Physics 111

Exam #1

September 30, 2022

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $\left|\vec{p}\right| \approx m\left|\vec{v}\right| = (5kg)\times\left(2\,\frac{m}{s}\right) = 10\,\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 6 points.

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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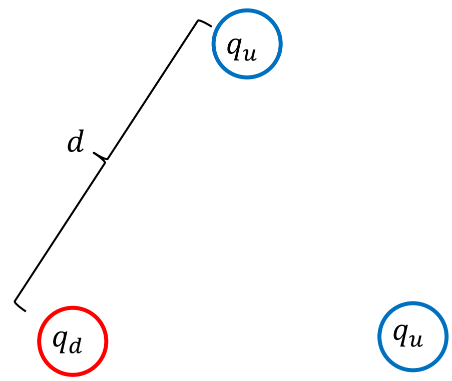
1. When we've done problem involving protons, we've assumed them to be point charges, meaning that they are a point in space with no extent. Protons are not points, but a bound state consisting of three quarks. Suppose that you have two "up" quarks with a charge of $q_u = +\frac{2}{3}e$ and one "down" quark with charge $q_d = -\frac{1}{3}e$ located on the vertices of an equilateral triangle. The distance between each charge is $d = 2.6 \times 10^{-15} m$ as shown on the right.



   a. What is the electrostatic force on the down quark due to the two up quarks? Assume that the quarks are point charges.

$$\vec{F}_{net,d} = \vec{F}_{d,u} + \vec{F}_{d,u}$$

$$F_{net,d,x} = \frac{kq_u q_d}{d^2}\cos\theta + \frac{kq_u q_d}{d^2} = \frac{k\left(\frac{2}{3}e\right)\left(\frac{1}{3}e\right)}{d^2}[\cos\theta + 1] = \frac{2ke^2}{9d^2}[1 + \cos 60]$$

$$F_{net,d,x} = 1.5 \times \frac{2 \times 9 \times 10^9 \frac{Nm^2}{C^2} \times \left(1.6 \times 10^{-19} C\right)^2}{9 \times \left(2.6 \times 10^{-15} m\right)^2} = 11.4N$$

$$F_{net,d,y} = \frac{kq_u q_d}{d^2}\sin\theta = \frac{2ke^2}{9d^2}\sin 60$$

$$= 0.866 \times \frac{2 \times 9 \times 10^9 \frac{Nm^2}{C^2} \times (1.6 \times 10^{-19} C)^2}{9 \times (2.6 \times 10^{-15} m)^2}$$

$$F_{net,d,y} = 6.6N$$

$$F_{net,d} = \sqrt{F_{net,d,x}^2 + F_{net,d,y}^2} = \sqrt{(11.4N)^2 + (6.6N)^2} = 13.2N$$

$$\phi = \tan^{-1}\frac{F_{net,d,y}}{F_{net,d,x}} = \tan^{-1}\frac{6.6N}{11.4N} = 30^0$$

b. What is the net electric field at the down quark's location due to the two up quarks?

$$\vec{F}_{net,d} = q\vec{E}_{net,d} \rightarrow \left|\vec{E}_{net,d}\right| = \left|\frac{\vec{F}_{net,d}}{q_d}\right| = \frac{13.2N}{\frac{1}{3}\times 1.6\times 10^{-19}C} = 2.5\times 10^{20}\frac{N}{C}$$

The direction of the electric field is opposite to the direction of the net force since the down quark has a negative charge. $\phi = 30^0$ below the negative x-axis or 210^0 from the positive x-axis.

c. What is the value of the electric potential (voltage) at the down quark's location due the two up quarks?

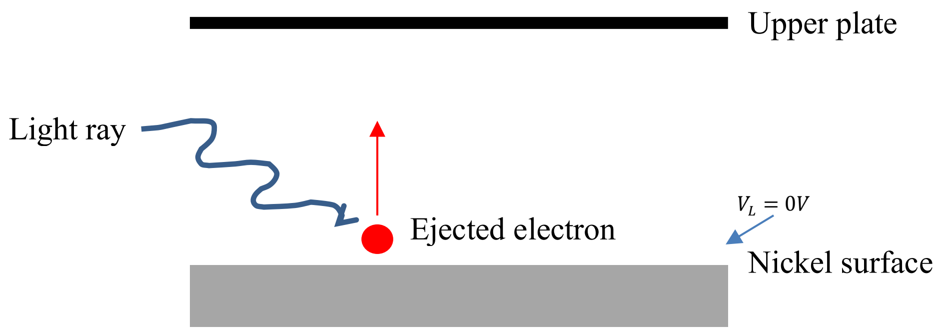
$$V_d = V_{u,d} + V_{u,d} = \frac{kq_u}{d} + \frac{kq_u}{d} = \frac{2kq_u}{d} = \frac{4ke}{3d} = \frac{4\times 9\times 10^9\frac{Nm^2}{C^2}\times 1.6\times 10^{-19}C}{3\times 2.6\times 10^{-15}m}$$

$$V_d = 7.4\times 10^5 V$$

d. How much work would it take to bring the down quark in from very far away and place it at its current location? Assume that the two up quarks are already at their locations when you bring the down quark in.

$$W = -q\Delta V = -\left(-\frac{1}{3}e\right)\Delta V = \frac{1}{3}\times 1.6\times 10^{-19}C\times 7.4\times 10^5 V = 3.94\times 10^{-14}J$$

2. The photoelectric effect (which we'll talk more about in class later in the term) is a phenomenon in which light incident on a metal surface can eject valence electrons from atoms in the metal surface. To measure the speed of the ejected electrons we use the setup shown below where a charged plate is placed above a metal surface, taken here to be nickel, at a potential of $V_L = 0V$. The ejected electron heads toward the charged upper plate, slows down, and if the potential on the upper plate is just right, comes to rest and then returns to the metal surface.



a. Explain what the sign of the charge on the upper plate is, what the direction of the electric field between the nickel metal surface and the upper plate is, and the potential on the upper plate compared to the lower plate. Be sure to address why your answers are as you state them to earn full credit.

   1. When the electron is ejected that leaves the metal with a net positive e charge. To repel the electron from the upper plate, the upper plate must have a negative charge on it.

   2. Since the nickel metal surface has a net positive charge on it and the upper plate a net negative charge on it, the electric field points from the nickel metal surface (the positive plate) to the upper plate (the negative plate).

   3. Since the electric field points along decreasing electric potentials, the upper plate must be at a lower electric potential compared to the lower plate (the nickel metal surface). Thus, the electric potential must be a negative value compared to the potential on the lower plate.

b. Suppose that the upper plate has a potential of $|V_u| = 7.28V$ on it. What is the speed of the fastest electron ejected from the nickel surface that can be repelled just before striking the upper plate?

$$W = -q\Delta V = -q[V_{upper} - V_{lower}] = \Delta K = \tfrac{1}{2}mv_{fy}^2 - \tfrac{1}{2}mv_{iy}^2 = -\tfrac{1}{2}mv_{iy}^2$$

$$v_{iy} = \sqrt{\frac{2e[V_{upper} - V_{lower}]}{m}} = \sqrt{\frac{2 \times (-1.6 \times 10^{-19}C) \times [-7.28V - 0V]}{9.11 \times 10^{-31}kg}}$$

$$v_{iy} = 1.6 \times 10^6 \tfrac{m}{s}$$

c. Suppose that the nickel metal surface and the upper charged plate are separated by a distance $d = 10mm$. How much charge in magnitude was placed on the upper plate? Assume the upper plate is square with sides of length $L = 20cm$.

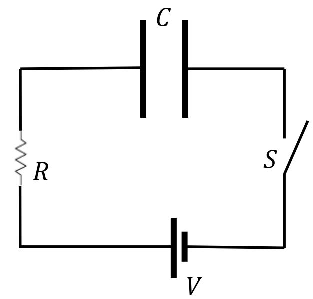
$$E = \frac{Q}{\varepsilon_0 A} = -\frac{\Delta V}{\Delta y} \rightarrow Q = -\varepsilon_0 A \frac{\Delta V}{\Delta y}$$

$$Q = -8.85 \times 10^{-12} \tfrac{C^2}{Nm^2} \times (0.2m)^2 \times \frac{[-7.28V - 0V]}{10 \times 10^{-3}m} = 2.6 \times 10^{-10}C = 0.26nC$$

d. What is the change in electric potential energy for the ejected electron between the nickel metal surface and when it comes momentarily to rest at the upper plate, in eV?

$$W = -q\Delta V = -\Delta U_e \rightarrow \Delta U_e = q\Delta V = -e \times [-7.28V - 0V] = +7.28eV$$

3. A resistor R, uncharged capacitor C, and a battery V are connected to an open switch S as shown below.



   a. When the switch S is closed the capacitor begins to charge through the resistor R. The resistance of the resistor is found to be $R = 100k\Omega$ and you desire the time constant of the circuit to be $\tau = 28s$. What is the capacitance of the capacitor C that should be wired in the circuit?

$$\tau = RC \to c = \frac{\tau}{R} = \frac{28s}{100000\Omega} = 2.8 \times 10^{-4}F = 280\mu F$$

   b. Suppose that the resistor was made of tungsten $\left({}^{181}_{73}W, \rho_W = 19280\frac{kg}{m^3}, \rho = 5.6 \times 10^{-8}\Omega m, m_W = 181\frac{g}{mol}\right)$ wire with a diameter of $4mm$. At the instant the voltage across the capacitor was $9V$, what was the drift velocity of the charge carriers in the tungsten wire? Assume that the resistor and capacitor are connected to a $30V$ battery and that tungsten donates two charge carriers per atom.

$$V_R = V - V_C = IR \to I = \frac{V - V_C}{R} = \frac{30V - 9V}{100000\Omega} = 2.1 \times 10^{-4}A$$

$$I = neAv_d \to v_d = \frac{I}{neA} = \frac{2.1 \times 10^{-4}A}{1.28 \times 10^{29}m^{-3} \times 1.6 \times 10^{-19}C \times \pi(2 \times 10^{-3}m)^2}$$

$$\to v_d = 8.2 \times 10^{-10}\frac{m}{s} = 0.82\frac{nm}{s}$$

where, $n = \left[\frac{\rho_W}{m_W}N_A\right] \times 2 = 2 \times \frac{19280\frac{kg}{m^3}}{0.181\frac{kg}{mol}} \times 6.02 \times 10^{23} = 1.28 \times 10^{29}m^{-3}$

c. Suppose the capacitor was constructed out of two parallel circular metal plates filled with a glass as a dielectric ($\kappa = 4$) separated by a distance $d = 2\mu m$. What was the radius of the circular capacitor plates?

$$C = \frac{\kappa\varepsilon_0 A}{d} = \frac{\kappa\varepsilon_0 \pi r^2}{d} \rightarrow r = \sqrt{\frac{Cd}{\kappa\pi\varepsilon_0}} = \sqrt{\frac{2.8 \times 10^{-4} F \times 2 \times 10^{-6} m}{4\pi \times 8.85 \times 10^{-12} \frac{C^2}{Nm^2}}} = 2.2m$$

d. When the capacitor is fully charged the battery is removed. The capacitor is then connected to the resistor and allowed to discharge. At what time will the charge remaining on the capacitor be 15% of the initial charge?

$$Q(t) = Q_{max} e^{-\frac{t}{RC}} = 0.15 Q_{max} \rightarrow t = -RC \ln \frac{Q(t)}{Q_{max}} = -28s \ln 0.15 = 53.1s$$

Physics 111 Formula Sheet

Electrostatics

$F = k\frac{q_1 q_2}{r^2}$

$\vec{F} = q\vec{E}; \quad E_{pc} = k\frac{q}{r^2}; \quad E_{plate} = \frac{q}{\epsilon_0 A}$

$E = -\frac{\Delta V}{\Delta x}$

$V = k\frac{q}{r}$

$U_e = k\frac{q_1 q_2}{r} = qV$

$W = -q\Delta V = -\Delta U_e = \Delta K$

Magnetism

$\vec{F} = q\vec{v} \times \vec{B} \rightarrow F = qvB\sin\theta$

$\vec{F} = I\vec{L} \times \vec{B} \rightarrow F = ILB\sin\theta$

$V_{Hall} = wv_d B$

$B = \frac{\mu_0 I}{2\pi r}$

$\varepsilon = \Delta V = -N\frac{\Delta\phi_B}{\Delta t}$

$\phi_B = BA\cos\theta$

Electric Circuits - Capacitors

$Q = CV; \quad C = \frac{\kappa\epsilon_0 A}{d}$

$C_{parallel} = \sum_{i=1}^{N} C_i$

$\frac{1}{C_{series}} = \sum_{i=1}^{N}\frac{1}{C_i}$

$Q_{charging}(t) = Q_{max}\left(1 - e^{-\frac{t}{\tau}}\right)$

$Q_{discharging}(t) = Q_{max}e^{-\frac{t}{\tau}}$

$I(t) = I_{max}e^{-\frac{t}{\tau}} = \frac{Q_{max}}{\tau}e^{-\frac{t}{\tau}}$

$\tau = RC$

$U_C = \frac{1}{2}qV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$

Electric Circuits - Resistors

$I = \frac{\Delta Q}{\Delta t}$

$I = neAv_d; \quad n = \frac{\rho N_A}{m}$

$V = IR$

$R = \frac{\rho L}{A}$

$R_{series} = \sum_{i=1}^{N} R_i$

$\frac{1}{R_{parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$

$P = \frac{\Delta E}{\Delta t} = IV = I^2 R = \frac{V^2}{R}$

Light as a Wave

$c = f\lambda$

$S(t) = \frac{\text{Energy}}{\text{time}\times\text{Area}} = c\epsilon_0 E^2(t) = c\frac{B^2(t)}{\mu_0}$

$I = S_{avg} = \frac{1}{2}c\epsilon_0 E_{max}^2 = c\frac{B_{max}^2}{2\mu_0}$

$P = \begin{cases} \frac{S}{c}; & \text{absorbed} \\ \frac{2S}{c}; & \text{reflected} \end{cases}$

$S = S_0\cos^2\theta$

$v = \frac{c}{n}$

$\theta_{\text{incident}} = \theta_{\text{reflected}}$

$n_1\sin\theta_1 = n_2\sin\theta_2$

$P = \frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i}$

$M = -\frac{d_i}{d_0}; \quad |M| = \frac{h_i}{h_0}$

Light as a Particle/Relativity

$E = hf = \frac{hc}{\lambda}$

$K_{max} = hf - \phi$

$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$

$\frac{1}{E'} = \frac{1}{E} + \frac{(1-\cos\phi)}{E_{rest}}; \quad E_{rest} = mc^2$

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$p = \gamma mv$

$E_{total} = E_{rest} + K = \gamma mc^2$

$K = (\gamma - 1)mc^2$

$E_{total}^2 = p^2c^2 + m^2c^4$

Nuclear Physics

$N = N_0 e^{-\lambda t}$

$m = m_0 e^{-\lambda t}$

$A = A_0 e^{-\lambda t}$

$A = \lambda N$

$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

Constants

$g = 9.8 \frac{m}{s^2}$

$1e = 1.6 \times 10^{-19} C$

$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$

$\epsilon_0 = 8.85 \times 10^{-12} \frac{c^2}{Nm^2}$

$1eV = 1.6 \times 10^{-19} J$

$\mu_0 = 4\pi \times 10^{-7} \frac{Tm}{A}$

$c = 3 \times 10^8 \frac{m}{s}$

$h = 6.63 \times 10^{-34} Js = 4.14 \times 10^{-15} eVs$

$N_A = 6.02 \times 10^{23}$

$1u = 1.66 \times 10^{-27} kg = 931.5 \frac{MeV}{c^2}$

$m_p = 1.67 \times 10^{-27} kg = 937.1 \frac{MeV}{c^2}$

$m_n = 1.69 \times 10^{-27} kg = 948.3 \frac{MeV}{c^2}$

$m_e = 9.11 \times 10^{-31} kg = 0.511 \frac{MeV}{c^2}$

Physics 110 Formulas

$\vec{F} = m\vec{a}; \quad F_G = \frac{GM_1 m_2}{r^2}; \quad F_s = -ky; \quad a_c = \frac{v^2}{r}$

$W = -\Delta U_g - \Delta U_s = \Delta K$

$U_g = mgy$

$U_s = \frac{1}{2} k y^2$

$K = \frac{1}{2} m v^2$

$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2} \vec{a} t^2$

$\vec{v}_f = \vec{v}_i + \vec{a} t$

$v_f^2 = v_i^2 + 2 a_r \Delta r$

Common Metric Units

$nano\ (n) = 10^{-9}$

$micro\ (\mu) = 10^{-6}$

$milli\ (m) = 10^{-3}$

$centi\ (c) = 10^{-2}$

$kilo\ (k) = 10^{3}$

$mega\ (M) = 10^{6}$

Geometry/Algebra

Circles:	$A = \pi r^2$	$C = 2\pi r = \pi$
Spheres:	$A = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$
Triangles:	$A = \frac{1}{2} bh$	
Quadratics:	$ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$