Physics 111

Exam #1

January 28, 2022

Name____________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $\left|\vec{p}\right| \approx m\left|\vec{v}\right| = (5kg)\times\left(2\,\frac{m}{s}\right) = 10\,\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 6 points.

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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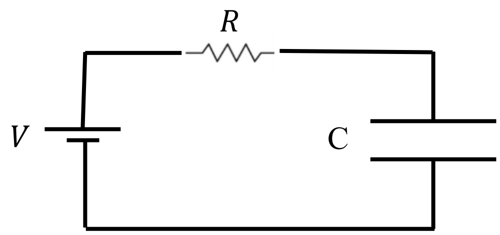
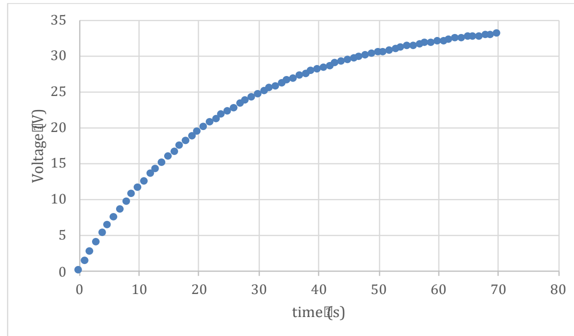
1. A $50k\Omega$ resistor is connected to an initially uncharged capacitor of capacitance C and the combination wired in series to a battery. At a time $t_i = 0s$, the battery is connected to the circuit, and the capacitor begins to charge through the resistor. Data are taken on the potential (in Volts) across the capacitor as a function of time (in seconds) and are shown in the graph below.

$$V(t) = 35(1 - e^{-0.04t})$$





a. What is the capacitance of the capacitor and how much energy is stored in the capacitor when it is fully charged?

$$\tau = RC = \frac{1}{0.04s^{-1}} \rightarrow C = \frac{\tau}{R} = \frac{1}{0.04s^{-1} \times 50000\Omega} = 5 \times 10^{-4}F$$

$$U_e = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(5 \times 10^{-4}F)(35V)^2 = 0.3063J = 306.3mJ$$

b. What is the potential difference across the resistor at a time $t_f = 10s$? Hint: For best results, use the equation provided and not the graph.

$$V = V_R + V_C \rightarrow V_R = V - V_C = 35V - 35V(1 - e^{-0.04t})$$
$$V_R = 35V - 35V\left(1 - e^{-0.04s^{-1} \times 10s}\right) = 23.5V$$

c. When the capacitor is fully charged, how much charge Q is on a plate and what is the magnitude of the electric field between the plates if the plates have an effective area $0.5m^2$?

$$Q = CV = 5 \times 10^{-5}F \times 35V = 0.0175C = 17.5mC$$

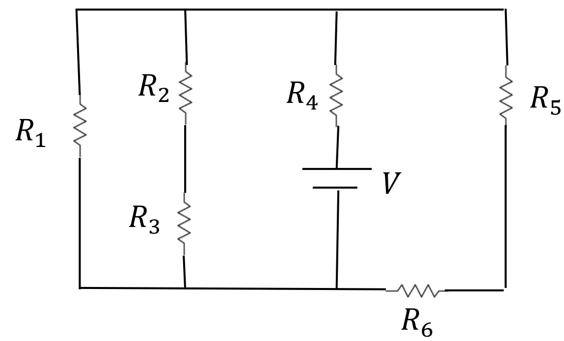
$$E = \frac{Q}{\epsilon_0 A} = \frac{0.0175C}{8.85 \times 10^{-12}\frac{C^2}{Nm^2} \times 0.5m^2} = 3.96 \times 10^9 c$$

d. Suppose that when the capacitor is fully charged, the battery is removed. The resistor, connected in series to the initially fully charged capacitor, is allowed to discharge through the resistor. As current flows through the resistor, energy is dissiapted across the resistor as heat. What is the expression for the energy dissipated per second across the resistor as heat?

$$P = \frac{V^2}{R} = \frac{\left(V_{max}e^{-\frac{t}{RC}}\right)^2}{R} = \frac{V_{max}^2 e^{-\frac{2t}{RC}}}{R} = \frac{(35V)^2}{50000\Omega}e^{-2\times 0.04t}$$

$$P = \frac{V_{max}^2 e^{-\frac{2t}{RC}}}{R} = 0.025We^{-0.08t}$$

2. Suppose that you have the circuit shown on the right in which some identical resistors ($R = 100\Omega$) are connected to a $V = 10V$ battery.



a. What is the equivalent resistance of the circuit and the total current produced by the battery?

R_2 and R_2 in series $R_{23} = R_2 + R_3 = 100\Omega + 100\Omega = 200\Omega$.

R_5 and R_6 in series $R_{56} = R_5 + R_6 = 100\Omega + 100\Omega = 200\Omega$.

R_1, R_{23}, and R_{56} in parallel $\frac{1}{R_{12356}} = \frac{1}{R_1} + \frac{1}{R_{23}} + \frac{1}{R_{56}} = \frac{1}{100\Omega} + \frac{1}{200\Omega} + \frac{1}{200\Omega}$.
$R_{12356} = 50\Omega$

R_4 and R_{12356} in series $R_{123456} = R_{eq} = R_4 + R_{12356} = 100\Omega + 50\Omega = 150\Omega$.

$I_t = \frac{V}{R_{eq}} = \frac{10V}{150\Omega} = 0.0714A = 71.4mA$

b. What is the current in resistor R_1?

$V_{R4} = I_T R_4 = 0.0667A \times 100\Omega = 6.67V$
$V_{R123} = V_{R1} = V - V_{R4} = 10V - 6.67V = 3.33V$

$I_{R_1} = \frac{V_{R1}}{R_1} = \frac{3.33V}{100\Omega} = 0.033A = 33mA$

c. Suppose that the resistors were constructed out of gold wire ($\rho = 2.2 \times 10^{-8} \Omega m$) with a circular cross-section (diameter $d = 0.25mm$). What is the electric field (magnitude and direction) in the resistor R_1?

$$R = \frac{\rho L}{A} \rightarrow L = \frac{RA}{\rho} = \frac{100\Omega \times \pi \left(0.125 \times 10^{-3} m\right)^2}{2.2 \times 10^{-8} \Omega m} = 223.1m$$

$E = \left|-\frac{\Delta V}{\Delta x}\right| = \frac{3.33V}{223.1m} = 0.0149\frac{V}{m}$ in the direction of the conventional current.

d. What is the drift velocity of the charge carriers in the gold wire resistor R_1? Assume that the density of gold is $\rho_{Au} = 19300\frac{kg}{m^3}$, the molar mass $197\frac{g}{mol}$, and that gold donates 2 charge carriers per gold atom.

$$I = neAv_d \rightarrow v_d = \frac{I}{neA}$$

$$n = \frac{\rho_{Au} N \rho_A}{m_{Au}} \times 2 = \frac{19300\frac{kg}{m^3} \times 6.02 \times 10^{23} \frac{Au\ atoms}{mol}}{0.197\frac{kg}{mol}} \times 2\frac{charge\ carriers}{Au\ atom} = 1.18 \times 10^{29}$$

$$v_d = \frac{0.033A}{1.18 \times 10^{29} \times \pi (0.125 \times 10^{-3} m)^2} = 3.6 \times 10^{-5}\frac{m}{s} = 3.6\frac{\mu m}{s}$$

3. A calcium ion (${}^{40}_{20}Ca^{+2}$) is accelerated from rest through a potential difference ΔV_{acc}. The calcium ion starts off very far from an Einsteinium nucleus comes to rest when it just "touches" the nucleus of Einsteinium nucleus (${}^{252}_{99}Es$).

   a. Through what potential difference, ΔV_{acc}, were the calcium ions accelerated? Assume that the nuclear radii of ${}^{40}_{20}Ca^{+2}$ and ${}^{252}_{99}Es$ are $3.5 \times 10^{-15}m$ and $6.4 \times 10^{-15}m$ respectively.

$$W = -q\Delta V = -\Delta U_e = -(q_{Ca})\Delta V = -\left[\frac{k(q_{Ca})(99e)}{r_{Ca} + r_{Es}} + \frac{k(q_{Ca})(99e)}{r_i}\right]$$

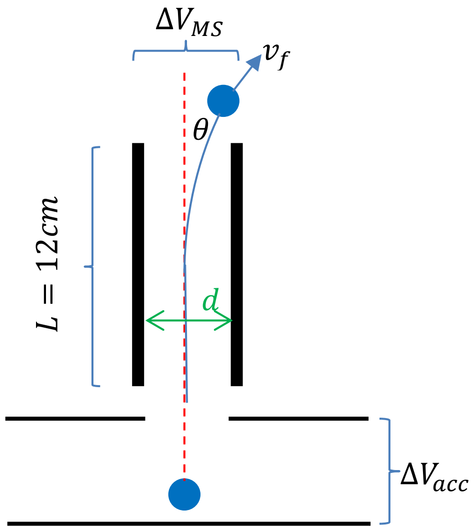
$$\Delta V = \frac{99ke}{r_{Ca} + r_{Es}} = \frac{99 \times 9 \times 10^9 \frac{Nm^2}{C^2} \times 1.6 \times 10^{-19}C}{3.5 \times 10^{-15}m + 6.4 \times 10^{-15}m} = 1.44 \times 10^7 V$$

   b. What is the speed of the calcium ions after they were accelerated through a potential difference ΔV_{acc}?

$$W = -q\Delta V = \Delta K = \tfrac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{-2q\Delta V}{m}}$$

$$v = \sqrt{\frac{-2q\Delta V}{m}} = \sqrt{\frac{-2 \times 2 \times 1.6 \times 10^{-19}C \times (-1.44 \times 10^7 V)}{40 \times 1.66 \times 10^{-27}kg}} = 1.18 \times 10^7 \frac{m}{s}$$

c. In general, any sample of ions that you accelerate is not usually pure, meaning there are a mixture of ions rather than a single species of ion. A mass spectrometer, MS, is a device for separating mixtures of ions based on their changes and masses and is used to filter the calcium from the other ion species in the accelerated beam. What is the deflection angle θ, of a calcium ion, if the potential difference across the mass spectrometer region is $\Delta V_{MS} = -5000V$? Assume that the spacing between the mass spectrometer plates is $d = 1.0cm$.



$$F = qE = ma_y \rightarrow a_y = a = \frac{qE}{m} = \frac{2eE}{m}$$

$$E = -\frac{\Delta V}{\Delta x} = -\frac{(-5000V)}{0.01m} = 5 \times 10^5 \frac{V}{m}$$

$$v_{iy} = \frac{\Delta y}{\Delta t} = \frac{L}{t} \rightarrow t = \frac{L}{v_{iy}} = \frac{0.12m}{1.18\times10^7 \frac{m}{s}} = 1.02 \times 10^{-8} s$$

$$v_{fx} = v_{ix} + a_x t = a_x t = \frac{2eE}{m} t$$

$$v_{fx} = \frac{2\times1.6\times10^{-19}C\times5\times10^5\frac{V}{m}}{40\times1.66\times10^{-27}kg} \times 1.02 \times 10^{-8}s = 2.5 \times 10^4 \frac{m}{s}$$

$$v_{fy} = v_{iy} + a_y t = v_{iy} = 1.18 \times 10^7 \frac{m}{s}$$

$$\tan\theta = \frac{v_{fx}}{v_{fy}} \rightarrow \theta = \tan^{-1}\frac{v_{fx}}{v_{fy}} = \tan^{-1}\frac{2.5\times10^4\frac{m}{s}}{1.18\times10^7\frac{m}{s}} = 0.12^0$$

d. What is the horizontal displacement of a calcium ion after it leaves the mass spectrometer?

$$x_f = x_i + v_{ix} t + \frac{1}{2} a_x t^2 = \frac{2eE}{m} t^2$$

$$x_f = \frac{2 \times 1.6 \times 10^{-19} C \times 5 \times 15^5 \frac{V}{m}}{40 \times 1.66 \times 10^{-27} kg} \times (1.02 \times 10^{-8} s)^2 = 0.0005m$$

Physics 111 Formula Sheet

Electrostatics

$F = k\frac{q_1 q_2}{r^2}$

$\vec{F} = q\vec{E}$; $E_{pc} = k\frac{q}{r^2}$; $E_{plate} = \frac{q}{\epsilon_0 A}$

$E = -\frac{\Delta V}{\Delta x}$

$V = k\frac{q}{r}$

$U_e = k\frac{q_1 q_2}{r} = qV$

$W = -q\Delta V = -\Delta U_e = \Delta K$

Magnetism

$\vec{F} = q\vec{v} \times \vec{B} \rightarrow F = qvB\sin\theta$

$\vec{F} = I\vec{L} \times \vec{B} \rightarrow F = ILB\sin\theta$

$B = \frac{\mu_0 I}{2\pi r}$

$\varepsilon = \Delta V = -N\frac{\Delta\phi_B}{\Delta t}$

$\phi_B = BA\cos\theta$

Electric Circuits - Capacitors

$Q = CV$; $C = \frac{\kappa\epsilon_0 A}{d}$

$C_{parallel} = \sum_{i=1}^{N} C_i$

$\frac{1}{C_{series}} = \sum_{i=1}^{N}\frac{1}{C_i}$

$Q_{charging}(t) = Q_{max}\left(1 - e^{-\frac{t}{\tau}}\right)$

$Q_{discharging}(t) = Q_{max}e^{-\frac{t}{\tau}}$

$I(t) = I_{max}e^{-\frac{t}{\tau}} = \frac{Q_{max}}{\tau}e^{-\frac{t}{\tau}}$

$\tau = RC$

$U_C = \frac{1}{2}qV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$

Electric Circuits - Resistors

$I = \frac{\Delta Q}{\Delta t}$

$I = neAv_d$; $n = \frac{\rho N_A}{m}$

$V = IR$

$R = \frac{\rho L}{A}$

$R_{series} = \sum_{i=1}^{N} R_i$

$\frac{1}{R_{parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$

$P = \frac{\Delta E}{\Delta t} = IV = I^2 R = \frac{V^2}{R}$

Light as a Wave

$c = f\lambda$

$S(t) = \frac{\text{Energy}}{\text{time}\times\text{Area}} = c\epsilon_0 E^2(t) = c\frac{B^2(t)}{\mu_0}$

$I = S_{avg} = \frac{1}{2}c\epsilon_0 E_{max}^2 = c\frac{B_{max}^2}{2\mu_0}$

$P = \begin{cases} \frac{S}{c}; & \text{absorbed} \\ \frac{2S}{c}; & \text{reflected} \end{cases}$

$S = S_0\cos^2\theta$

$v = \frac{c}{n}$

$\theta_{\text{incident}} = \theta_{\text{reflected}}$

$n_1\sin\theta_1 = n_2\sin\theta_2$

$P = \frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i}$

$M = -\frac{d_i}{d_0}$; $|M| = \frac{h_i}{h_0}$

Light as a Particle/Relativity

$E = hf = \frac{hc}{\lambda}$

$K_{max} = hf - \phi$

$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$

$\frac{1}{E'} = \frac{1}{E} + \frac{(1-\cos\phi)}{E_{rest}}$; $E_{rest} = mc^2$

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$p = \gamma mv$

$E_{total} = E_{rest} + K = \gamma mc^2$

$K = (\gamma - 1)mc^2$

$E_{total}^2 = p^2c^2 + m^2c^4$

Nuclear Physics

$N = N_0 e^{-\lambda t}$

$m = m_0 e^{-\lambda t}$

$A = A_0 e^{-\lambda t}$

$A = \lambda N$

$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

Constants

$g = 9.8\frac{m}{s^2}$

$1e = 1.6 \times 10^{-19} C$

$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$

$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$

$1eV = 1.6 \times 10^{-19} J$

$\mu_0 = 4\pi \times 10^{-7} \frac{Tm}{A}$

$c = 3 \times 10^8 \frac{m}{s}$

$h = 6.63 \times 10^{-34} J \cdot s = 4.14 \times 10^{-15} eV \cdot s$

$N_A = 6.02 \times 10^{23}$

$1u = 1.66 \times 10^{-27} kg = 931.5 \frac{MeV}{c^2}$

$m_p = 1.67 \times 10^{-27} kg = 937.1 \frac{MeV}{c^2}$

$m_n = 1.69 \times 10^{-27} kg = 948.3 \frac{MeV}{c^2}$

$m_e = 9.11 \times 10^{-31} kg = 0.511 \frac{MeV}{c^2}$

Physics 110 Formulas

$\vec{F} = m\vec{a}; \quad F_G = \frac{GM_1 m_2}{r^2}; \quad F_s = -ky; \quad a_c = \frac{v^2}{r}$

$W = -\Delta U_g - \Delta U_s = \Delta K$

$U_g = mgy$

$U_s = \frac{1}{2} k y^2$

$K = \frac{1}{2} m v^2$

$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a} t^2$

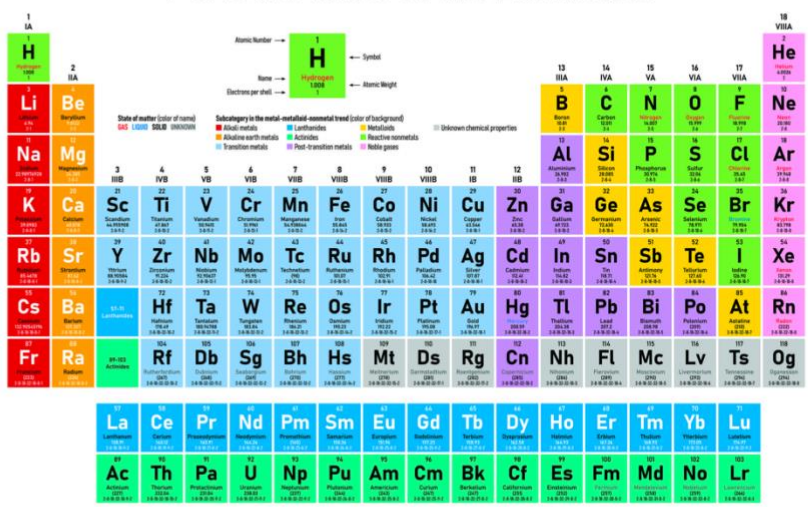
$\vec{v}_f = \vec{v}_i + \vec{a} t$

$v_f^2 = v_i^2 + 2a_r \Delta r$

Geometry/Algebra

Circles:	$A = \pi r^2$	$C = 2\pi r = \pi$
Spheres:	$A = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$
Triangles:	$A = \frac{1}{2} bh$	
Quadratics:	$ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	

Periodic Table of the Elements



https://www.wuwm.com/post/periodic-table-elements-turns-150#stream/0