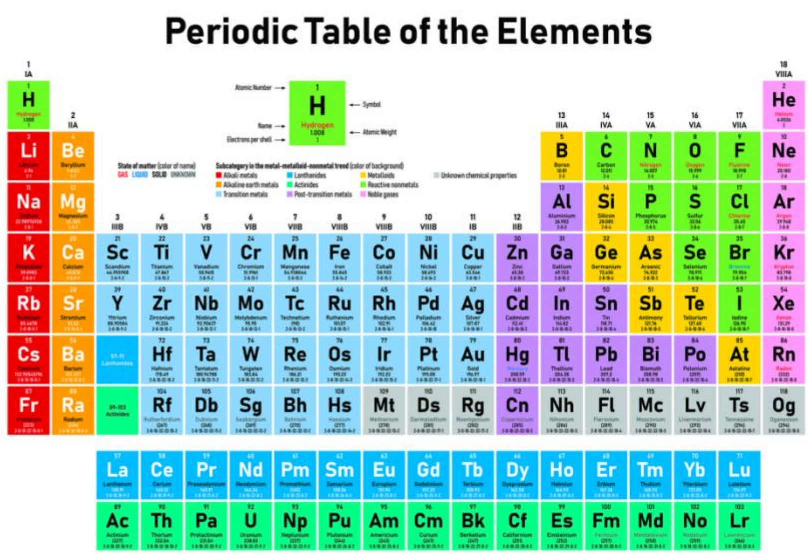
$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a} t^2$

$\vec{v}_f = \vec{v}_i + \vec{a} t$

$v_f^2 = v_i^2 + 2 a_r \Delta r$

Geometry/Algebra

Circles:	$A = \pi r^2$	$C = 2\pi r = \pi$
Spheres:	$A = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$
Triangles:	$A = \frac{1}{2} bh$	
Quadratics:	$ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	



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