Physics 111

Exam #1

January 26, 2018

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg)\times(2\,\frac{m}{s}) = 10\,\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 3 points and each free-response part is worth 9 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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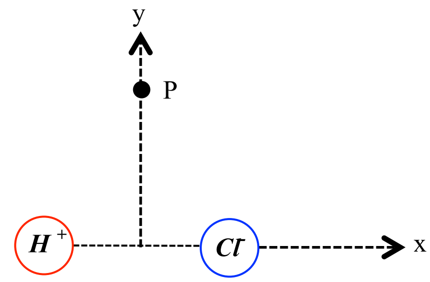
1. Hydrochloric acid is a moderately strong acid composed of a hydrogen ion (H^+) and a chlorine (Cl^-) ion covalently bonded together. This configuration is called an electric dipole and the separation between the hydrogen and chlorine ions is $s = 0.127nm$.



a. Suppose that a Cu^{+2} ion were placed at the point $P = (x, y) = (0,25)nm$. What are the net electric field and net electric force that the Cu^{+2} ion would feel?

The electric field from the ion would point away from along the line joining the point and while the electric field due to the ion would point toward the ion along the line joining the ion and the point. Since these charges carry equal magnitudes of charge and are equidistant from the point the vertical components of the electric field (and hence electric force) vanish by symmetry. Thus there is only an x-component to the electric field and force.

$$E_x = \frac{kq_+}{r_+^2}\cos\theta + \frac{kq_-}{r_-^2}\cos\theta = \frac{2kq}{r^2}\cos\theta = \frac{2kq}{r^2}\left(\frac{s/2}{r}\right) = \frac{kqs}{r^3}$$

$$= \frac{kqs}{r^3} = \frac{9\times10^9 \frac{Nm^2}{C^2} \times 1.6\times10^{-19}C \times 0.127\times10^{-9}m}{\left(\sqrt{\left(\frac{0.127\times10^{-9}m}{2}\right)^2 + \left(25\times10^{-9}m\right)^2}\right)^3} = 1.2\times10^4 \frac{N}{C}$$

$$F_x = qE_x = 2eE_x = 2\times1.6\times10^{-19}C \times 1.7\times10^4 \frac{N}{C} = 3.8\times10^{-15}N$$

b. How much work would be done to bring in the Cu^{+2} ion from very far away and place it at the point $P = (x, y) = (0,25)nm$?

$$V_P = V_{+q} + V_{-q} = \frac{kq}{r} - \frac{kq}{r} = 0V$$

$$W = -q\Delta V = -2e\left[0V - 0V\right] = 0J$$

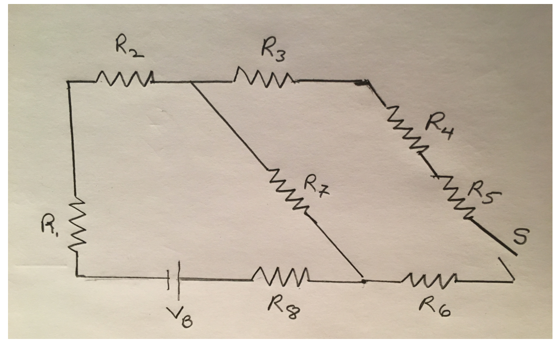
c. Assuming that each of the three charges in the distribution above are brought in from very far away and placed in their respective locations, the electric potential energy of the assembled charge distribution is given by

1. $U = q_{Cu^{+2}} k \left[\frac{q_{H^+}}{r^2} + \frac{q_{Cl^-}}{r^2} \right] = \frac{4ke^2}{r^2}$.
2. $U = kq_{Cu^{+2}} \left[\frac{q_{H^+}}{r} + \frac{q_{Cl^-}}{r} \right] = \frac{2ke^2}{r}$.
3. $U = kq_{Cu^{+2}} \left[\frac{q_{H^+}}{r} + \frac{q_{Cl^-}}{r} \right] = \frac{ke^2}{r}$.
4. $U = \frac{ke^2}{s}$.
5. $U = 0$.

d. Suppose that the Cu^{+2} ion were released from rest at the point $P = (x, y) = (0, 25)nm$. The motion of the ion would most likely be

1. to move down along the y-axis toward origin.
2. to move up along the y-axis away from origin.
3. to move to the left along the x-axis and then toward the H^+ ion.
4. to move to the right along the x-axis and then toward the Cl^- ion.
5. to move along the x-axis and keep moving along the x-axis until the Cu^{+2} ion is very far from the H^+ and Cl^- ions.

2. Circuits
   a. Consider the resistor circuit on the right in which a collection of resistors is wired in combination to a battery. The battery is rated at $V_B = 10V$ and each resistor is $R = 100\Omega$. What is the equivalent resistance of the circuit and the total current produced by the battery? Assume that the switch S is closed for this question.



Resistors R_3, R_4, R_5, and R_6 are in series and their equivalent resistance is $R_{3456} = R_3 + R_4 + R_5 + R_6 = 400\Omega$.

Resistors R_7 and R_{3456} are in parallel and their equivalent resistance is

$$\frac{1}{R_{34567}} = \frac{1}{R_7} + \frac{1}{R_{3456}} = \frac{1}{100\Omega} + \frac{1}{400\Omega} = \frac{5}{400\Omega} \rightarrow R_{34567} = 80\Omega .$$

Resistors R_1, R_2, R_{34567}, and R_8 are in series and their equivalent resistance is

$$R_{eq} = R_{1234568} = R_1 + R_2 + R_{34567} + R_8 = 380\Omega$$

The total current is $I_{total} = \dfrac{V}{R_{eq}} = \dfrac{10V}{380\Omega} = 0.026A$.

   b. What is the total power developed by the battery and what is the energy dissipated across resistor R_7 if the circuit is energized (connected to the battery) for 1 minute? Assume that the switch S is closed for this question.

The power developed by the battery is given by

$$P = IV = \frac{V^2}{R} = I^2 R = (0.026A)^2 \times 380\Omega = 0.257W = 257mW .$$

To determine the energy dissipated across R_7 we need to determine the power lost across R_7 and then to multiply this by the time.

The potential difference across R_1, R_2, and R_8 is given by

$V_{R_1} = V_{R_2} = V_{R_8} = I_{total} R_1 = 0.026A \times 100\Omega = 2.6V$. The potential difference across is given by

$$V = V_1 + V_2 + V_{34567} + V_8 \rightarrow V_{34567} = V_7 = V - (V_1 + V_2 + V_8) = 10V - 2.6V \times 3 = 2.2V .$$

Therefore the energy dissipated is given by

$$P_7 = \frac{E_7}{t} \rightarrow E_7 = P_7 t = \left(\frac{V_7^2}{R_7}\right) t = \left(\frac{(2.2V)^2}{100\Omega}\right) 60s = 2.9J .$$

c. Suppose that the switch S in the circuit above were left open. In this case the total current and the power produced by the battery would

1. Both the total current and the power developed by the battery would increase.
2. The total current would decrease and the power developed by the battery would increase.
3. Both the total current and the power developed by the battery would remain constant at their original values.
4. The total current would increase and the power developed by the battery would decrease.
5. The total current would decrease and the power developed by the battery would decrease.

d. Suppose that you wanted to build a resistor-capacitor circuit utilizing the equivalent resistance you calculated in part a. You decide to build your air filled capacitor out of two parallel metal plates of area A and separation between the plates d. You calculate the capacitance to be C_o and you charge the capacitor with a battery of potential difference V_o at which point you remove the battery from your capacitor. After you built and charged this capacitor you decide that you want a different value for the capacitance. Instead of rebuilding from scratch a new capacitor you realize that you can get the desired value of the capacitance you want by grabbing one of the capacitor plates and pulling it away from the other plate so that the new separation between the plates is s, where $s > d$. This capacitor has a capacitance C_n. The work you did on the capacitor is given by

1. $W = \dfrac{Q^2}{2\varepsilon_0 A}(s-d)$.
2. $W = \dfrac{Q^2}{2\varepsilon_0 A}(d-s)$.
3. $W = \dfrac{Q^2}{\varepsilon_0 A}(s-d)$.
4. $W = \dfrac{Q^2}{\varepsilon_0 A}(d-s)$.
5. none of the above.

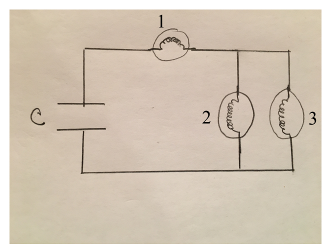
3. Speaking of capacitors, a parallel plate capacitor is constructed out of two circular metal plates of diameter $D = 0.5m$. The plates are separated by $d = 5mm$ and an unknown dielectric material has been inserted between the plates.

   a. If the capacitor were charged by a battery such that a charge of $Q = 4.6\mu C$ were placed on each plate and an eclectic field $E = 27500\frac{N}{C}$ were produced between the plates. What is the value of the dielectric constant of the unknown material placed between the plates of the capacitor?

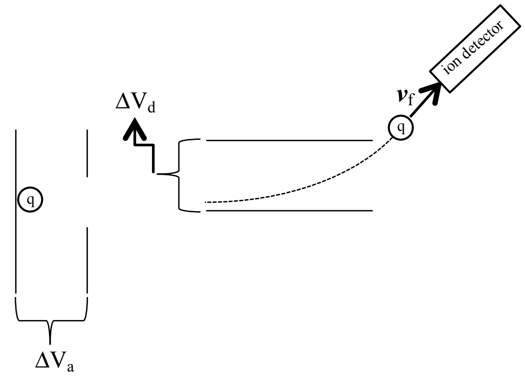
$$Q = CV = \left(\frac{\kappa\varepsilon_0 A}{d}\right)V = \left(\frac{\kappa\varepsilon_0 A}{d}\right)Ed = \kappa\varepsilon_0 AE$$

$$\rightarrow \kappa = \frac{Q}{\varepsilon_0 AE} = \frac{4.6\times10^{-6}C}{8.85\times10^{-12}\frac{C^2}{Nm^2}\times\pi\times(0.25m)^2\times27500\frac{N}{C}} = 96.3$$

   b. Suppose that the fully charged capacitor from part a was connected to the following circuit where the resistances are all identical light bulbs each with resistance *R*. When the capacitor is connected to the circuit, the capacitor begins to discharge through the light bulbs. Which of the following is/are true?



      1. The light bulbs light and each of the light bulbs remain at a constant brightness at all times.
      2. The light bulbs light and the brightness of each of the light bulbs decreases with time.
      3. The light bulbs light and light bulbs 2 and 3 are brighter than light bulb 1.
      4. The light bulbs light and light bulbs 2 and 3 are less bright (dimmer) than light bulb 1.
      5. There is not enough information to determine the brightness of the bulbs and what happens to the brightness over time.



Consider the setup on the right where two sets of capacitors are used. The capacitor on the left is called the accelerator and has a potential difference ΔV_a across the plates while the capacitor on the right is called the deflector and has a potential difference of ΔV_d across its plates. In parts, c and d, the actual values of both ΔV_a and ΔV_d are unknown. A charge $+q$ is accelerated from rest in the accelerator and then enters the deflector. The charge $+q$ is deflected in the y-direction over a distance $\Delta y = y$ by the electric field between the deflector plates and the deflector plates are separated by an amount d, where $d > y$.

c. To detect the accelerated ions from the accelerator, the potential difference across the deflector ΔV_d is changed until a signal is seen in the ion detector. When this signal is seen, it is determined that the y-component of the velocity of the ions is v_{fy}. In terms of q, y, d, v_{fy}, and the mass of the charge m, what is the expression for the potential difference across the deflector and which deflector plate, the upper or lower, is at the higher electric potential?

The work done on the charge in the deflector region is given by $W = -q\Delta V_y = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}m\left(v_{fy}^2 + v_{fx}^2\right) - \frac{1}{2}mv_i^2$, where ΔV_y is the potential difference over which the charge is accelerated in the electric field.

Since there are no forces in the horizontal direction, $v_{fx} = v_{ix} = v_i$. Thus $W = -q\Delta V_y = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}mv_{fy}^2$.

The electric field between the deflector plates is constant so we can relate ΔV_d to ΔV_y through $E = -\dfrac{\Delta V_y}{y} = -\dfrac{\Delta V_d}{d} \rightarrow \Delta V_y = \dfrac{y}{d}\Delta V_d$.

The difference in potential across the deflector plates is therefore

$$W = -q\Delta V_y = -q\left(\frac{y}{d}\Delta V_d\right) = \tfrac{1}{2}mv_{fy}^2 \rightarrow \Delta V_d = -\frac{mdv_{fy}^2}{2qy}.$$

d. Which of the following expressions below give the angle, with respect to the horizontal, at which the ion detector was set?

1. $\tan\theta = \dfrac{y}{d}\left(\dfrac{\Delta V_d}{\Delta V_a}\right)$. 2. $\tan\theta = \dfrac{\Delta V_d}{\Delta V_a}$. 3. $\tan\theta = \sqrt{\dfrac{\Delta V_d}{\Delta V_a}}$. 4. $\tan\theta = \sqrt{\dfrac{y}{d}\left(\dfrac{\Delta V_d}{\Delta V_a}\right)}$.

Physics 111 Equation Sheet

Electric Forces, Fields and Potentials

$$\vec{F} = k\frac{Q_1Q_2}{r^2}\hat{r}$$
$$\vec{E} = \frac{\vec{F}}{q}$$
$$\vec{E}_Q = k\frac{Q}{r^2}\hat{r}$$
$$PE = k\frac{Q_1Q_2}{r}$$
$$V(r) = k\frac{Q}{r}$$
$$E_x = -\frac{\Delta V}{\Delta x}$$
$$W = -q\Delta V = -q\left[V_f - V_i\right]$$

Magnetic Forces and Fields

$$F = qvB\sin\theta$$
$$F = ILB\sin\theta$$
$$\tau = NIAB\sin\theta = \mu B\sin\theta$$
$$PE = -\mu B\cos\theta$$
$$B = \frac{\mu_0 I}{2\pi r}$$
$$\varepsilon_{induced} = -N\frac{\Delta\phi_B}{\Delta t} = -N\frac{\Delta(BA\cos\theta)}{\Delta t}$$

Constants

$g = 9.8\,\frac{m}{s^2}$

$1e = 1.6\times10^{-19}C$

$k = \frac{1}{4\pi\varepsilon_o} = 9\times10^{9}\,\frac{Nm^2}{C^2}$

$\varepsilon_o = 8.85\times10^{-12}\,\frac{C^2}{Nm^2}$

$1eV = 1.6\times10^{-19}J$

$\mu_o = 4\pi\times10^{-7}\,\frac{Tm}{A}$

$c = 3\times10^{8}\,\frac{m}{s}$

$h = 6.63\times10^{-34}Js$

$m_e = 9.11\times10^{-31}kg = \frac{0.511MeV}{c^2}$

$m_p = 1.67\times10^{-27}kg = \frac{937.1MeV}{c^2}$

$m_n = 1.69\times10^{-27}kg = \frac{948.3MeV}{c^2}$

$1amu = 1.66\times10^{-27}kg = \frac{931.5MeV}{c^2}$

$N_A = 6.02\times10^{23}$

$Ax^2 + Bx + C = 0 \rightarrow x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$

Electric Circuits

$$I = \frac{\Delta Q}{\Delta t}$$
$$V = IR = I\left(\frac{\rho L}{A}\right)$$
$$R_{series} = \sum_{i=1}^{N} R_i$$
$$\frac{1}{R_{parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$$
$$P = IV = I^2R = \frac{V^2}{R}$$
$$Q = CV = \left(\frac{\kappa\varepsilon_0 A}{d}\right)V = (\kappa C_0)V$$
$$W = U = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}$$
$$Q_{\text{charge}}(t) = Q_{\max}\left(1 - e^{-\frac{t}{RC}}\right)$$
$$Q_{\text{discharge}}(t) = Q_{\max}e^{-\frac{t}{RC}}$$
$$C_{parallel} = \sum_{i=1}^{N} C_i$$
$$\frac{1}{C_{series}} = \sum_{i=1}^{N}\frac{1}{C_i}$$

Light as a Particle & Relativity

$$E = hf = \frac{hc}{\lambda} = pc$$
$$KE_{\max} = hf - \phi = eV_{stop}$$
$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\phi)$$
$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$p = \gamma mv$$
$$E_{total} = KE + E_{rest} = \gamma mc^2$$
$$E_{total}^2 = p^2c^2 + m^2c^4$$
$$E_{rest} = mc^2$$
$$KE = (\gamma - 1)mc^2$$

Geometry

Circles: $C = 2\pi r = \pi D \qquad A = \pi r^2$

Triangles: $A = \tfrac{1}{2}bh$

Spheres: $A = 4\pi r^2 \qquad V = \tfrac{4}{3}\pi r^3$

Light as a Wave

$$c = f\lambda = \frac{1}{\sqrt{\varepsilon_o\mu_o}}$$
$$S(t) = \frac{energy}{time \times area} = c\varepsilon_o E^2(t) = c\frac{B^2(t)}{\mu_0}$$
$$I = S_{avg} = \tfrac{1}{2}c\varepsilon_o E_{\max}^2 = c\frac{B_{\max}^2}{2\mu_0}$$
$$P = \frac{S}{c} = \frac{Force}{Area}$$
$$S = S_o\cos^2\theta$$
$$v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{n}$$
$$\theta_{inc} = \theta_{refl}$$
$$n_1\sin\theta_1 = n_2\sin\theta_2$$
$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$
$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$
$$M_{total} = \prod_{i=1}^{N} M_i$$
$$d\sin\theta = m\lambda \text{ or } \left(m+\tfrac{1}{2}\right)\lambda$$
$$a\sin\phi = m'\lambda$$

Nuclear Physics

$$E_{binding} = \left(Zm_p + Nm_n - m_{rest}\right)c^2$$
$$\frac{\Delta N}{\Delta t} = -\lambda N_o \rightarrow N(t) = N_o e^{-\lambda t}$$
$$A(t) = A_o e^{-\lambda t}$$
$$m(t) = m_o e^{-\lambda t}$$
$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

Misc. Physics 110 Formulae

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\Delta(m\vec{v})}{\Delta t} = m\vec{a}$$
$$\vec{F} = -k\vec{y}$$
$$\vec{F}_C = m\frac{v^2}{R}\hat{r}$$
$$W = \Delta KE = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right) = -\Delta PE$$
$$PE_{gravity} = mgy$$
$$PE_{spring} = \tfrac{1}{2}ky^2$$
$$x_f = x_i + v_{ix}t + \tfrac{1}{2}a_x t^2$$
$$v_{fx} = v_{ix} + a_x t$$
$$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x$$