## Physics 111

## Exam \#1

January 30, 2015

Name $\qquad$

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}|=(5 \mathrm{~kg}) \times\left(2 \frac{\mathrm{~m}}{\mathrm{~s}}\right)=10 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 3 points and each free-response part is worth 7 points

| Problem \#1 | $/ 24$ |
| :---: | :---: |
| Problem \#2 | $/ 24$ |
| Problem \#3 | $/ 24$ |
| Total | $/ 72$ |

I affirm that I have carried out my academic endeavors with full academic honesty.

1. Suppose that you place a collection of three sodium ions $\left(\mathrm{Na}^{+}\right)$, one at a time at the following locations. $q_{1}=N a^{+}$is located at the point $(x, y)=(0,0), q_{2}=N a^{+}$is located at the point $(x, y)=(0, a)$, and $q_{3}=N a^{+}$is located at the point $(x, y)=(a, a)$,.
a. How much work (in eV ) would be done to move a chlorine ion $\left(\mathrm{Cl}^{-}\right)$ion into point $P=(a, 0)$ from very far away? Assume that $a=0.5 \mathrm{~nm}$.

$$
\begin{aligned}
& W=-q \Delta V=-q\left[\Delta V_{q_{1} C l}+\Delta V_{q_{2} C l}+\Delta V_{q_{3} C l}\right]=-(-e)\left[\left(k \frac{e}{a}-0\right)+\left(k \frac{e}{a}-0\right)+\left(k \frac{e}{\sqrt{2} a}-0\right)\right] \\
& W=k \frac{e^{2}}{a}\left(2+\frac{1}{\sqrt{2}}\right)=9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{C^{2}} \times \frac{\left(1.6 \times 10^{-19} \mathrm{C}\right)^{2}}{0.5 \times 10^{-9} \mathrm{~m}}\left(2+\frac{1}{\sqrt{2}}\right) \\
& W=1.25 \times 10^{-18} J \times \frac{1 \mathrm{eV}}{1.6 \times 10^{-19} \mathrm{~J}}=7.8 \mathrm{eV}
\end{aligned}
$$

b. If the chlorine ion were released from rest at $P=(a, 0)$, its motion would most likely be
(1.) toward the point $(0, a)$ along the line joining the chlorine ion and the point.
2. toward the point $(0,0)$ along the line joining the chlorine ion and the point.
3. toward the point $(a, a)$ along the line joining the chlorine ion and the point.
4. away from the point $(0, a)$ along the line joining the chlorine ion and the point.
5. difficult to determine since all of the forces on the chlorine ion change as the ion moves.
c. What is the magnitude of the electric force on the chlorine ion located at point $P=(a, 0)$ due to the three sodium ions?

$$
\begin{aligned}
\begin{aligned}
F_{\text {net }, x} & =-F_{q_{1} C l}-F_{q_{2} C l} \cos \theta=-k \frac{e^{2}}{a^{2}}-k \frac{e^{2}}{a^{2}} \cos 45=-9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{\mathrm{C}^{2}}\left(\frac{1.6 \times 10^{-19} \mathrm{C}}{0.5 \times 10^{-9} \mathrm{~m}}\right)^{2}(1+\cos 45) \\
& =-1.57 \times 10^{-9} \mathrm{~N}
\end{aligned} \\
\begin{aligned}
F_{\text {net,yy}} & =F_{q_{3} C l}+F_{q_{2} C l} \sin \theta=k \frac{e^{2}}{a^{2}}+k \frac{e^{2}}{a^{2}} \sin 45=9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{\mathrm{C}^{2}}\left(\frac{1.6 \times 10^{-19} \mathrm{C}}{0.5 \times 10^{-9} \mathrm{~m}}\right)^{2}(1+\sin 45) \\
& =1.57 \times 10^{-9} \mathrm{~N} \\
F_{\text {net }} & =\sqrt{F_{\text {net }, x}^{2}+F_{\text {net,yy}}^{2}}=\sqrt{2\left(1.57 \times 10^{-9} \mathrm{~N}\right)^{2}}=2.2 \times 10^{-9} \mathrm{~N}
\end{aligned}
\end{aligned}
$$

d. What is the electric field at point $P=(a, 0)$ due to the three sodium ions?
$\vec{F}=q \vec{E} \rightarrow|\vec{E}|=\left|\frac{\vec{F}}{q}\right|=\frac{2.2 \times 10^{-9} \mathrm{~N}}{1.6 \times 10^{-19} \mathrm{C}}=1.4 \times 10^{10} \frac{\mathrm{~N}}{\mathrm{C}}$ directed away from the point $P=(a, 0)$ at an angle $45^{\circ}$ of below the x -axis.
2. Consider the circuit in which a collection of resistors is wired in combination to a battery. The battery is rated at $V_{B}=10 \mathrm{~V}$ and each resistor is $R=100 \Omega$.
a. What is the equivalent resistance of the circuit and the total current produced by the battery?
$R_{3}$ and $R_{4}$ are in parallel so
$\frac{1}{R_{34}}=\frac{1}{R_{3}}+\frac{1}{R_{4}}=\frac{2}{100 \Omega} \rightarrow R_{34}=50 \Omega$.

$R_{34}$ is in series with $R_{2}$;
$R_{234}=R_{2}+R_{34}=100 \Omega+50 \Omega=150 \Omega$.
$R_{6}$ and $R_{7}$ are in parallel so $\frac{1}{R_{67}}=\frac{1}{R_{6}}+\frac{1}{R_{7}}=\frac{2}{100 \Omega} \rightarrow R_{67}=50 \Omega$.
$R_{67}$ is in series with $R_{8} ; R_{678}=R_{8}+R_{67}=100 \Omega+50 \Omega=150 \Omega$.
$R_{234}$ and $R_{678}$ are in parallel so $\frac{1}{R_{234678}}=\frac{1}{R_{234}}+\frac{1}{R_{678}}=\frac{2}{150 \Omega} \rightarrow R_{234678}=75 \Omega$.
$R_{234678}$ is in series with $R_{5}$ and $R_{1}$;
$R_{e q}=R_{12345678}=R_{1}+R_{5}+R_{234678}=100 \Omega+100 \Omega+75 \Omega=275 \Omega$.
Therefore the total current produced by the battery is
$V=I R \rightarrow I_{\text {total }}=\frac{V_{B}}{R_{e q}}=\frac{10 \mathrm{~V}}{275 \Omega}=0.036 \mathrm{~A}=36 \mathrm{~mA}$.
b. Suppose that at time $t_{i}=0 s$ the battery is connected to the circuit. How much energy is dissipated as heat across the resistor $R_{1}$ after a time $\Delta t=10 \mathrm{~s}$ ?

$$
P=\frac{\Delta E}{\Delta t} \rightarrow \Delta E=P \Delta t=I_{\text {total }}^{2} R_{e q} \Delta t=(0.036 \mathrm{~A})^{2} \times 100 \Omega \times 10 \mathrm{~s}=1.3 \mathrm{~J} .
$$

c. The actual current in any circuit is the flow of negative charges, or the electrons.

Suppose that the total current (flow of electrons) flows from the battery through resistor $R_{5}$. The potential energy of the electrons as they flow through resistor $R=100 \Omega$
1 increases.
2. decreases
3. remains the same
4. is unable to be determined from the information given.
d. Suppose instead of the circuit above, you have the following circuit. The battery (rated at $\left.V_{B}=10 \mathrm{~V}\right)$ is connected to a resistor ( $R=10 \mathrm{k} \Omega$ ) and three capacitors.
Each capacitor is $C=10,000 \mu F$. How long will it take to accumulate $63.2 \%$ of the total charge on the capacitor?
$C_{2}$ and $C_{3}$ are in parallel;
$C_{23}=C_{2}+C_{3}=2 \times 10000 \mu F=20000 \mu F$.
$C_{1}$ and $C_{23}$ are in series;
$\frac{1}{C_{e q}}=\frac{1}{C_{123}}=\frac{1}{C_{1}}+\frac{1}{C_{23}}=\frac{1}{10000 \mu F}+\frac{1}{20000 \mu F}=\frac{3}{20000 \mu F} \rightarrow C_{e q}=C_{123}=6667 \mu F=0.0067 F$

The time it will take to accumulate $63.2 \%$ of the total charge is the time constant.
We have $\tau=R C=10000 \Omega \times 0.0067 F=66.7 \mathrm{~s}$.
An alternative way would be:

$$
Q(t)=0.632 Q_{\max }=Q_{\max }\left(1-e^{-\frac{t}{R C}}\right) \rightarrow t=-R C \ln (1-0.632)=66.7 \mathrm{~s} .
$$

3. An air-filled capacitor is constructed out of two parallel-circular plates of diameter $d=50 \mathrm{~cm}$. A charge $+Q$ is placed on the left plate while a charge of $-Q$ is placed on the right plate. The right plate has a hole cut into its center. An electric field exists between the plates with magnitude $E=1 \times 10^{6} \frac{N}{C}$.
a. Suppose that an alpha particle (a helium nucleus ${ }_{2}^{4} \mathrm{He}$ ) is accelerated from rest at the left plate toward the right plate with the hole in it. Starting from Newton's $2^{\text {nd }}$ law of motion, what are the acceleration of the alpha particle and what will the speed of the alpha particle be when it exits the hole if the plates are separated by $2 m$ ?

$$
\vec{F}=q \vec{E}=m \vec{a} \rightarrow|\vec{a}|=\left|\frac{q \vec{E}}{m}\right|=\frac{2 \times 1.6 \times 10^{-19} \mathrm{C} \times 1 \times 10^{6} \frac{\mathrm{~V}}{\mathrm{~m}}}{4 \times 1.67 \times 10^{-27} \mathrm{~kg}}=4.8 \times 10^{13} \frac{\mathrm{~m}}{\mathrm{~s}^{2}} \text { away from }
$$ the left plate toward the right plate with the hole in it.

The speed of the alpha particle will be:

$$
v_{f}^{2}=v_{i}^{2}+2 a \Delta x \rightarrow v_{f}=\sqrt{2 a d}=\sqrt{2 \times 4.8 \times 10^{13} \frac{\mathrm{~m}}{\mathrm{~s}^{2}} \times 2 \mathrm{~m}}=1.4 \times 10^{7} \frac{\mathrm{~m}}{\mathrm{~s}} .
$$

b. How much charge is on each capacitor plate?

$$
\begin{aligned}
& Q=C V \text { where } \\
& C=\frac{\kappa \varepsilon_{0} A}{d}=\frac{1 \times 8.85 \times 10^{-12} \frac{C^{2}}{N m^{2}} \times\left(\pi(0.25 \mathrm{~m})^{2}\right)}{2 \mathrm{~m}}=8.7 \times 10^{-13} \mathrm{~F}=87 \mathrm{pF} . \text { Thus, } \\
& Q=C V=8.7 \times 10^{-13} F \times\left(1 \times 10^{6} \frac{V}{m} \times 2 \mathrm{~m}\right)=1.7 \times 10^{-6} \mathrm{C}=1.7 \mu C .
\end{aligned}
$$

c. Suppose that when the alpha particle exits the hole in the right capacitor plate, it enters a region of uniform magnetic field $B=0.37 T$ directed out of the page at you. The angle between the incident alpha particle and the magnetic field is unknown. If the alpha particle traverses a circle with diameter $d=79 \mathrm{~cm}$, at what angle with respect to the magnetic field did the alpha particle enter the field?

$$
\begin{aligned}
& F_{B}=q v_{\perp} B=m \frac{v_{\perp}^{2}}{R} \rightarrow \frac{q R B}{m}=v \sin \theta \Rightarrow \sin \theta=\frac{q R B}{m v} \\
& \sin \theta=\frac{q R B}{m v}=\frac{\left(2 \times 1.6 \times 10^{-19} \mathrm{C}\right) \times 0.395 \mathrm{~m} \times 0.37 \mathrm{~T}}{\left(4 \times 1.67 \times 10^{-27} \mathrm{~kg}\right) \times 1.4 \times 10^{7} \frac{\mathrm{~m}}{\mathrm{~s}}}=0.5 \rightarrow \theta=30^{\circ}
\end{aligned}
$$

d. Suppose that the electric field between the capacitor plates were to increase (say by adding more charge to the plates) while keeping the plates the same size and keeping the separation the same. If the electric field increased by a factor of 10 , which change would most likely occur?

1. The time for one orbit of the alpha particle in the magnetic field would increase.
2. The time for one orbit of the alpha particle in the magnetic field would decrease.
3. The speed of the alpha particle would decrease.
4. The radius of the alpha particles orbit would decrease.
5. The angle between the incident alpha particle and the magnetic field would increase.

## Physics 111 Equation Sheet

Electric Forces, Fields and Potentials

$$
\begin{aligned}
& \vec{F}=k \frac{Q_{1} Q_{2}}{r^{2}} \hat{r} \\
& \vec{E}=\frac{\vec{F}}{q} \\
& \vec{E}_{Q}=k \frac{Q}{r^{2}} \hat{r} \\
& P E=k \frac{Q_{1} Q_{2}}{r} \\
& V(r)=k \frac{Q}{r} \\
& E_{x}=-\frac{\Delta V}{\Delta x} \\
& W=-q \Delta V
\end{aligned}
$$

Magnetic Forces and Fields
$F=q v B \sin \theta$
$F=I l B \sin \theta$
$\tau=N I A B \sin \theta=\mu B \sin \theta$
$P E=-\mu B \cos \theta$
$B=\frac{\mu_{0} I}{2 \pi r}$
$\varepsilon_{\text {induced }}=-N \frac{\Delta \phi_{B}}{\Lambda t}=-N \frac{\Delta(B A \cos \theta)}{\Lambda t}$

## Constants

$g=9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$
$1 e=1.6 \times 10^{-19} \mathrm{C}$
$k=\frac{1}{4 \pi \varepsilon_{o}}=9 \times 10^{9} \frac{\mathrm{~N} \mathrm{~N}^{2}}{\mathrm{c}^{2}}$
$\varepsilon_{o}=8.85 \times 10^{-12} \frac{\mathrm{c}^{2}}{\mathrm{Nm}^{2}}$
$1 \mathrm{eV}=1.6 \times 10^{-19} \mathrm{~J}$
$\mu_{o}=4 \pi \times 10^{-7} \frac{7 m}{A}$
$c=3 \times 10^{8} \frac{\mathrm{~m}}{\mathrm{~s}}$
$h=6.63 \times 10^{-34} \mathrm{Js}$
$m_{e}=9.11 \times 10^{-31} \mathrm{~kg}=\frac{0.511 \mathrm{MeV}}{c^{2}}$
$m_{p}=1.67 \times 10^{-27} \mathrm{~kg}=\frac{937.1 \mathrm{MeV}}{c^{2}}$
$m_{n}=1.69 \times 10^{-27} \mathrm{~kg}=\frac{948.3 \mathrm{MeV}}{c^{2}}$
$1 \mathrm{amu}=1.66 \times 10^{-27} \mathrm{~kg}=\frac{931.5 \mathrm{MeV}}{\mathrm{c}^{2}}$
$N_{A}=6.02 \times 10^{23}$
$A v^{2} \perp R v \perp C-n \perp v-B \pm \sqrt{B^{2}-4 A C}$

$$
\begin{aligned}
& I=\frac{\Delta Q}{\Delta t} \\
& V=I R=I\left(\frac{\rho L}{A}\right)
\end{aligned}
$$

$$
R_{\text {series }}=\sum_{i=1}^{N} R_{i}
$$

$$
\frac{1}{R_{\text {parallel }}}=\sum_{i=1}^{N} \frac{1}{R_{i}}
$$

$$
P=I V=I^{2} R=\frac{V^{2}}{R}
$$

$$
Q=C V=\left(\frac{\kappa \varepsilon_{0} A}{d}\right) Y=\left(\kappa C_{0}\right) V
$$

$$
P E=\frac{1}{2} Q V=\frac{1}{2} C V^{2}=\frac{Q^{2}}{2 C}
$$

$$
Q_{\text {charge }}(t)=Q_{\max }\left(1-e^{-\frac{t}{R C}}\right)
$$

$$
Q_{\text {discharge }}(t)=Q_{\max } e^{-\frac{t}{R C}}
$$

$$
C_{\text {parallel }}=\sum_{i=1}^{N} C_{i}
$$

$$
\frac{1}{C_{\text {series }}}=\sum_{i=1}^{N} \frac{1}{C_{i}}
$$

Light as a Particle \& Relativity

$$
\begin{aligned}
& E=h f=\frac{h c}{\lambda}=p c \\
& K E_{\max }=h f-\phi=e V_{\text {stop }} \\
& \Delta \lambda=\frac{h}{m_{e} c}(1-\cos \phi) \\
& \gamma=\frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}} \\
& p=\gamma m v \\
& E_{\text {total }}=K E+E_{\text {rest }}=\gamma m c^{2} \\
& E_{\text {total }}^{2}=p^{2} c^{2}+m^{2} c^{4} \\
& E_{\text {rest }}=m c^{2}
\end{aligned}
$$

$$
K E=(\gamma-1) m c^{2}
$$

## Geometry

Circles: $C=2 \pi r=\pi D \quad A=\pi r^{2}$
Triangles: $A=\frac{1}{2} b h$
Spheres: $A=4 \pi r^{2} \quad V=\frac{4}{3} \pi r^{3}$

Light as a Wave

$$
\begin{aligned}
& c=f \lambda=\frac{1}{\sqrt{\varepsilon_{o} \mu_{o}}} \\
& S(t)=\frac{\text { energ } y}{\text { time } \times \text { area }}=c \varepsilon_{o} E^{2}(t)=c \frac{B^{2}(t)}{\mu_{0}} \\
& I=S_{\text {avg }}=\frac{1}{2} c \varepsilon_{o} E_{\max }^{2}=c \frac{B_{\max }^{2}}{2 \mu_{0}} \\
& P=\frac{S}{c}=\frac{\text { Force }}{\text { Area }} \\
& S=S_{o} \cos ^{2} \theta \\
& v=\frac{1}{\sqrt{\varepsilon \mu}}=\frac{c}{n} \\
& \theta_{\text {inc }}=\theta_{\text {refl }} \\
& n_{1} \sin \theta_{1}=n_{2} \sin \theta_{2} \\
& \frac{1}{f}=\frac{1}{d_{o}}+\frac{1}{d_{i}} \\
& M=\frac{h_{i}}{h_{o}}=-\frac{d_{i}}{d_{o}} \\
& M_{\text {total }}=\prod_{i=1}^{N} M_{i} \\
& d \sin \theta=m \lambda \text { or }\left(m+\frac{1}{2}\right) \lambda \\
& a \sin \phi=m^{\prime} \lambda
\end{aligned}
$$

Nuclear Physics

$$
\begin{aligned}
& E_{\text {binding }}=\left(Z m_{p}+N m_{n}-m_{r ब t}\right) c^{2} \\
& \frac{\Delta N}{\Delta t}=-\lambda N_{o} \rightarrow N(t)=N_{o} e^{-\lambda t} \\
& A(t)=A_{o} e^{-\lambda t} \\
& m(t)=m_{o} e^{-\lambda t} \\
& t_{\frac{1}{2}}=\frac{\ln 2}{\lambda}
\end{aligned}
$$

Misc. Physics 110
Formulae
$\vec{F}=\frac{\Delta \vec{p}}{\Delta t}=\frac{\Delta(m v)}{\Delta t}=m \vec{a}$
$\vec{F}=-k \vec{y}$
$\vec{F}_{C}=m \frac{v^{2}}{R} \hat{r}$
$W=\Delta K E=\frac{1}{2} m\left(v_{f}^{2}-v_{i}^{2}\right)=-\Delta P E$
$P E_{g r a v i t y}=m g y$
$P E_{\text {spring }}=\frac{1}{2} k y^{2}$
$x_{f}=x_{i}+v_{i x} t+\frac{1}{2} a_{x} t^{2}$
$v_{f x}=v_{i x}+a_{x} t$
$v_{f x}^{2}=v_{i x}^{2}+2 a_{x} \Delta x$

