Physics 111

Exam #1

February 5, 2021

Name_____

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m |\vec{v}| = (5kg) \times (2\frac{m}{s}) = 10 \frac{kg \cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple-choice questions are worth 3 points and each free-response part is worth 7 points.

| Problem #1 | /24 |
|------------|-----|
| Problem #2 | /24 |
| Problem #3 | /24 |
| Total | /72 |

I affirm that I have carried out my academic endeavors with full academic honesty.

1. Point Charges

Water is a polar molecule composed to two hydrogen ions (H^+) and an oxygen ion (O^{-2}) as shown below.

a. How much work was done to assemble this distribution of charges and how much energy is stored in the system as a potential energy? Assume each charge is brought in separately from very far away and placed in their current positions, where the separation between each hydrogen ion and the oxygen ion is $s = 9.6 \times 10^{-11} m$. Note the distance between the two hydrogen ions may or may not be *s*.

$$W = -\Delta U_e = W_0 + W_H + W_H$$

$$W_0 = 0$$

$$W_H = -(e) \left[\frac{-2ke}{s}\right] = \frac{2ke^2}{s}$$

$$W_H = -(e) \left[\frac{-2ke}{s}\right] - (e) \left[\frac{ke}{2x}\right]$$

$$W_{net} = W_0 + W_H + W_H = \frac{ke^2}{s} \left[4 - \frac{1}{2\sin 52.3}\right]$$

$$= \frac{9 \times 10^{9} \frac{Nm^2}{C^2} (1.6 \times 10^{-19} C)^2}{9.6 \times 10^{-11} m} [3.36]$$

$$W_{net} = 8.1 \times 10^{-18} J = -\Delta U_e$$

$$\Delta U_e = -8.1 \times 10^{-18} J$$

Where, $\sin \theta = \frac{x}{s} \rightarrow x = s \sin \theta = s \sin \frac{104.5}{2} = s \sin 52.3$

b. What is the net electric field at the location of the hydrogen ion on the left due to the oxygen ion and the hydrogen ion on the right?

$$\begin{split} E_{net,x} &= -E_H + E_{0x} = -\frac{ke}{(2x)^2} + \frac{2ke}{s^2} \cos 37.7 = \frac{ke}{s^2} \left[-\frac{1}{(2\sin 52.3)^2} + 2\cos 37.7 \right] \\ E_{net,x} &= \frac{9 \times 10^9 \frac{Nm^2}{C^2} \times 1.6 \times 10^{-19} C}{(9.6 \times 10^{-11} m)^2} [1.18] = 2.5 \times 10^{11} \frac{N}{C} \\ E_{net,y} &= -E_{0y} = -\frac{2ke}{s^2} \sin 37.7 \\ E_{net,y} &= -\frac{9 \times 10^9 \frac{Nm^2}{C^2} \times 1.6 \times 10^{-19} C}{(9.6 \times 10^{-11} m)^2} [0.61] = -1.9 \times 10^{11} \frac{N}{C} \\ E_{net} &= \sqrt{E_{net,x}^2 + E_{net,y}^2} @ \phi = \tan^{-1} \left(\frac{E_{net,y}}{E_{net,x}}\right) \\ E_{net} &= 3.14 \times 10^{11} \frac{N}{C} @ \phi = -37.2^0 \end{split}$$

c. What is the net electric force on the hydrogen ion located on the left due to the oxygen ion and the hydrogen ion on the right?

 $\vec{F}_{net} = q\vec{E}_{net} \rightarrow F_{net} = 1.6 \times 10^{-19} C \times 3.14 \times 10^{11} \frac{N}{C} @ \phi = -37.2^{0}$ $F_{net} = 5.0 \times 10^{-8} N @ \phi = 37.2^{0} \text{ below the positive x-axis in the direction of the}$

electric field.

- d. A point charge (+Q) of mass m hangs vertically at rest at the end of a light insulating rope of length L. Suddenly a uniform electric field \vec{E} is turned on and the point chage starts to move. The point charge comes to rest when the string makes an angle θ with respect to the vertical as shown below. Which of the following gives the magnitude and direction of the uniform electric field?
 - 1.) The magnitude of the electric field is $E = \frac{mg}{\rho} \tan \theta$ and the direction is to the right. 2. The magnitude of the electric field is $E = \frac{m}{o} \tan \theta$ θ and the direction is to the right. 3. The magnitude of the electric field is $E = \frac{mg}{\rho} \tan \theta$

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and the direction is to the left.

- 4. The magnitude of the electric field is $E = \frac{m}{\rho} \tan \theta$ and the direction is to the left.
- 5. None of the above gives the correct magnitude and direction for the electric field.

2. Capacitors

Dielectric constants are often tabulated in books, but first they must be measured. To measure the values of dielectric constants of various materials we construct a parallelplate capacitor and measure the capacitance of the system using a digital multimeter. Suppose that you wanted to determine the dielectric constant for paper, and you construct your capacitor out of two thin parallel copper plates of area $A = 0.060m^2$. Sheets of paper (of the same cross-sectional area as the plates) are placed between the copper plates and the capacitance is measured as a function of the plate separation due to changing the thickness of the stack of paper as shown in the figure below. Data on the capacitance (*C*) versus the number of sheets of paper used are given in the table and the data are plotted on the graph.



a. Based on the data and graph above, what is the dielectric constant for paper?

$$C = \frac{\kappa \varepsilon_0 A}{d} \rightarrow slope = \kappa \varepsilon_0 A \rightarrow \kappa = \frac{slope}{\varepsilon_0 A} = \frac{0.1867 \times 10^{-11} F}{8.85 \times 10^{-12} \frac{C^2}{Nm^2} \times 0.060 m^2} = 3.52$$

b. Suppose that you construct a capacitor using the same copper plates of area $A = 0.060m^2$ and *one* sheet of paper as the dielectric material? You connect your capacitor to a 36V battery and some resistors as shown below. The capacitor is initially uncharged and when switch *S* is closed, the capacitor begins to charge through the resistors in the circuit. What is the maximum current produced by the battery, the time constant that characterizes the circuit, and the expression for the current produced by the battery as a function of time? If you cannot determine the dielectric constant from part a, use $\kappa = 3$.

 80Ω

80Ω

$$1500sheets = 0.136m \rightarrow 1sheet = 9.1 \times 10^{-5}m$$

$$V = 5$$

$$C = \frac{\kappa \varepsilon_0 A}{d} = \frac{3.52 \times 8.85 \times 10^{-12} \frac{C^2}{Nm^2} \times 0.060m^2}{9.1 \times 10^{-5}m} = 2.1 \times 10^{-8}F$$

$$R_{eq} = R + \left(\frac{1}{R} + \frac{1}{R}\right)^{-1} = 1.5R = 1.5 \times 80\Omega = 120\Omega$$

$$I_{max} = \frac{V}{r_{eq}} = \frac{36V}{120\Omega} = 0.3A = 300mA$$

$$\tau = R_{eq}C = 120\Omega \times 2.1 \times 10^{-8}F = 2.52 \times 10^{-6}s = 2.5\mu s$$

$$I(t) = \frac{\Delta Q(t)}{\Delta t} = \frac{Q_{max}e^{-\frac{T}{\tau}}}{t} = I_{max}e^{-\frac{T}{\tau}} = 0.3Ae^{-\frac{t}{2.5\mu s}}$$

c. After the switch has been closed for a long time, how much total charge and energy were stored in the capacitor?

$$Q_{max} = CV_{max} = 2.1 \times 10^{-8} F \times 36V = 7.6 \times 10^{-7} C$$
$$U_e = \frac{1}{2} CV_{max}^2 = \frac{1}{2} \times 2.1 \times 10^{-8} F (36V)^2 = 1.36 \times 10^{-5} J = 13.6 \mu J$$

- d. Suppose that you measure the time it takes for the capacitor used in part c to reach 50% of the maximum potential across the plates and you find that the value you measure is longer than what is predicted by theory? Which of the following would provide an explanation for why the measured value for this time is larger than what is predicted?
 - 1. The effective resistance of the circuit is smaller than what's predicted by theory.
 - 2) The effective capacitance is larger than what's predicted by theory.
 - 3. The effective capacitance is smaller than what's predicted by theory.
 - 4. The time constant of the circuit is always smaller than what's predicted by theory.
 - 5. None of the above adequately explain why the measured and predicted times are different.

3. Electric Circuits

Consider the circuit shown below in which several identical resistors each with resistance $R = 100\Omega$ are connected to a 10V battery, with the exception of resistor R_1 , the value of which is unknown. To determine the value of the unknown resistor several different potential differences were selected from the battery and the total current that was produced by the battery were measured. The data of the battery potential versus the total current are plotted below.

a. What is the value of the unknown resistor R_1 ?

 R_5 and R_6 are in series. $R_{56} = R_5 + R_6 = 100\Omega + 100\Omega = 200\Omega$ R_3 and R_4 are in series. $R_{34} = R_3 + R_4 = 100\Omega + 100\Omega = 200\Omega$

 $\begin{aligned} &R_{34} \text{ and } R_{56} \text{ are in parallel.} \\ &\frac{1}{R_{3456}} = \frac{1}{R_{34}} + \frac{1}{R_{56}} = \frac{1}{200\Omega} + \frac{1}{200\Omega} = \frac{2}{200\Omega} \\ &R_{3456} = 100\Omega \end{aligned}$

 R_2 , R_{3456} , and R_7 are in series $R_{234567} = R_2 + R_{3456} + R_7$ $R_{234567} = 100\Omega + 100\Omega + 100\Omega = 300\Omega$

 R_1 and R_{234567} are in parallel and this combination is the equivalent resistance of the circuit

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_{234567}} \rightarrow \frac{1}{62.2\Omega} = \frac{1}{R_1} + \frac{1}{300\Omega}$$

$$R_1 = 78.5\Omega$$



b. Suppose that the circuit above were wired to a 10V battery. How much total current is produced by the battery and how much energy per unit time is dissipated across resistor R_1 as heat?

$$I_{total} = \frac{V}{R_{eq}} = \frac{10V}{62.2\Omega} = 0.161A = 161mA$$

$$V_{R_1} = V = 10V$$

$$P = \frac{V_{R_1}^2}{R_1} = \frac{(10V)^2}{78.5\Omega} = 1.27W$$

c. What is the current that flows in the branches containing resistors R_4 and R_5 ?

$$\begin{split} &V_{234567} = V = V_{R_2} + V_{R_{3456}} + V_{R_7} \rightarrow V_{R_{3456}} = V - V_{R_2} - V_{R_7} = 10V - 2 \times 3.3V \\ &V_{R_{3456}} = 3.34V \end{split}$$

$$\begin{split} &V_{R_{3456}} = V_{R_{34}} = V_{R_{56}} = 3.34V \\ &V_{R_{34}} = I_{R_{34}}R_{34} \rightarrow I_{R_{34}} = \frac{V_{R_{34}}}{R_{34}} = \frac{3.34V}{200\Omega} = 0.0167A = 16.7mA \\ &V_{R_{56}} = I_{R_{56}}R_{56} \rightarrow I_{R_{56}} = \frac{V_{R_{56}}}{R_{56}} = \frac{3.34V}{200\Omega} = 0.0167A = 16.7mA \\ &Where \ I_{234567} = \frac{V_{R_{234567}}}{R_{234567}} = \frac{10V}{300\Omega} = 0.0333A = 33.3mA \text{ and } V_{R_2} = V_{R_7} = I_{234567}R_2 = 0.033A \times 100\Omega = 3.3V \end{split}$$

- d. Suppose that you have a segment of wire of length L, cross-sectional area A and is made out of a metal with resistivity ρ . The segment of wire is connected to a battery (with potential difference V_A) and a current I_A flows through the wire that is measured with an ammeter. The wire segment is disconnected from the first battery and a second battery (with potential difference $V_B > V_A$) is connected. With this second battery, current I_B flows through the wire segment measured using the same ammeter. Which of the following gives the ratio of the drift velocity $\left(\frac{v_{d,B}}{v_{d,A}}\right)$ of the charge carriers in the wire when battery V_B is connected compared to battery V_A ?
 - 1. $\frac{v_{d,B}}{v_{d,A}} = \frac{I_A}{I_B}.$ 2. $\frac{v_{d,B}}{v_{d,A}} = \frac{I_B}{I_A}.$ 3. $\frac{v_{d,B}}{v_{d,A}} = I_A I_B.$

 - 4. $\frac{v_{d,B}}{v_{d,A}} = 1$, independent of the batteries used and currents produced.
 - 5. None of the above gives the correct ratio of the drift velocities in the wire.

Physics 111 Equation Sheet

R_{series}

Electric Forces, Fields and Potentials

$$\vec{F} = k \frac{Q_1 Q_2}{r^2} \hat{r}$$
$$\vec{E} = \frac{\vec{F}}{q}$$
$$\vec{E}_Q = k \frac{Q}{r^2} \hat{r}$$
$$PE = k \frac{Q_1 Q_2}{r}$$
$$V(r) = k \frac{Q}{r}$$
$$E_x = -\frac{\Delta V}{\Delta x}$$
$$W = -q \Delta V = -q \left[V_f - V_i \right]$$

Magnetic Forces and Fields $F = q v B \sin \theta$ $F = IlB\sin\theta$ $\tau = NIAB\sin\theta = \mu B\sin\theta$ $PE = -\mu B\cos\theta$ $B = \frac{\mu_0 I}{2\pi r}$ $\varepsilon_{in\,d\,u\,ced} = -N \frac{\Delta \phi_B}{\Delta t} = -N \frac{\Delta (BA\cos\theta)}{\Delta t}$

Constants

 $g = 9.8 \frac{m}{s^2}$ $1e = 1.6 \times 10^{-19} C$ $k = \frac{1}{4\rho e_o} = 9 \times 10^9 \frac{Nm^2}{C^2}$ $e_{o} = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$ $1eV = 1.6 \times 10^{-19} J$ $m_{o} = 4p \times 10^{-7} \frac{Tm}{4}$ $c = 3 \times 10^8 \frac{m}{s}$ $h = 6.63 \times 10^{-34} Js$ $m_e = 9.11 \times 10^{-31} kg = \frac{0.511 MeV}{c^2}$ $m_p = 1.67 \times 10^{-27} kg = \frac{937.1 MeV}{c^2}$ $m_n = 1.69 \times 10^{-27} kg = \frac{948.3 MeV}{c^2}$ $1amu = 1.66 \times 10^{-27} kg = \frac{931.5 \, MeV}{c^2}$ $N_A = 6.02 \times 10^{23}$ $Ax^{2} + Bx + C = 0 \rightarrow x = \frac{-B \pm \sqrt{B^{2} - 4AC}}{2A}$

Electric CircuitsLight as a Wave
$$I = \frac{DQ}{Dt} = NeAv_d$$
 $c = f\lambda = \frac{1}{\sqrt{\varepsilon_o \mu_o}}$ $V = IR = I\left(\frac{f'L}{A}\right)$ $S(t) = \frac{energy}{time \times area} = c\varepsilon_o E^2(t) = c\frac{B^2(t)}{\mu_0}$ $R_{series} = \sum_{i=1}^{N} R_i$ $I = S_{avg} = \frac{1}{2} c\varepsilon_o E_{max}^2 = c\frac{B_{max}^2}{2\mu_0}$ $\frac{1}{R_{parallel}} = \sum_{i=1}^{N} \frac{1}{R_i}$ $P = \frac{S}{c} = \frac{Force}{Area}$ $P = IV = I^2 R = \frac{V^2}{R}$ $S = S_o \cos^2 \theta$ $Q = CV = \left(\frac{k\theta_0 A}{d}\right)V = (kC_0)V$ $v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{n}$ $\theta_{inc} = \theta_{refl}$ $n_1 \sin \theta_1 = n_2 \sin \theta_2$ $Q_{charge}(t) = Q_{max}\left(1 - e^{-\frac{t}{RC}}\right)$ $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$ $Q_{discharge}(t) = Q_{max}e^{-\frac{t}{RC}}$ $M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$ $C_{parallel} = \sum_{i=1}^{N} C_i$ $d \sin \theta = m\lambda$ or $(m + \frac{1}{2})\lambda$ $a \sin \phi = m'\lambda$ $d \sin \phi = m'\lambda$

Light as a Particle & Relativity

$$E = hf = \frac{hc}{\lambda} = pc$$

$$KE_{max} = hf - \phi = eV_{stop}$$

$$\Delta \lambda = \frac{h}{m_e c} (1 - \cos \phi)$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$p = \gamma mv$$

$$E_{total} = KE + E_{rest} = \gamma mc^2$$

$$E_{total}^2 = p^2 c^2 + m^2 c^4$$

$$E_{rest} = mc^2$$

$$KE = (\gamma - 1)mc^2$$

Geometry

Circles: $C = 2\rho r = \rho D$ $A = \rho r^2$ *Triangles*: $A = \frac{1}{2}bh$ Spheres: $A = 4\rho r^2$ $V = \frac{4}{3}\rho r^3$

Nuclear Physics

$$E_{binding} = (Zm_p + Nm_n - m_{nst})c^2$$

$$\frac{\Delta N}{\Delta t} = -\lambda N_o \rightarrow N(t) = N_o e^{-\lambda t}$$

$$A(t) = A_o e^{-\lambda t}$$

$$m(t) = m_o e^{-\lambda t}$$

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

Misc. Physics 110 Formulae

$$\vec{F} = \frac{\Delta \vec{p}}{\Delta t} = \frac{\Delta (mv)}{\Delta t} = m\vec{a}$$

$$\vec{F} = -k\vec{y}$$

$$\vec{F}_{c} = m\frac{v^{2}}{R}\hat{r}$$

$$W = \Delta KE = \frac{1}{2}m(v_{f}^{2} - v_{i}^{2}) = -\Delta PE$$

$$PE_{gravity} = mgy$$

$$PE_{spring} = \frac{1}{2}ky^{2}$$

$$x_{f} = x_{i} + v_{ix}t + \frac{1}{2}a_{x}t^{2}$$

$$v_{fx} = v_{ix} + a_{x}t$$

$$v_{fx}^{2} = v_{ix}^{2} + 2a_{x}\Delta x$$