## Physics 111

Exam \#1

January 20, 2023

Name $\qquad$

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}|=(5 \mathrm{~kg}) \times\left(2 \frac{\mathrm{~m}}{\mathrm{~s}}\right)=10 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 6 points.

| Problem \#1 | $/ 24$ |
| :---: | :---: |
| Problem \#2 | $/ 24$ |
| Problem \#3 | $/ 24$ |
| Total | $/ 72$ |

I affirm that I have carried out my academic endeavors with full academic honesty.

1. Two equal and opposite point-charges are placed at equal distances above and below the x-axis. The point charges have magnitude $|q|=e$ and are separated by a distance $d=2 \mathrm{~nm}$ between their centers as shown on the right.
a. How much work (in electron volts) did it take to assemble this collection of point charges? Assume that each point-charge was brought in one at a time from very far away and placed at their final locations.

$W_{\text {net }}=W_{+q}+W_{-q}$
$W_{+q}=0$
$W_{-q}=-q \Delta V=-(-e)\left[\frac{k e}{d}-0\right]=\frac{k e^{2}}{d}=\frac{9 \times 10^{9} \frac{N m^{2}}{C^{2}}\left(1.6 \times 10^{-19} \mathrm{C}\right)^{2}}{2 \times 10^{-9} \mathrm{~m}} \times \frac{1 \mathrm{eV}}{1.6 \times 10^{-19} \mathrm{~J}}$
$W_{-q}=0.72 \mathrm{eV}$
$W_{\text {net }}=0 \mathrm{eV}+0.72 \mathrm{eV}=0.72 \mathrm{eV}$
b. At a point $P=(x, y)=(10,0) n m$, what is the net electric field?

$$
\begin{aligned}
& E_{P x}=E_{+q x}-E_{-q x}=\frac{k q}{r^{2}} \cos \theta-\frac{k q}{r^{2}} \cos \theta=0 \\
& E_{P y}=-E_{+q y}-E_{-q x}=-\frac{k q}{r^{2}} \sin \theta-\frac{k q}{r^{2}} \sin \theta=-\frac{2 k q}{r^{2}} \sin \theta \\
& E_{P y}=-\frac{2 k q}{r^{2}} \sin \theta=-\frac{2 \times 9 \times 10^{9} \frac{N m^{2}}{C^{2}} \times 1.6 \times 10^{-19} C}{1.01 \times 10^{-16} \mathrm{~m}^{2}} \times 0.1=-2.9 \times 10^{6} \frac{\mathrm{~N}}{\mathrm{C}}
\end{aligned}
$$

where, $r^{2}=\left(\frac{d}{2}\right)^{2}+x^{2}=\left(1 \times 10^{-9} \mathrm{~m}\right)^{2}+\left(10 \times 10^{-9} \mathrm{~m}\right)^{2}=1.01 \times 10^{-16} \mathrm{~m}^{2}$
and $\sin \theta=\frac{d}{2 r}=\frac{2 \times 10^{-9} \mathrm{~m}}{2 \times 1.01 \times 10^{-9} \mathrm{~m}}=0.1$
So, the net electric field points in the negative $y$-direction and has a magnitude
$2.9 \times 10^{6} \frac{N}{c}$.
c. Suppose that a point-charge $q=-3 e$ was placed at point $P$. If the charge had a mass $m=14 u$, where $1 u=1.66 \times 10^{-27} \mathrm{~kg}$, what would be the initial acceleration of the charge?

Since $q=-3 e$ is negative, it will accelerate opposite to the direction of the electric field. Thus, the direction of the acceleration will be in the positive ydirection.

The magnitude of the acceleration is given by Newton's $2^{\text {nd }}$ law:

$$
F=q E=m a \rightarrow a=\frac{q E}{m}=\frac{3 \times 1.6 \times 10^{-19} \mathrm{C} \times 2.9 \times 10^{6} \frac{\mathrm{~N}}{\mathrm{C}}}{14 u \times \frac{1.66 \times 10^{-27} \mathrm{~kg}}{1 u}}=5.95 \times 10^{13} \frac{\mathrm{~m}}{\mathrm{~s}^{2}}
$$

d. Suppose that you have the situation below in which two positive point charges $(+e)$ are located a distance $d$ apart and are also located equidistant above and below the $x$-axis. A small negative charge $(-e)$ is placed at the midpoint between the two positive point charges and is given a small kick to the right directly along the x -axis. In as much detail as possible, describe the resulting motion of the negative charge. The two positive point charges cannot move.

Since the charge that's being kicked is negative, when kicked to the right will interact with the net electric field from the positive charges, along the x -axis which points in the positive x direction and will feel a force opposite to the electric field. This will cause the negative charge to slow down, come to rest, and then accelerate in the negative x -direction. As the negative charge passes the
 origin heading left, it will again interact with the net electric field (from the positive charges) which will now point in the negative x-direction. The negative charge will feel a force opposite to the electric field and will slow down, come to rest, and then move in the positive $x$-direction. This motion will continue, and the negative charge will oscillate about the y -axis along the x -axis.
2. Ion beams generated by particle accelerators are routinely used in materials analysis, where the composition of an unknown material needs to be determined. Suppose that you have the accelerator shown below in which an alpha particle (a helium nucleus ${ }_{2}^{4} \mathrm{He}$ ) is accelerated from rest near the left plate and that the alpha particle will eventually exit through the hole in the right plate. A potential difference $\Delta V=70 \mathrm{~V}$ exists across the plates and the plates are separated by a distance $d=2 m$.

a. When the alpha particle exits through the hole on the right plate, what will be its speed?

$$
\begin{aligned}
W & =-q \Delta V=\Delta K=\frac{1}{2} m v_{f}^{2}-\frac{1}{2} m v_{i}^{2}=\frac{1}{2} m v_{f}^{2} \\
v_{f} & =\sqrt{\frac{2 W}{m}}=\sqrt{\frac{2 \times\left(-2 \times 1.6 \times 10^{-19} \mathrm{C}\right) \times(0 \mathrm{~V}-70 \mathrm{~V})}{4 \times 1.67 \times 10^{-27} \mathrm{~kg}}} \\
v_{f} & =8.37 \times 10^{\frac{4}{s}} \frac{m}{s}
\end{aligned}
$$

b. Suppose that after the alpha particle exits the hole in the right plate with speed $v_{i}$ determined from part a, it passes between a second set of parallel plates shown below. The plates are square with sides of length $L=15 \mathrm{~cm}$ and are separated by a distance $d=8 \mathrm{~cm}$. An 80 V battery across theses plates was used to create an electric field between the plates. What was the magnitude and direction of the electric field between the plates and what plate (the upper or lower plate) is at the higher electric potential? Be sure to explain your choice for the higher potential plate.


Since the alpha particle has a positive charge and its motion is up the plane of the page, the electric field must point up the page from the lower plate to the upper plate so that the net force on the alpha particle is up. The electric field points along decreasing electric potentials, so the lower plate must be at the higher electric potential.

The magnitude (and direction) of the electric field is given by $E=-\frac{\Delta V}{\Delta y}=$ $-\frac{(0 \mathrm{~V}-80 \mathrm{~V})}{0.08 \mathrm{~m}}=1000 \frac{\mathrm{~V}}{\mathrm{~m}}$ up.
c. What is the magnitude and direction of the final velocity of the alpha particle when it leaves the plates on the right?

The x-component of the final velocity is a constant of the motion since the only force that acts on the alpha particle is due to the electric field between the plates and points in the y-direction. Thus, $v_{f x}=v_{i x}=8.4 \times 10^{4} \frac{\mathrm{~m}}{\mathrm{~s}}$.

The y -component of the final velocity is determined by $v_{f y}=v_{i y}+a_{y} t=a_{y} t$.
The time to cross the plates of length $L$ is given by

$$
x_{f}=x_{i}+v_{i x} t+\frac{1}{2} a_{x} t^{2}=v_{i x} t \rightarrow t=\frac{L}{v_{i x}}=\frac{0.15}{8.37 \times 10^{4} \frac{m}{s}}=1.8 \times 10^{-6} s .
$$

The acceleration of the alpha particle in the electric field is given by $F_{y}=q E=m a \rightarrow a=\frac{2 e E}{m}=\frac{2 \times 1.6 \times 10^{-19} \mathrm{C} \times 1000 \frac{N}{C}}{4 \times 1.67 \times 10^{-27} \mathrm{~kg}}=4.8 \times 10^{10} \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$

The vinal y-component of the velocity is $v_{f y}=a_{y} t=4.8 \times 10^{10} \frac{\mathrm{~m}}{s^{2}} \times 1.8 \times$ $10^{-6} s=8.6 \times 10^{4} \frac{m}{s}$

The magnitude and direction of the alpha's final velocity is given by
$v_{f}=\sqrt{v_{f x}^{2}+v_{f y}^{2}}=\sqrt{\left(8.4 \times 10^{4} \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}+\left(8.6 \times 10^{4} \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}}=1.2 \times 10^{5} \frac{\mathrm{~m}}{\mathrm{~s}}$
$\tan \phi=\frac{v_{f y}}{v_{f x}} \rightarrow \phi=\tan ^{-1} \frac{v_{f y}}{v_{f x}}=\tan ^{-1} \frac{8.6 \times 10^{4} \frac{\mathrm{~m}}{s}}{8.4 \times 10^{4} \frac{\mathrm{~m}}{s}}=45.7^{0}$
d. The alpha particles that emerge from the system in part c are incident on atoms of calcium. How close to the nucleus of a calcium atom does the alpha particle come? Hints: Assume that the proton starts very far away from a calcium nucleus and that calcium has 20 protons in its nucleus.

The calcium nucleus can be treated like a point charge so that the electric potential is given by $V=\frac{k Q}{r}$. As the alpha particle (with its positive charge) approaches the calcium nucleus it interacts with the electric field from the calcium. The electric field of the calcium does work on the alpha particle. The work done is

$$
\begin{aligned}
& W=-q \Delta V=\Delta K \rightarrow-(2 e)\left[\frac{k(20 e)}{r_{f}}-\frac{k(20 e)}{r_{i}}\right]=\frac{1}{2} m v_{f}^{2}-\frac{1}{2} m v_{i}^{2} \\
& \rightarrow \frac{40 k e^{2}}{r_{f}}=\frac{1}{2} m v_{i}^{2} \rightarrow r_{f}=\frac{80 k e^{2}}{m v_{i}^{2}}=\frac{80 \times 9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{C^{2}}\left(1.6 \times 10^{-19} \mathrm{C}\right)^{2}}{4 \times 1.67 \times 10^{-27} \mathrm{~kg}\left(1.2 \times 10^{5} \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}} \\
& r_{f}=1.9 \times 10^{-10} \mathrm{~m}=0.19 \mathrm{~nm}
\end{aligned}
$$

3. A $m=1 g$ point charge is suspended at the end of an insulating cord of length $L=$ 55 cm . An external electric field is turned on and the point charge is observed to be in equilibrium this uniform horizontal electric field of magnitude $|\vec{E}|=15000_{\bar{C}}^{N}$ when the pendulum's position is 12 cm above its lowest vertical position.

a. What is the magnitude of the tension force in the cord in this configuration?

In the y-direction: $0=F_{T} \cos \theta-m g \rightarrow F_{T}=\frac{m g}{\cos \theta}=\frac{0.001 \mathrm{~kg} \times 9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}}{\cos 38.6}=0.0125 \mathrm{~N}$
b. What is the magnitude and sign of the point charge on the end of the cord? Be sure to explain fully your choice for the sign of the point charge and why you chose this as the sign.

In the x-direction: $0=-F_{T} \sin \theta+q E \rightarrow q=\frac{F_{T} \sin \theta}{E}=\frac{0.0125 \mathrm{~N} \sin 38.6}{15000 \frac{N}{C}}=5.2 \times$ $10^{-7} C=0.52 \mu C$

Since the point charge moved opposite to the direction of the electric field from hanging vertical, the point charge must be negative since it felt an electric force in the direction opposite to the electric field.
c. Suppose that the electric field were generated by a set of parallel circular metal plates with diameter $D=75 \mathrm{~cm}$ that have equal and opposite charges on them. If the capacitance of the system was $C=3 p F=3 \times 10^{-12} F$ and the space between the plates is filled with air, how far apart were the plates spaced?
$C=\frac{\kappa \varepsilon_{0} A}{d} \rightarrow d=\frac{\kappa \varepsilon_{0} A}{C}=\frac{1 \times 8.85 \times 10^{-12} \frac{C^{2}}{\mathrm{Nm}^{2}} \times \pi\left(\frac{0.75 \mathrm{~m}}{2}\right)^{2}}{3 \times 10^{-12} F}=1.3 \mathrm{~m}$
d. What was the voltage of the battery that was used to charge these plates?

Method 1: $E=\frac{Q}{\varepsilon_{0} A}=\frac{C V}{\varepsilon_{0} A} \rightarrow V=\frac{\varepsilon_{0} A E}{C}=\frac{8.85 \times 10^{-12} \frac{C^{2}}{N m^{2}} \times \pi\left(\frac{0.75 \mathrm{~m}}{2}\right)^{2} \times 1.5 \times 10^{5} \frac{N}{C}}{3 \times 10^{-12} F}$ $V=1.96 \times 10^{4} V$

Method 2: $E=-\frac{\Delta V}{\Delta x} \rightarrow \Delta V=V=1.5 \times 10^{4} \frac{N}{C} \times 1.3 \mathrm{~m}=1.95 \times 10^{4} V$

Electrostatics
$F=k \frac{q_{1} q_{2}}{r^{2}}$
$\vec{F}=q \vec{E} ; \quad E_{p c}=k \frac{q}{r^{2}} ; \quad E_{\text {plate }}=\frac{q}{\epsilon_{0} A}$
$E=-\frac{\Delta V}{\Delta x}$
$V_{p c}=k \frac{q}{r}$
$U_{e}=k \frac{q_{1} q_{2}}{r}=q V$
$W=-q \Delta V=-\Delta U_{e}=\Delta K$
Electric Circuits - Capacitors
$Q=C V ; \quad C=\frac{\kappa \epsilon_{0} A}{d}$
$C_{\text {parallel }}=\sum_{i=1}^{N} C_{i}$
$\frac{1}{C_{\text {series }}}=\sum_{i=1}^{N} \frac{1}{C_{i}}$
$Q_{\text {charging }}(t)=Q_{\max }\left(1-e^{-\frac{t}{\tau}}\right)$
$Q_{\text {discharging }}(t)=Q_{\max } e^{-\frac{t}{\tau}}$
$I(t)=I_{\max } e^{-\frac{t}{\tau}}=\frac{Q_{\max }}{\tau} e^{-\frac{t}{\tau}}$
$\tau=R C$
$U_{C}=\frac{1}{2} q V=\frac{1}{2} C V^{2}=\frac{Q^{2}}{2 C}$
Light as a Wave
$c=f \lambda$
$S(t)=\frac{\text { Energy }}{\text { time } \times \text { Area }}=c \epsilon_{0} E^{2}(t)=c \frac{B^{2}(t)}{\mu_{0}}$
$I=S_{a v g}=\frac{1}{2} c \epsilon_{0} E_{\text {max }}^{2}=c \frac{B_{\text {max }}^{2}}{2 \mu_{0}}$
$P= \begin{cases}\frac{s}{c} ; & \text { absorbed } \\ \frac{2 S}{c} ; & \text { reflected }\end{cases}$
$S=S_{0} \cos ^{2} \theta$
$v=\frac{c}{n}$
$\theta_{\text {incident }}=\theta_{\text {reflected }}$
$n_{1} \sin \theta_{1}=n_{2} \sin \theta_{2}$
$P=\frac{1}{f}=\frac{1}{d_{0}}+\frac{1}{d_{i}}$
$M=-\frac{d_{i}}{d_{0}} ; \quad|M|=\frac{h_{i}}{h_{0}}$

Magnetism
$\vec{F}=q \vec{v} \times \vec{B} \rightarrow F=q v B \sin \theta$
$\vec{F}=I \vec{L} \times \vec{B} \rightarrow F=I L B \sin \theta$
$V_{\text {Hall }}=w v_{d} B$
$B=\frac{\mu_{0} I}{2 \pi r}$
$\varepsilon=\Delta V=-N \frac{\Delta \phi_{B}}{\Delta t}$
$\phi_{B}=B A \cos \theta$
Electric Circuits - Resistors
$I=\frac{\Delta Q}{\Delta t}$
$I=n e A v_{d} ; \quad n=\frac{\rho N_{A}}{m}$
$V=I R$
$R=\frac{\rho L}{A}$
$R_{\text {series }}=\sum_{i=1}^{N} R_{i}$
$\frac{1}{R_{\text {parallel }}}=\sum_{i=1}^{N} \frac{1}{R_{i}}$
$P=\frac{\Delta E}{\Delta t}=I V=I^{2} R=\frac{V^{2}}{R}$

Light as a Particle/Relativity
$E=h f=\frac{h c}{\lambda}$
$K_{\text {max }}=h f-\phi$
$\Delta \lambda=\lambda^{\prime}-\lambda=\frac{h}{m c}(1-\cos \phi)$
$\frac{1}{E^{\prime}}=\frac{1}{E}+\frac{(1-\cos \phi)}{E_{\text {rest }}} ; \quad E_{\text {rest }}=m c^{2}$
$\gamma=\frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}}$
$p=\gamma m v$
$E_{\text {total }}=E_{\text {rest }}+K=\gamma m c^{2}$
$K=(\gamma-1) m c^{2}$
$E_{\text {total }}^{2}=p^{2} c^{2}+m^{2} c^{4}$

Nuclear Physics

$$
\begin{aligned}
& N=N_{0} e^{-\lambda t} \\
& m=m_{0} e^{-\lambda t} \\
& A=A_{0} e^{-\lambda t} \\
& A=\lambda N \\
& t_{\frac{1}{2}}=\frac{\ln 2}{\lambda}
\end{aligned}
$$

## Constants

$g=9.8 \frac{m}{s^{2}}$
$1 e=1.6 \times 10^{-19} \mathrm{C}$
$k=\frac{1}{4 \pi \epsilon_{0}}=9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{\mathrm{C}^{2}}$
$\epsilon_{0}=8.85 \times 10^{-12} \frac{\mathrm{c}^{2}}{\mathrm{Nm}^{2}}$
$1 \mathrm{eV}=1.6 \times 10^{-19} \mathrm{~J}$
$\mu_{0}=4 \pi \times 10^{-7 \frac{T m}{A}}$
$c=3 \times 10^{8} \frac{\mathrm{~m}}{\mathrm{~s}}$
$h=6.63 \times 10^{-34} \mathrm{Js}=4.14 \times 10^{-15} \mathrm{eVs}$
$N_{A}=6.02 \times 10^{23}$
$1 u=1.66 \times 10^{-27} \mathrm{~kg}=931.5 \frac{\mathrm{MeV}}{\mathrm{c}^{2}}$
$m_{p}=1.67 \times 10^{-27} \mathrm{~kg}=937.1 \frac{\mathrm{MeV}}{\mathrm{c}^{2}}$
$m_{n}=1.69 \times 10^{-27} \mathrm{~kg}=948.3 \frac{\mathrm{MeV}}{\mathrm{c}^{2}}$
$m_{e}=9.11 \times 10^{-31} \mathrm{~kg}=0.511 \frac{\mathrm{MeV}}{\mathrm{c}^{2}}$

Physics 110 Formulas

$$
\begin{aligned}
& \vec{F}=m \vec{a} ; \quad F_{G}=\frac{G M_{1} m_{2}}{r^{2}} ; \quad F_{s}=-k y ; \quad a_{c}=\frac{v^{2}}{r} \\
& W=-\Delta U_{g}-\Delta U_{s}=\Delta K \\
& U_{g}=m g y \\
& U_{s}=\frac{1}{2} k y^{2} \\
& K=\frac{1}{2} m v^{2} \\
& \vec{r}_{f}=\vec{r}_{i}+\vec{v}_{i} t+\frac{1}{2} \vec{a} t^{2} \\
& \vec{v}_{f}=\vec{v}_{i}+\vec{a} t \\
& v_{f}^{2}=v_{i}^{2}+2 a_{r} \Delta r
\end{aligned}
$$

## Common Metric Units

$$
\begin{aligned}
& \text { nano }(n)=10^{-9} \\
& \text { micro }(\mu)=10^{-6} \\
& \operatorname{milli}(m)=10^{-3} \\
& \operatorname{centi}(c)=10^{-2} \\
& \operatorname{kilo}(k)=10^{3} \\
& \operatorname{mega}(M)=10^{6}
\end{aligned}
$$

## Geometry/Algebra

| Circles: | $A=\pi r^{2}$ | $C=2 \pi r=\pi$ |
| :--- | :--- | :--- |
| Spheres: | $A=4 \pi r^{2} \quad V=\frac{4}{3} \pi r^{3}$ |  |
| Triangles: | $A=\frac{1}{2} b h$ |  |
| Quadratics: | $a x^{2}+b x+c=0 \rightarrow x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$ |  |

## PERIODIC TABLE OF ELEMENTS



