Physics 111

Exam #1

January 27, 2017

Name_____

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg) \times (2\frac{m}{s}) = 10\frac{kg \cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 3 points and each free-response part is worth 7 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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1. Potassium (K^+) and chlorine (Cl^-) ions can form an ionic bond. Suppose that a K^+ ion is located at the point $(x,y)=(0,0)$ while a Cl^- ion is located at the point $(x,y)=(0,d)$. Further let the spacing between the K^+ and Cl^- ions be $d=0.63nm$.

   a. How much work (in eV) would be done to assemble this collection of charges if each charge is brought in from very far away?

$$W_1 = 0J$$

$$W_2 = -q\Delta V = -(-e)\left[\frac{ke}{r} - 0\right] = \frac{ke^2}{r}$$

$$W_{net} = W_1 + W_2 = \frac{ke^2}{r} = -\frac{9\times10^9 \frac{Nm^2}{C^2}\times\left(1.6\times10^{-19}C\right)^2}{0.63\times10^{-9}m} = 3.66\times10^{-19}J$$

$$W = 3.66\times10^{-19}J\times\frac{1eV}{1.6\times10^{-19}J} = 2.29eV$$

   b. What is the electric field at a point $P=(0,-d)$ due to potassium and chlorine ions?

$$E_{net,x} = 0$$

$$E_{net,y} = E_{Cl^-} - E_{K^+} = \frac{kQ}{(2d)^2} - \frac{kQ}{d^2} = \frac{ke}{4d^2} - \frac{ke}{d^2} = -\frac{3ke}{4d^2}$$

$$E_{net,y} = -\frac{3\times9\times10^9 \frac{Nm^2}{C^2}\times1.6\times10^{-19}C}{4\times\left(0.63\times10^{-9}m\right)^2} = -2.7\times10^9 \tfrac{N}{C}$$

$$E_{net} = \sqrt{E_{net,x}^2 + E_{net,y}^2} = \sqrt{\left(0\tfrac{N}{C}\right)^2 + \left(-2.7\times10^9\tfrac{N}{C}\right)^2} = 2.7\times10^9\tfrac{N}{C}$$

$$\phi = \tan^{-1}\left(\frac{E_{net,y}}{E_{net,y}}\right) = \tan^{-1}\left(\frac{-2.7\times10^9\frac{N}{C}}{0\frac{N}{C}}\right) = -90^0$$

Thus the net field has a magnitude of $2.7\times10^9\frac{N}{C}$ and points 90^0 below the positive x-axis, or in the negative y-direction.

c. Suppose that another chlorine ion was placed at the point $P = (0, -d)$ what force would the chlorine ion feel?

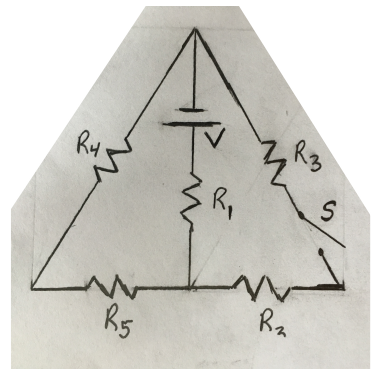
$F_{net} = qE_{net} = eE_{net} = 1.6 \times 10^{-19} C \times 2.7 \times 10^{9} \frac{N}{C} = 4.32 \times 10^{-10} N$ in magnitude and since this is an negative charge it will feel a force in the direction opposite to the electric field. Thus the direction of the force will be at $\theta = 90^{0}$ above the x-axis or in the positive y-direction.

d. Imagine that you had three point charges each $-q$ placed at the corners of an equilateral triangle. If the three point charges were simultaneously released from rest which of the following may be true?

1. The three charges will move toward each other and the work done on the charges will be negative.
2. The three charges will move apart and the work done on the charges will be negative.
3. The three charges will move toward each other and the work done on the charges will be positive.
4. The three charges will move apart and the work done on the charges will be positive.
5. The charges will move and work will be done, but the direction and the sign of the work done cannot be determined from the information given.

2. Consider the circuit in which a collection of resistors is wired in combination to a battery. The battery is rated at $V_B = 10V$ and each resistor is $R = 100\Omega$. Assume that switch S is closed for parts a - c.

   a. What is the equivalent resistance of the circuit and the total current produced by the battery?



R_2 & R_3 Series

$$R_{23} = R_2 + R_3 = 100\Omega + 100\Omega = 200\Omega$$

R_4 & R_5 Series

$$R_{45} = R_4 + R_5 = 100\Omega + 100\Omega = 200\Omega$$

R_{23} & R_{45} Parallel

$$\frac{1}{R_{2345}} = \frac{1}{200\Omega} + \frac{1}{200\Omega} = \frac{2}{200\Omega} \rightarrow R_{2345} = 100\Omega$$

R_1 & R_{2345} Series

$$R_{12345} = R_1 + R_{2345} = 100\Omega + 100\Omega = 200\Omega$$

$$R_{eq} = R_{12345} = 200\Omega$$

$$I_{total} = \frac{V}{R_{12345}} = \frac{10V}{200\Omega} = 0.05A = 50mA$$

   b. What are the current through resistor R_2 and the potential drop across resistor R_5?

Define the currents I_L and I_R to be the current in the left and right branches respectively. The current in the left branch is given by $I_L = \frac{V_L}{R_{45}}$ and the current in the right branch is given by $I_R = \frac{V_R}{R_{23}}$. The left and right branches are in parallel and the potential across these branches is given by $V_L = V_R = V_B - I_{total}R_1 = 10V - (0.05A \times 100\Omega) = 5V$. Thus,

$$I_L = \frac{V_L}{R_{45}} = \frac{5V}{200\Omega} = 0.025A = 25mA \text{ and } I_R = \frac{V_R}{R_{23}} = \frac{5V}{200\Omega} = 0.025A = 25mA.$$

Therefore the current through R_2 is $I_2 = I_R = 0.025A = 25mA$. The potential drop across R_5 is $V_5 = I_L R_5 = 0.025A \times 100\Omega = 2.5V$.

c. How much energy (per unit time) is produced by the battery and after a time $\Delta t = 5s$ how much energy is dissipated as heat by resistor R_3?

For the battery:

$$P_B = I_{total} V_B = I_{total}^2 R_{eq} = \frac{V_B^2}{R_{eq}}$$

$$P_B = (0.05A \times 10V) = (0.05A)^2 \times 200\Omega = \frac{(10V)^2}{200\Omega} = 0.5W$$

For R_3:

$$P_3 = I_R V_3 = I_R^2 R_3 = \frac{V_3^2}{R_3}$$

$$P_3 = (0.025A \times 2.5V) = (0.025A)^2 \times 100\Omega = \frac{(2.5V)^2}{100\Omega} = 0.0625W$$

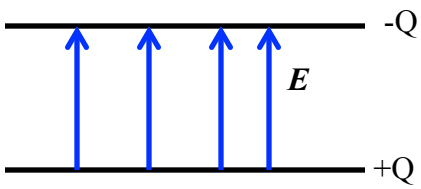
$$\rightarrow P_3 = \frac{\Delta E_3}{\Delta t} \rightarrow \Delta E_3 = P_3 \Delta t = 0.0625W \times 5s = 0.31J$$

d. Suppose that the switch, S were now opened. In this case the equivalent resistance R_{eq} of the circuit and the total current I_{total} produced by the battery would change according to which of the following?

1. $R_{eq} \downarrow$ and $I_{total} \uparrow$.
2. $R_{eq} \downarrow$ and $I_{total} \downarrow$.
3. $R_{eq} \uparrow$ and $I_{total} \uparrow$.
4. $R_{eq} \uparrow$ and $I_{total} \downarrow$.

3. Suppose that you have a parallel-plate capacitor that has been fully charged by a battery and further that there is a positive charge per unit area on the lower surface with a value $+8\times10^{-4}\,\frac{C}{m^2}$ and an equal layer of negative charge per unit area on the upper surface with a value $-8\times10^{-4}\,\frac{C}{m^2}$. The two parallel plates are separated by $d = 7nm$ and the space between the plates is filled with a dielectric with dielectric constant $\kappa = 9$.

   a. From the information given above, what is the magnitude and direction of the electric field between the plates? To specify the direction use the diagram given below and draw the direction of the electric field.



$$Q = CV = CEd = \left(\frac{\kappa\varepsilon_0 A}{d}\right)Ed \rightarrow E = \frac{Q}{\kappa\varepsilon_0 A}$$

$$E = \frac{Q}{\kappa\varepsilon_0 A} = \frac{8\times10^{-4}\,\frac{C}{m^2}}{9\times8.85\times10^{-12}\,\frac{C^2}{Nm^2}} = 1.0\times10^{7}\,\frac{N}{C}$$

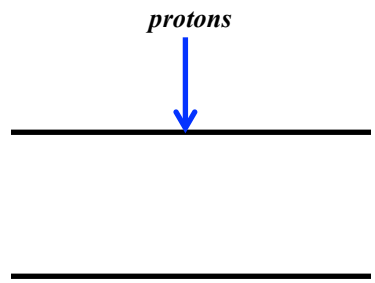
And the direction is from the positive plate to the negative plate, or vertically up.

   b. What is the difference in potential across the plates? Indicate by labeling on the diagram above which plate is at the higher potential and which is at the lower potential.

$$E = -\frac{\Delta V}{\Delta y} \rightarrow \Delta V = -Ed = -1\times10^{7}\,\frac{V}{m}\times7\times10^{-9}\,m = -0.0703V = -70.3mV$$

And the electric field points along decreasing electric potentials so the lower plate is at the higher potential and the upper plate is at the lower potential.

c. Suppose that you had a source of protons that you could inject through a small hole in the upper plate. What speed would the protons need to be injected at if they you wanted the protons to just barely touch the lower plate?



$$W = -\Delta U = -q\Delta V = \Delta K$$

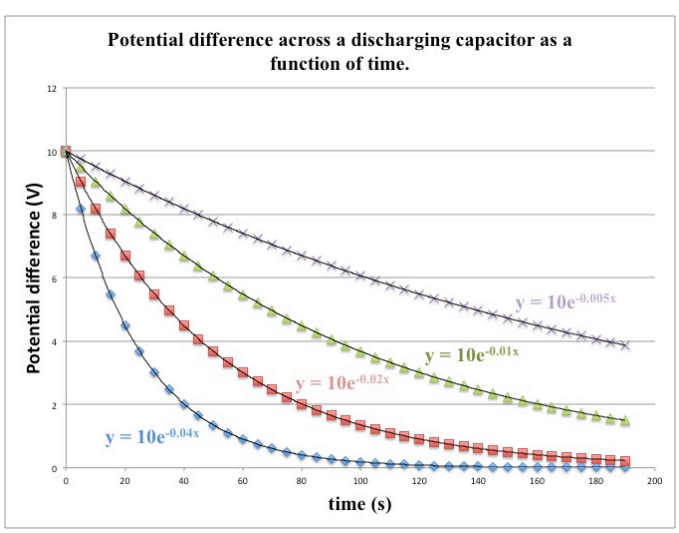
$$-q\left[V_{lower} - V_{upper}\right] = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$qV_{upper} = -\tfrac{1}{2}mv_i^2 \rightarrow v_i = \sqrt{-\frac{2qV_{upper}}{m}}$$

$$v_i = \sqrt{-\frac{2qV_{upper}}{m}} = \sqrt{-\frac{2\times 1.6\times 10^{-19}C}{1.67\times 10^{-27}kg}(-0.0703V)} = 3670\tfrac{m}{s}$$

d. Suppose that you had four capacitors (each with a different capacitance) that were each fully charged individually using a battery with potential difference V. The battery is then removed from each fully charged capacitor. Next each individual capacitor is separately connected in series to a resistor (and each resistor has a resistance R). The capacitors discharge through their individual resistor and plots of the potential difference across the discharging capacitors as a function of time are given below. From the graph below, which curve has a time constant of $\tau = 50s$?

1. The blue curve.
2. The red curve.
3. The green curve.
4. The purple curve.
5. It is not possible to determine which curve has a $\tau = 50s$ time constant with out more information.



Physics 111 Equation Sheet

Electric Forces, Fields and Potentials

$$\vec{F} = k\frac{Q_1Q_2}{r^2}\hat{r}$$

$$\vec{E} = \frac{\vec{F}}{q}$$

$$\vec{E}_Q = k\frac{Q}{r^2}\hat{r}$$

$$PE = k\frac{Q_1Q_2}{r}$$

$$V(r) = k\frac{Q}{r}$$

$$E_x = -\frac{\Delta V}{\Delta x}$$

$$W = -q\Delta V$$

Magnetic Forces and Fields

$$F = qvB\sin\theta$$

$$F = IlB\sin\theta$$

$$\tau = NIAB\sin\theta = \mu B\sin\theta$$

$$PE = -\mu B\cos\theta$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\varepsilon_{induced} = -N\frac{\Delta\phi_B}{\Delta t} = -N\frac{\Delta(BA\cos\theta)}{\Delta t}$$

Constants

$g = 9.8\,\frac{m}{s^2}$

$1e = 1.6\times10^{-19}C$

$k = \frac{1}{4\pi\varepsilon_o} = 9\times10^{9}\,\frac{Nm^2}{C^2}$

$\varepsilon_o = 8.85\times10^{-12}\,\frac{C^2}{Nm^2}$

$1eV = 1.6\times10^{-19}J$

$\mu_o = 4\pi\times10^{-7}\,\frac{Tm}{A}$

$c = 3\times10^{8}\,\frac{m}{s}$

$h = 6.63\times10^{-34}Js$

$m_e = 9.11\times10^{-31}kg = \frac{0.511MeV}{c^2}$

$m_p = 1.67\times10^{-27}kg = \frac{937.1MeV}{c^2}$

$m_n = 1.69\times10^{-27}kg = \frac{948.3MeV}{c^2}$

$1amu = 1.66\times10^{-27}kg = \frac{931.5MeV}{c^2}$

$N_A = 6.02\times10^{23}$

$Ax^2 + Bx + C = 0 \rightarrow x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$

Electric Circuits

$$I = \frac{\Delta Q}{\Delta t}$$

$$V = IR = I\left(\frac{\rho L}{A}\right)$$

$$R_{series} = \sum_{i=1}^{N} R_i$$

$$\frac{1}{R_{parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$$

$$P = IV = I^2R = \frac{V^2}{R}$$

$$Q = CV = \left(\frac{\kappa\varepsilon_0 A}{d}\right)V = (\kappa C_0)V$$

$$W = U = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}$$

$$Q_{\text{charge}}(t) = Q_{\max}\left(1 - e^{-\frac{t}{RC}}\right)$$

$$Q_{\text{discharge}}(t) = Q_{\max}e^{-\frac{t}{RC}}$$

$$C_{parallel} = \sum_{i=1}^{N} C_i$$

$$\frac{1}{C_{series}} = \sum_{i=1}^{N}\frac{1}{C_i}$$

Light as a Particle & Relativity

$$E = hf = \frac{hc}{\lambda} = pc$$

$$KE_{\max} = hf - \phi = eV_{stop}$$

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\phi)$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$p = \gamma mv$$

$$E_{total} = KE + E_{rest} = \gamma mc^2$$

$$E_{total}^2 = p^2c^2 + m^2c^4$$

$$E_{rest} = mc^2$$

$$KE = (\gamma - 1)mc^2$$

Geometry

Circles: $C = 2\pi r = \pi D$ $\quad A = \pi r^2$

Triangles: $A = \tfrac{1}{2}bh$

Spheres: $A = 4\pi r^2$ $\quad V = \tfrac{4}{3}\pi r^3$

Light as a Wave

$$c = f\lambda = \frac{1}{\sqrt{\varepsilon_o\mu_o}}$$

$$S(t) = \frac{energy}{time\times area} = c\varepsilon_o E^2(t) = c\frac{B^2(t)}{\mu_0}$$

$$I = S_{avg} = \tfrac{1}{2}c\varepsilon_o E_{\max}^2 = c\frac{B_{\max}^2}{2\mu_0}$$

$$P = \frac{S}{c} = \frac{Force}{Area}$$

$$S = S_o\cos^2\theta$$

$$v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{n}$$

$$\theta_{inc} = \theta_{refl}$$

$$n_1\sin\theta_1 = n_2\sin\theta_2$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$M_{total} = \prod_{i=1}^{N} M_i$$

$$d\sin\theta = m\lambda \text{ or } (m+\tfrac{1}{2})\lambda$$

$$a\sin\phi = m'\lambda$$

Nuclear Physics

$$E_{binding} = (Zm_p + Nm_n - m_{rest})c^2$$

$$\frac{\Delta N}{\Delta t} = -\lambda N_o \rightarrow N(t) = N_o e^{-\lambda t}$$

$$A(t) = A_o e^{-\lambda t}$$

$$m(t) = m_o e^{-\lambda t}$$

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

Misc. Physics 110 Formulae

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\Delta(mv)}{\Delta t} = m\vec{a}$$

$$\vec{F} = -k\vec{y}$$

$$\vec{F}_C = m\frac{v^2}{R}\hat{r}$$

$$W = \Delta KE = \tfrac{1}{2}m(v_f^2 - v_i^2) = -\Delta PE$$

$$PE_{gravity} = mgy$$

$$PE_{spring} = \tfrac{1}{2}ky^2$$

$$x_f = x_i + v_{ix}t + \tfrac{1}{2}a_x t^2$$

$$v_{fx} = v_{ix} + a_x t$$

$$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x$$