Physics 111

Exam #1

September 22, 2023

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}| = (5kg)\times\left(2\,\frac{m}{s}\right) = 10\,\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 6 points.

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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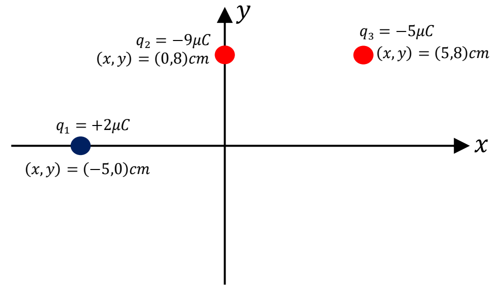
1. Three point-charges are assembled as shown below where each charge is brought in from very far away.

   a. How much work did it take to assemble this collection of point charges?



$$W_1 = 0J$$

$$W_2 = -q_2\Delta V_1 = -q_2\left[\frac{kq_1}{r_{12}} - 0\right] = -(-9\times10^{-6}C)\left[\frac{9\times10^9\frac{Nm^2}{C^2}\times 2\times10^6 C}{\sqrt{(0.05m)^2+(0.08m)^2}}\right] = 1.72J$$

$$W_3 = -q_3\Delta V_1 - q_3\Delta V_2 = -q_3\left[\frac{kq_1}{r_{13}} - 0\right] - q_3\left[\frac{kq_2}{r_{23}} - 0\right]$$

$$W_3 = -(-5\times10^{-6}C)\left[\frac{9\times10^9\frac{Nm^2}{C^2}\times 2\times10^6 C}{\sqrt{(0.1m)^2+(0.08m)^2}}\right] - (-5\times10^{-6}C)\left[\frac{9\times10^9\frac{Nm^2}{C^2}\times(-9\times10^6 C)}{0.05m}\right]$$

$$W_3 = 0.74J$$

$$W_{net} = W_1 + W_2 + W_3 = 0J + 1.72J + 0.74J = 2.46J$$

   b. At a point $P = (x,y) = (0,0)cm$, what is the net electric field in magnitude and direction?

$$E_{net\ P,x} = E_1 + E_{3x} = E_1 + E_3\cos\theta = \frac{kq_2}{r_{2P}^2} + \frac{kq_3}{r_{3P}^2}\cos\theta$$

$$E_{net\ P,x} = 9\times10^9\frac{Nm^2}{C^2}\left[\frac{2\times10^{-6}C}{(0.05m)^2} + \frac{5\times10^{-6}C}{(0.094m)^2}\left(\frac{0.05m}{0.094m}\right)\right] = 9.91\times10^6\frac{N}{C}$$

$$E_{net\ P,y} = E_2 + E_{3y} = E_2 + E_3\sin\theta = \frac{kq_2}{r_{2P}^2} + \frac{kq_3}{r_{3P}^2}\sin\theta$$

$$E_{net\ P,y} = 9\times10^9\frac{Nm^2}{C^2}\left[\frac{9\times10^{-6}C}{(0.08m)^2} + \frac{5\times10^{-6}C}{(0.094m)^2}\left(\frac{0.08m}{0.094m}\right)\right] = 1.70\times10^7\frac{N}{C}$$

$$E_{net,P} = \sqrt{E_{net\ P,x}^2 + E_{net\ P,y}^2} = \sqrt{\left(9.91\times10^6\frac{N}{C}\right)^2 + \left(1.70\times10^7\frac{N}{C}\right)^2}$$

$$E_{net,P} = 1.97\times10^7\frac{N}{C}$$

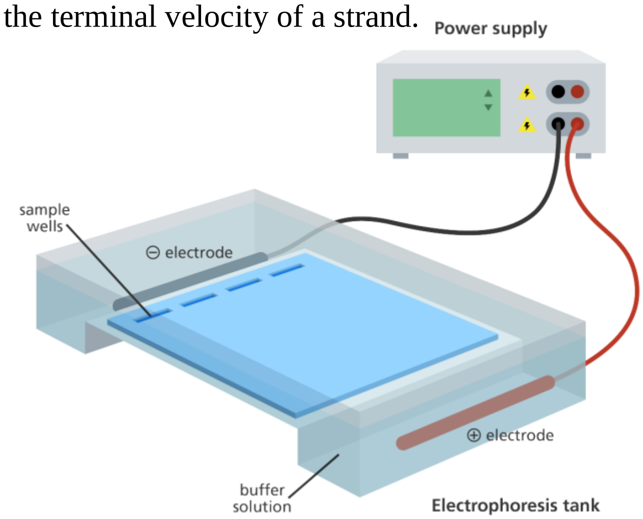
$$\tan\phi = \frac{E_{net\ P,y}}{E_{net\ P,x}} = \frac{1.70\times10^7\frac{N}{C}}{9.91\times10^6\frac{N}{C}} = 1.715 \rightarrow \phi = 59.8^0$$

c. Suppose that a point-charge $q_4 = -3\mu C$ was placed at point P, what net force, in magnitude and direction, would q_4 feel?

The magnitude of the force:
$F_{q_4} = q_4 E_{net} = 3 \times 10^{-6} C \times 1.7 \times 10^7 \frac{N}{C} = 51N$

The direction of the force is opposite to the direction of the electric field. $\phi = 59.8^0 + 180^0 = 239.8^0$ from the positive x-axis.

d. We've said that if you put a charge q into an electric field $\vec{E}$, it will feel a force $\vec{F} = q\vec{E}$. This is a useful technique that can be exploited in something called gel electrophoresis. The main idea of gel electrophoresis is to separate masses of DNA, RNA or protein chains by molecular (or chain) weight based on how far the chain moves through the gel in the presence of an external electric field. The basic setup of gel electrophoresis is shown below. As the segemnt of say DNA moves through the gel due to its charge (assumed for this problem to be $q = -e$ for all chains) it is subject to a drag force $F_d = Cv$, opposite to its velocity v, where C is a constant called the drag coefficient that depends on mass. Suppose that you have a solution of various masses (chain lengths) and the solution of various DNA chains move through the gel at a constant velocity, called the terminal velocity. The DNA sample is inserted into the left most end and the samples migrate toward the right end as they interact with the applied electric field. This interaction separates out the masses into bands that can be used (with a standard) to determine molecular chain weights. Using the figure below, explain the direction of the electric field in the gel, the motion of the DNA strands through the gel, and where you would expect short, medium, and long chain DNA strands end up (in relation to the sample well) and why? Be sure to explain your answer using as many physics ideas as possible and determine an expression for the terminal velocity of a strand.



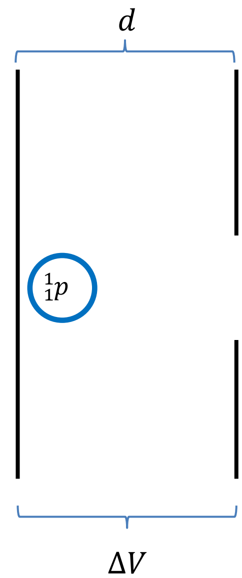
https://www.yourgenome.org/facts/what-is-gel-electrophoresis/

The electric field points from the positive electrode to the negative electrode and since all the DNA chains are negatively charged, they feel a force toward the positive electrode. From the forces we can determine the terminal velocity:

$F_e - F_d = ma_x = 0 \rightarrow qE = Cv \rightarrow v = \frac{d}{t} = \frac{eE}{C} \rightarrow d = \frac{eEt}{C}$.

Since the drag coefficient depends on mass, the larger masses travel a smaller distance through the gel and the lighter masses travel further in the gel. The larger the mass the closer to the negative electrode and the smaller the mass the closer to the positive electrode. To determine masses, we need to run a set of standards.

2. Ion beams generated by particle accelerators are routinely used in materials analysis, where the composition of an unknown material needs to be determined. Suppose that you have the accelerator shown below in which a proton is accelerated from rest near the left plate and that the proton exits through the hole in the right plate. A potential difference ΔVexists across the plates and the plates are separated by a distance $d = 2m$.



a. If the speed of the proton through the hole on the right plate is $v = 2.1 \times 10^7 \frac{m}{s}$, what was ΔV?

$$W = -q\Delta V = \Delta K = \frac{1}{2}mv_f^2 \rightarrow \Delta V = \frac{mv_f^2}{2q} = \frac{1.67 \times 10^{-27}kg\left(2.1 \times 10^7 \frac{m}{s}\right)^2}{2 \times 1.6 \times 10^{-19}C}$$

$$\Delta V = -2.3 \times 10^6 V = -2.3MV$$

b. After the proton is accelerated through the potential difference in part a, it is directed at a target composed of a single element with atomic number Z. The proton comes to rest at a distance $r = 4.63 \times 10^{-14}m$ from the nucleus of an atom of the unknown element. What was the element that the target was made from?

$$W = -q\Delta V = -q\left[\frac{kQ}{r_f} - \frac{kQ}{r_i}\right] = \Delta K = 0 - \frac{1}{2}mv_i^2$$

$$\frac{kZe^2}{r_f} = \frac{1}{2}mv_i^2 \rightarrow Z = \frac{mr_f v_i^2}{2ke^2}$$

$$Z = \frac{1.67 \times 10^{-27}kg \times 4.63 \times 10^{-14}m \times \left(2.1 \times 10^7 \frac{m}{s}\right)^2}{2 \times 9 \times 10^9 \frac{Nm^2}{C^2} \times (1.6 \times 10^{-19}C)^2} = 74 \rightarrow Z = W$$

c. The accelerator is constructed out of two parallel circular metal plates of diameter $20cm$. What is the capacitance of the system and how much charge was on a plate of this capacitor?

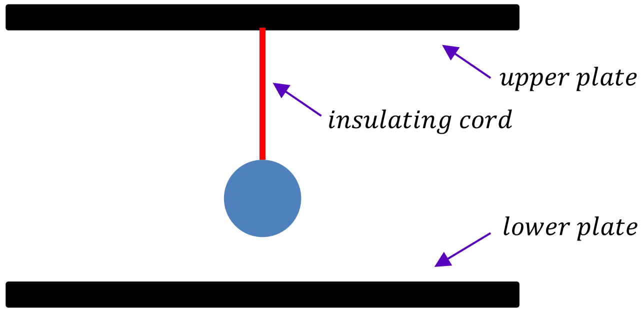
$$C = \frac{\kappa\varepsilon_0 A}{d} = \frac{1 \times 8.85 \times 10^{-12}\frac{C^2}{Nm^2} \times \pi(0.1m)^2}{2m} = 1.39 \times 10^{-13}F$$
$$Q = CV = 1.39 \times 10^{-13}F \times 2.3 \times 10^6 V = 3.2 \times 10^{-7}C$$

d. Suppose the proton was accelerated through the accelerator from left to right, where the right most circular plate has a small hole so the protons can escape. What is the magnitude and direction of the electric field that accelerated the protons between the plates?

In magnitude the electric field is:
$$E = \frac{Q}{\varepsilon_0 A} = \frac{3.2 \times 10^{-7}C}{8.85 \times 10^{-12}\frac{C^2}{Nm^2} \times \pi(0.1m)^2} = 1.15 \times 10^6 \frac{N}{C}$$
The direction of the electric field is from left to right to accelerate the proton to the right. Since the proton has a positive charge, it must accelerate in the direction of the electric field.

The magnitude and direction of the electric field can also be calculated from
$$E = -\frac{\Delta V}{\Delta x} = -\frac{(-2.3 \times 10^6 V)}{2m} = +1.15 \times 10^6 \frac{V}{m}$$

3. A point-charge with mass $m = 400g$ and charge $q = -0.5\mu C$ is suspended at the end of a light insulating cord as shown below. The point-charge is suspended between two metal plates of equal and opposite charge. The tension in the insulating cord is measured to be $8.6N$.



a. What is the magnitude of the assumed constant electric field between the plates?

Assuming the electric force points up we have

$$F_e + F_T - F_w = ma_y = 0 \to F_e = F_w - F_T = mg - F_T$$

$$F_e = 0.4kg \times 9.8\frac{m}{s^2} - 8.6N = -4.68N$$

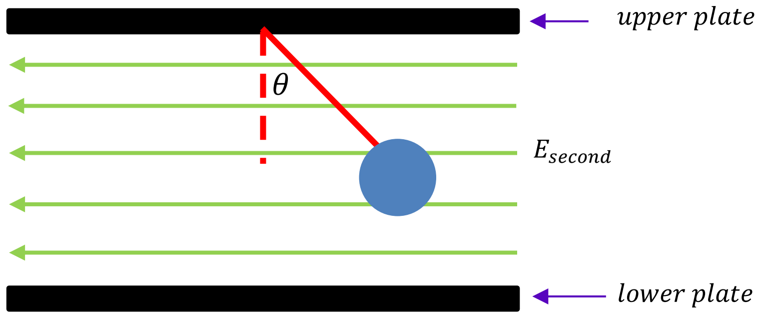
$$F_e = qE \to E = \frac{F_e}{q} = \frac{-4.68N}{-0.5 \times 10^{-6}C} = 9.4 \times 10^6\frac{N}{C}$$

The magnitude of the electric field $E = 9.4 \times 10^6\frac{N}{C}$

b. Explain the direction of the electric field needed in part a to achieve this situation.

The tension force is larger than the weight, so we need another force (F_e) in the negative y-direction. And, since the charge is negative and a negative charge experiences a force opposite to the direction of the electric field, the electric field must point in the positive y-direction, as shown in part a. Since the electric field points in the positive y-direction and points from positive charges to negative charges, the lower plate must be positive and the upper plate negative.

c. Suppose that in addition to the electric field in part a, a second electric field (E_{second}) were applied the system as shown below. This second electric field makes the point-charge rise through an angle of $\theta = 28^0$ measured with respect to the vertical. What is the tension in the rope in this situation?



x direction:

$$F_{E_{second}} - F_T \sin\theta = ma_x = 0$$

y direction:

$$F_T \cos\theta - F_W - F_e = ma_y = 0 \rightarrow F_T = \frac{F_e + F_W}{\cos\theta} = \frac{4.68N + 0.4kg \times 9.8\frac{m}{s^2}}{\cos 28} = 9.7N$$

d. What is the magnitude of the second electric field, E_{second}?

$$F_{E_{second}} - F_T \sin\theta = ma_x = 0 \rightarrow F_{E_{second}} = F_T \sin\theta = 9.7N \sin 28 = 4.6N$$

$$F_{E_{second}} = qE_{second} \rightarrow E_{second} = \frac{F_{E_{second}}}{q} = \frac{4.6N}{0.5 \times 10^{-6}C} = 9.2 \times 10^6 \frac{N}{C}$$

Physics 111 Formula Sheet

Electrostatics

$F = k\frac{q_1 q_2}{r^2}$

$\vec{F} = q\vec{E}; \quad E_{pc} = k\frac{q}{r^2}; \quad E_{plate} = \frac{q}{\epsilon_0 A}$

$E = -\frac{\Delta V}{\Delta x}$

$V_{pc} = k\frac{q}{r}$

$U_e = k\frac{q_1 q_2}{r} = qV$

$W = -q\Delta V = -\Delta U_e = \Delta K$

Magnetism

$\vec{F} = q\vec{v} \times \vec{B} \rightarrow F = qvB\sin\theta$

$\vec{F} = I\vec{L} \times \vec{B} \rightarrow F = ILB\sin\theta$

$V_{Hall} = wv_d B$

$B = \frac{\mu_0 I}{2\pi r}$

$\varepsilon = \Delta V = -N\frac{\Delta\phi_B}{\Delta t}$

$\phi_B = BA\cos\theta$

Electric Circuits - Capacitors

$Q = CV; \quad C = \frac{\kappa\epsilon_0 A}{d}$

$C_{parallel} = \sum_{i=1}^{N} C_i$

$\frac{1}{C_{series}} = \sum_{i=1}^{N}\frac{1}{C_i}$

$Q_{charging}(t) = Q_{max}\left(1 - e^{-\frac{t}{\tau}}\right)$

$Q_{discharging}(t) = Q_{max}e^{-\frac{t}{\tau}}$

$I(t) = I_{max}e^{-\frac{t}{\tau}} = \frac{Q_{max}}{\tau}e^{-\frac{t}{\tau}}$

$\tau = RC$

$U_C = \frac{1}{2}qV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$

Electric Circuits - Resistors

$I = \frac{\Delta Q}{\Delta t}$

$I = neAv_d; \quad n = \frac{\rho N_A}{m}$

$V = IR$

$R = \frac{\rho L}{A}$

$R_{series} = \sum_{i=1}^{N} R_i$

$\frac{1}{R_{parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$

$P = \frac{\Delta E}{\Delta t} = IV = I^2 R = \frac{V^2}{R}$

Light as a Wave

$c = f\lambda$

$S(t) = \frac{\text{Energy}}{\text{time}\times\text{Area}} = c\epsilon_0 E^2(t) = c\frac{B^2(t)}{\mu_0}$

$I = S_{avg} = \frac{1}{2}c\epsilon_0 E_{max}^2 = c\frac{B_{max}^2}{2\mu_0}$

$P = \begin{cases} \frac{S}{c}; & \text{absorbed} \\ \frac{2S}{c}; & \text{reflected} \end{cases}$

$S = S_0\cos^2\theta$

$v = \frac{c}{n}$

$\theta_{\text{incident}} = \theta_{\text{reflected}}$

$n_1\sin\theta_1 = n_2\sin\theta_2$

$P = \frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i}$

$M = -\frac{d_i}{d_0}; \quad |M| = \frac{h_i}{h_0}$

Light as a Particle/Relativity

$E = hf = \frac{hc}{\lambda}$

$K_{max} = hf - \phi$

$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$

$\frac{1}{E'} = \frac{1}{E} + \frac{(1-\cos\phi)}{E_{rest}}; \quad E_{rest} = mc^2$

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$p = \gamma mv$

$E_{total} = E_{rest} + K = \gamma mc^2$

$K = (\gamma - 1)mc^2$

$E_{total}^2 = p^2c^2 + m^2c^4$

Nuclear Physics

$N = N_0 e^{-\lambda t}$

$m = m_0 e^{-\lambda t}$

$A = A_0 e^{-\lambda t}$

$A = \lambda N$

$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

Constants

$g = 9.8\frac{m}{s^2}$

$1e = 1.6 \times 10^{-19} C$

$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$

$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$

$1eV = 1.6 \times 10^{-19} J$

$\mu_0 = 4\pi \times 10^{-7} \frac{Tm}{A}$

$c = 3 \times 10^8 \frac{m}{s}$

$h = 6.63 \times 10^{-34} Js = 4.14 \times 10^{-15} eVs$

$N_A = 6.02 \times 10^{23}$

$1u = 1.66 \times 10^{-27} kg = 931.5 \frac{MeV}{c^2}$

$m_p = 1.67 \times 10^{-27} kg = 937.1 \frac{MeV}{c^2}$

$m_n = 1.69 \times 10^{-27} kg = 948.3 \frac{MeV}{c^2}$

$m_e = 9.11 \times 10^{-31} kg = 0.511 \frac{MeV}{c^2}$

Physics 110 Formulas

$\vec{F} = m\vec{a}; \quad F_G = \frac{GM_1 m_2}{r^2}; \quad F_s = -ky; \quad a_c = \frac{v^2}{r}$

$W = -\Delta U_g - \Delta U_s = \Delta K$

$U_g = mgy$

$U_s = \frac{1}{2} k y^2$

$K = \frac{1}{2} m v^2$

$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a} t^2$

$\vec{v}_f = \vec{v}_i + \vec{a} t$

$v_f^2 = v_i^2 + 2a_r \Delta r$

Common Metric Units

$nano\ (n) = 10^{-9}$

$micro\ (\mu) = 10^{-6}$

$milli\ (m) = 10^{-3}$

$centi\ (c) = 10^{-2}$

$kilo\ (k) = 10^{3}$

$mega\ (M) = 10^{6}$

Geometry/Algebra

Circles: $A = \pi r^2$ $\quad C = 2\pi r = \pi$

Spheres: $A = 4\pi r^2$ $\quad V = \frac{4}{3}\pi r^3$

Triangles: $A = \frac{1}{2} bh$

Quadratics: $ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$