# Physics 111 

Exam \#1

January 24, 2014

Name $\qquad$

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}|=(5 \mathrm{~kg}) \times\left(2 \frac{\mathrm{~m}}{\mathrm{~s}}\right)=10 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 3 points and each free-response part is worth 9 points

| Problem \#1 | $/ 24$ |
| :---: | :---: |
| Problem \#2 | $/ 27$ |
| Problem \#3 | $/ 21$ |
| Total | $/ 72$ |

I affirm that I have carried out my academic endeavors with full academic honesty.

1. Suppose that you are given the system of point charges, where $q_{1}=+2 \mu C$ is located at the point $(x, y)=(0,0.5 m)$, while $q_{2}=-6 \mu C$ is located at the point $(x, y)=(0,-0.2 m)$.
a. What is the electric field at a point $P$ with coordinates $(x, y)=(0.7 m,-0.4 m)$ ?

The components of the net electric field vector are given as

$$
\begin{aligned}
& E_{n e t, x}=E_{1, x}-E_{2, x}=\frac{k q_{1}}{r_{1, P}^{2}} \cos \theta_{1}-\frac{k q_{2}}{r_{2, P}^{2}} \cos \theta_{2} \\
& E_{\text {net }, y}=-E_{1, y}+E_{2, y}=-\frac{k q_{1}}{r_{1, P}^{2}} \sin \theta_{1}+\frac{k q_{2}}{r_{2, P}^{2}} \sin \theta_{2}
\end{aligned}
$$

From the geometry of the system we can determine the distances between each charge and point P along with the values of each of the trig functions.

$$
\begin{aligned}
& \cos \theta_{1}=\frac{0.70}{1.14}=0.61 \quad \cos \theta_{2}=\frac{0.70}{0.72}=0.97 \\
& \sin \theta_{1}=\frac{0.90}{1.14}=0.79 \quad \sin \theta_{2}=\frac{0.20}{0.72}=0.28 \\
& r_{1, P}=\sqrt{(0.7 m)^{2}+(0.9 m)^{2}}=1.14 m \\
& r_{2, P}=\sqrt{(0.7 m)^{2}+(0.2 m)^{2}}=0.72 m
\end{aligned}
$$

Inserting the quantities into the expressions for the net horizontal and vertical components of the electric field we find

$$
\begin{aligned}
& E_{\text {net }, x}=9 \times 10^{9} \frac{\mathrm{Nm}}{\mathrm{C}^{2}}
\end{aligned}\left[\frac{2 \times 10^{-6} \mathrm{C}}{(1.14 \mathrm{~m})^{2}}(0.61)-\frac{6 \times 10^{-6} \mathrm{C}}{(0.72 \mathrm{~m})^{2}}(0.97)\right]=-9.24 \times 10^{4} \frac{\mathrm{~N}}{\mathrm{C}}
$$

Therefore the net electric field at point P is given as
$E_{\text {net }, P}=\sqrt{E_{\text {net }, x}^{2}+E_{\text {net }, y}^{2}} @ \phi=\tan ^{-1}\left(\frac{E_{\text {net }, y}}{E_{\text {net }, x}}\right) \rightarrow E_{\text {net }, P}=9.41 \times 10^{4} \frac{\mathrm{~N}}{\mathrm{C}} @ \phi=78.8^{\circ}$ above the negative x -axis.
b. How much work is required to bring in a third charge $q_{3}=-5 \mu \mathrm{C}$ and place it at point $P$ ?

$$
\begin{aligned}
& W=-q_{3} \Delta V_{P, 2}-q_{3} \Delta V_{P, 1}=-q_{3}\left[\left(\frac{k q_{2}}{r_{P, 2}}-0\right)+\left[\left(\frac{k q_{1}}{r_{P, 1}}-0\right)\right]\right]=-k q_{3}\left[\frac{q_{2}}{r_{P, 2}}+\frac{q_{1}}{r_{P, 1}}\right] \\
& W=-9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{\mathrm{C}^{2}} \times\left(-5 \times 10^{-6} C\right)\left[\frac{-6 \times 10^{-6} C}{0.72 \mathrm{C}}+\frac{2 \times 10^{-6} \mathrm{C}}{1.14 \mathrm{~m}}\right]=-0.3 \mathrm{~J}
\end{aligned}
$$

c. If $q_{3}$ were released from rest at point $P$, it would

1. accelerate in the direction of the net electric field at point $P$.
2.) accelerate in the direction opposite to the net electric field at point $P$.
2. feel no net force and thus remain at rest at point $P$.
3. feel no net force and continue moving at a constant velocity along the charges original direction of motion.
d. Suppose that $q_{3}$ were placed again at point $P$. If all three charges had identical masses and if the charges were released from rest simultaneously, when all three charges are very far away from each other their speeds would be given by
4. $v=\sqrt{\frac{k Q_{1} Q_{2} Q_{3}}{3 r m}}$.
5. $v=\sqrt{\frac{3 k Q_{1} Q_{2} Q_{3}}{r m}}$.
6. $v=\sqrt{\frac{6}{m}\left(\frac{Q_{1} Q_{2}}{r_{1,2}}+\frac{Q_{2} Q_{3}}{r_{2,3}}+\frac{Q_{1} Q_{3}}{r_{1,3}}\right)}$.

There was no correct answer given to this question so everyone got credit for the question. The correct answer should be $v=\sqrt{\frac{2 k}{3 m}\left(\frac{Q_{1} Q_{2}}{r_{1,2}}+\frac{Q_{2} Q_{3}}{r_{2,3}}+\frac{Q_{1} Q_{3}}{r_{1,3}}\right)}$.
7. A proton is accelerated from rest through a potential difference of $\Delta V_{a c c}=2.3 \mathrm{MV}$ as shown below.
a. How much work (in $e V$ and $J$ ) was done on the proton and what is its speed when it leaves the accelerating region?

The work done is given by $W=-q \Delta V=-(e)\left[0-V_{\text {acc }}\right]=e V_{\text {acc }}=2.3 \mathrm{MeV}$. This energy converts to $W=-q \Delta V=-(e)\left[0-V_{a c c}\right]=e V_{a c c}=2.3 \mathrm{MeV}$.
The work done is equal to the change in kinetic energy of the proton. Thus the final speed of the proton is given by

$$
W=\Delta K E=\frac{1}{2} m_{p} v_{p}^{2} \rightarrow v_{p}=\sqrt{\frac{2 W}{m_{p}}}=\sqrt{\frac{2 \times 3.68 \times 10^{-13} J}{1.67 \times 10^{-27} k g}}=2.1 \times 10^{7} \frac{\mathrm{~m}}{\mathrm{~s}}
$$



The proton that leaves the accelerator above and is directed vertically upwards and approaches a second set of capacitor plates angled at $\theta=37^{\circ}$ with respect to the horizontal as shown below. The proton enters this second set of capacitor plates through the left-most hole in the bottom plate. The capacitor plates are used to steer the proton by $90^{\circ}$ and make it leave through the right-most hole in the bottom plate.

b. What electric field is needed to make the proton enter through the left hole and exit though the right hole if the distance between the centers of the holes is $L=0.5 m$ and the plates are separated by $d=0.1 m$ ? (Hint: Since, the proton is so small, you can assume that it enters at the center of the left hole and exits at the center of the right hole.)

The second capacitor is inclined at $\theta=37^{\circ}$, the proton enters the left-most hole and its velocity vector makes a $\phi=90^{\circ}-37^{\circ}=53^{\circ}$ angle with respect to the lower capacitor plate. Thus the horizontal and vertical components of the initial velocity are
$v_{i x}=v_{i} \cos \phi=2.1 \times 10^{7} \frac{\mathrm{~m}}{\mathrm{~s}} \cos 53=1.26 \times 10^{7} \frac{\mathrm{~m}}{\mathrm{~s}}$
$v_{i y}=v_{i} \sin \phi=2.1 \times 10^{7} \frac{\mathrm{~m}}{\mathrm{~s}} \sin 53=1.68 \times 10^{7} \frac{\mathrm{~m}}{\mathrm{~s}}$
Assuming that the x -axis runs along the lower capacitor plate and the y -axis is perpendicular to the lower capacitor plate, we find the time that is needed for the charge to cover the distance $L$ along the lower capacitor plate between the holes. The time is given from the horizontal trajectory equation where the horizontal acceleration is zero. Thus, $x_{f}=x_{i}+v_{i x} t \rightarrow t=\frac{L}{v_{i x}}=\frac{0.5 \mathrm{~m}}{1.26 \times 10^{7} \frac{\mathrm{~m}}{\mathrm{~s}}}=3.96 \times 10^{-8} \mathrm{~s}$.
Then to calculate the electric field that is needed we use the vertical trajectory equation. Thus we have
$y_{f}=y_{i}+v_{i y} t+\frac{1}{2} a_{a} t^{2} \rightarrow 0=\left(v_{i y}+\frac{1}{2} a_{y} t\right) t \rightarrow\left\{\begin{array}{c}t=0 \\ \left(v_{i y}+\frac{1}{2} a_{y} t\right)=0\end{array}\right.$. The acceleration is
given by $a_{y}=\frac{F_{y}}{m_{p}}=\frac{-e E}{m_{p}}$. Combining these two results we can solve for the
electric field. We have
$0=\left(v_{i y}+\frac{1}{2} a_{y} t\right)=v_{i y}-\left(\frac{e E}{2 m_{p}}\right) t \rightarrow v_{i y}-\left(\frac{e E}{2 m_{p}}\right)\left(\frac{L}{v_{i x}}\right)$
$E=\frac{2 m_{p} v_{i y} v_{i x}}{e L}=\frac{2 \times 1.67 \times 10^{-27} \mathrm{~kg} \times 1.68 \times 10^{7} \frac{\mathrm{~m}}{\mathrm{~s}} \times 1.26 \times 10^{7} \frac{\mathrm{~m}}{\mathrm{~s}}}{1.6 \times 10^{-19} \mathrm{C} \times 0.5 \mathrm{~m}}=8.9 \times 10^{6} \frac{\mathrm{~N}}{\mathrm{C}}$
and the direction of the field is from the upper plate to the lower plate.
c. What is the charge $Q$ on either plate, if the plates each have an area $A=0.1 \mathrm{~m}^{2}$ and the space between the plates is filled with air with dielectric constant $\kappa=1$ ?

The charge is given by

$$
\begin{aligned}
& Q=C V=C(E d)=\frac{\kappa \varepsilon_{o} A E d}{d}=\kappa \varepsilon_{o} A E \\
& \therefore Q=8.85 \times 10^{-12} \frac{C^{2}}{N m^{2}} \times 0.1 \mathrm{~m}^{2} \times 8.9 \times 10^{6} \frac{N}{C}=7.9 \times 10^{-6} C=7.9 \mu C
\end{aligned}
$$

3. The Earth's atmosphere can act as a capacitor, with the ground acting as one plate, the clouds acting as the second plate and the space between the clouds and ground filled with air. Air is normally an insulating material, but under certain conditions can be made to conduct electricity, so that electric charge can flow from the clouds to the ground, in what we call a lightning strike. Assume that the clouds are uniformly distributed around the entire Earth at a fixed distance of 5000m ( $\sim 3 m i$ ) above the ground of area $4 \pi R_{\text {Earth }}^{2}$, where $R_{\text {Earth }}=6400 \mathrm{~km}$. Further, assume that the air between the clouds and the ground has a resistance taken to be $R=350 \Omega$.
a. Taking the upper negative plate to be the clouds and the lower positive plate to be the ground, what is the magnitude of the difference in potential that exists between the clouds and the ground if in a typical day a maximum of $5 \times 10^{5} \mathrm{C}$ of charge is spread over the surface of the Earth?

The capacitance is given as:

$$
C=\frac{\kappa \varepsilon_{0} A}{d}=\frac{8.85 \times 10^{-12} \frac{C^{2}}{N m^{2}} \times 4 \pi\left(6.4 \times 10^{6} \mathrm{~m}\right)^{2}}{5000 \mathrm{~m}}=0.91 \mathrm{~F} .
$$

The magnitude of the difference in potential that exists between the clouds and the ground is $Q=C V \rightarrow V=\frac{Q}{C}=\frac{5 \times 10^{5} \mathrm{C}}{0.91 \mathrm{~F}}=5.5 \times 10^{5} \mathrm{~V}$
b. Approximately how long would it take the Earth-cloud capacitor to discharge of $99 \%$ its total initial charge, $Q_{\text {max }}$ ? Further, assuming that the charge is immediately replenished as soon as the discharge process ends, how many lightning strikes are produced in a single day if each strike contains $25 C$ of charge?

The time to discharge $99 \%$ its total initial charge, $Q_{\max }$ is given as

$$
Q(t)=0.01 Q_{\max }=Q_{\max } e^{-\frac{t}{R C}} \rightarrow t=-R C \ln (0.01)=-300 \Omega \times 0.91 F \times \ln (0.01)=1467 \mathrm{~s}
$$

In this time we discharge $5 \times 10^{5} \mathrm{C}$ of charge by lightning strikes that contain 25 C of charge each. Thus we have $\frac{5 \times 10^{5} C}{1467 s} \times \frac{1 \text { strike }}{25 C} \sim 14 \frac{\text { strikes }}{s}$. Converting this to lightning strikes per day we have $14 \frac{\text { strikes }}{s} \times \frac{3600 s}{1 h r} \times \frac{24 h r}{1 d a y}=1.2 \times 10^{6} \frac{\text { strikes }}{\text { day }}$.
c. Of course if you've ever driven in the rain (and especially during a thunderstorm) you probably have had the occasion to set your car's windshield wipers to wipe the windows at intervals that match the amount of rainfall hitting the window. Your car has intermittent windshield wipers that control when the wiper actually moves across the window and you can select how quick or slow this occurs by using a RC circuit. A charging RC circuit controls the intermittent windshield wipers in your car by using the car's battery, which is rated at $V_{B}$. Suppose that for a particular setting, the wipers are triggered when the voltage across a capacitor C reaches $V_{C}$, where $V_{C} \leq V_{B}$. At this point the capacitor is quickly discharged (through a much smaller resistor) and the cycle repeats. What variable resistance $R$ should be used in a charging circuit if the wipers are to operate once every $t$ seconds, where $t$ is the amount of time between each wipe cycle of the wipers?

1. $R=\frac{t}{\ln \left(1-\frac{V_{C}}{V_{B}}\right) \times C}$
2. $R=\frac{-t}{\ln \left(1-\frac{V_{B}}{V_{C}}\right) \times C}$
3. $R=\frac{t}{\ln \left(1-\frac{V_{B}}{V_{C}}\right) \times C}$
(4.) $R=\frac{-t}{\ln \left(1-\frac{V_{C}}{V_{B}}\right) \times C}$

Just for reference, the charging/discharging circuit is given below. When charging the capacitor, the switch $S$ is connected to the battery (in the left most position), capacitor C and variable resistor $R$ (the one with the arrow through it). When the potential reaches a specified value, the switch $S$ moves to the right most position and discharges the capacitor through the wipers and the wiper motor moves the actual wipers. After the discharge the switch moves back to the left and the circuit charges the capacitor again. The switch moves back and forth at the time $t$ above. This circuit should look reasonably familiar and not all of the control circuitry is shown.


## Physics 111 Equation Sheet

Electric Forces, Fields and Potentials

$$
\begin{aligned}
& \vec{F}=k \frac{Q_{1} Q_{2}}{r^{2}} \hat{r} \\
& \vec{E}=\frac{\vec{F}}{q} \\
& \vec{E}_{Q}=k \frac{Q}{r^{2}} \hat{r} \\
& P E=k \frac{Q_{1} Q_{2}}{r} \\
& V(r)=k \frac{Q}{r} \\
& E_{x}=-\frac{\Delta V}{\Delta x} \\
& W=-q \Delta V
\end{aligned}
$$

Magnetic Forces and Fields
$F=q v B \sin \theta$
$F=I l B \sin \theta$
$\tau=N I A B \sin \theta=\mu B \sin \theta$
$P E=-\mu B \cos \theta$
$B=\frac{\mu_{0} I}{2 \pi r}$
$\varepsilon_{\text {induced }}=-N \frac{\Delta \phi_{B}}{\Delta t}=-N \frac{\Delta(B A \cos \theta)}{\Delta t}$

## Constants

$g=9.8 \frac{\mathrm{~m}}{s^{2}}$
$1 e=1.6 \times 10^{-19} \mathrm{C}$
$k=\frac{1}{4 \pi \varepsilon_{o}}=9 \times 10^{9} \frac{\mathrm{~N} \mathrm{~N}^{2}}{\mathrm{c}^{2}}$
$\varepsilon_{o}=8.85 \times 10^{-12} \frac{\mathrm{C}^{2}}{\mathrm{Nm} n^{2}}$
$1 \mathrm{eV}=1.6 \times 10^{-19} \mathrm{~J}$
$\mu_{o}=4 \pi \times 10^{-7} \frac{7}{A}$
$c=3 \times 10^{8} \frac{\mathrm{~m}}{\mathrm{~s}}$
$h=6.63 \times 10^{-34} \mathrm{Js}$
$m_{e}=9.11 \times 10^{-31} \mathrm{~kg}=\frac{0.511 \mathrm{MeV}}{c^{2}}$
$m_{p}=1.67 \times 10^{-27} \mathrm{~kg}=\frac{937.1 \mathrm{MeV}}{c^{2}}$
$m_{n}=1.69 \times 10^{-27} \mathrm{~kg}=\frac{948.3 \mathrm{MeV}}{c^{2}}$
$1 \mathrm{amu}=1.66 \times 10^{-27} \mathrm{~kg}=\frac{931.5 \mathrm{MeV}}{c^{2}}$
$N_{A}=6.02 \times 10^{23}$
$A x^{2}+B x+C=0 \rightarrow x=\frac{-B \pm \sqrt{B^{2}-4 A C}}{2 A}$

## Electric Circuits

$$
\begin{aligned}
& I=\frac{\Delta Q}{\Delta t} \\
& V=I R=I\left(\frac{\rho L}{A}\right)
\end{aligned}
$$

$$
R_{\text {series }}=\sum_{i=1}^{N} R_{i}
$$

$$
\frac{1}{R_{\text {parallel }}}=\sum_{i=1}^{N} \frac{1}{R_{i}}
$$

$$
P=I V=I^{2} R=\frac{V^{2}}{R}
$$

$$
Q=C V=\left(\frac{\kappa \varepsilon_{0} A}{d}\right) V=\left(\kappa C_{0}\right) V
$$

$$
P E=\frac{1}{2} Q V=\frac{1}{2} C V^{2}=\frac{Q^{2}}{2 C}
$$

$$
Q_{\text {charge }}(t)=Q_{\max }\left(1-e^{-\frac{t}{R C}}\right)
$$

$$
Q_{\text {discharge }}(t)=Q_{\max } e^{-\frac{t}{R C}}
$$

$$
C_{\text {parallel }}=\sum_{i=1}^{N} C_{i}
$$

$$
\frac{1}{C_{\text {series }}}=\sum_{i=1}^{N} \frac{1}{C_{i}}
$$

Light as a Particle \& Relativity

$$
\begin{aligned}
& E=h f=\frac{h c}{\lambda}=p c \\
& K E_{\max }=h f-\phi=e V_{\text {stop }} \\
& \Delta \lambda=\frac{h}{m_{e} c}(1-\cos \phi) \\
& \gamma=\frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}} \\
& p=\gamma m v \\
& E_{\text {total }}=K E+E_{\text {rest }}=\gamma m c^{2} \\
& E_{\text {total }}^{2}=p^{2} c^{2}+m^{2} c^{4} \\
& E_{\text {rest }}=m c^{2} \\
& K E=(\gamma-1) m c^{2}
\end{aligned}
$$

Geometry
Circles: $C=2 \pi r=\pi D \quad A=\pi r^{2}$
Triangles: $\quad A=\frac{1}{2} b h$
Spheres: $A=4 \pi r^{2} \quad V=\frac{4}{3} \pi r^{3}$

Light as a Wave

$$
\begin{aligned}
& c=f \lambda=\frac{1}{\sqrt{\varepsilon_{o} \mu_{o}}} \\
& S(t)=\frac{\text { energ } y}{\text { time } \times \text { area }}=c \varepsilon_{o} E^{2}(t)=c \frac{B^{2}(t)}{\mu_{0}} \\
& I=S_{\text {avg }}=\frac{1}{2} c \varepsilon_{o} E_{\max }^{2}=c \frac{B_{\max }^{2}}{2 \mu_{0}} \\
& P=\frac{S}{c}=\frac{\text { Force }}{\text { Area }} \\
& S=S_{o} \cos ^{2} \theta \\
& v=\frac{1}{\sqrt{\varepsilon \mu}}=\frac{c}{n} \\
& \theta_{\text {inc }}=\theta_{\text {ref } l} \\
& n_{1} \sin \theta_{1}=n_{2} \sin \theta_{2} \\
& \frac{1}{f}=\frac{1}{d_{o}}+\frac{1}{d_{i}} \\
& M=\frac{h_{i}}{h_{o}}=-\frac{d_{i}}{d_{o}} \\
& M_{\text {total }}=\prod_{i=1}^{N} M_{i} \\
& d \sin \theta=m \lambda \text { or }\left(m+\frac{1}{2}\right) \lambda \\
& a \sin \phi=m^{\prime} \lambda
\end{aligned}
$$

Nuclear Physics

$$
\begin{aligned}
& E_{\text {binding }}=\left(Z m_{p}+N m_{n}-m_{\text {rest }}\right) y^{2} \\
& \frac{\Delta N}{\Delta t}=-\lambda N_{o} \rightarrow N(t)=N_{o} e^{-\lambda t} \\
& A(t)=A_{o} e^{-\lambda t} \\
& m(t)=m_{o} e^{-\lambda t} \\
& t_{\frac{1}{2}}=\frac{\ln 2}{\lambda}
\end{aligned}
$$

Misc. Physics 110
Formulae
$\vec{F}=\frac{\Delta \vec{p}}{\Delta t}=\frac{\Delta(m v)}{\Delta t}=m \vec{a}$ $\vec{F}=-k \vec{y}$
$\vec{F}_{C}=m \frac{v^{2}}{R} \hat{r}$
$W=\Delta K E=\frac{1}{2} m\left(v_{f}^{2}-v_{i}^{2}\right)=-\Delta P E$
$P E_{g r a v i t y}=m g y$
$P E_{\text {spring }}=\frac{1}{2} k y^{2}$
$x_{f}=x_{i}+v_{i x} t+\frac{1}{2} a_{x} t^{2}$
$v_{f x}=v_{i x}+a_{x} t$
$v_{f x}^{2}=v_{i x}^{2}+2 a_{x} \Delta x$

