Physics 111

Exam #1

January 19, 2024

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}| = (5kg)\times\left(2\,\tfrac{m}{s}\right) = 10\,\tfrac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 6 points.

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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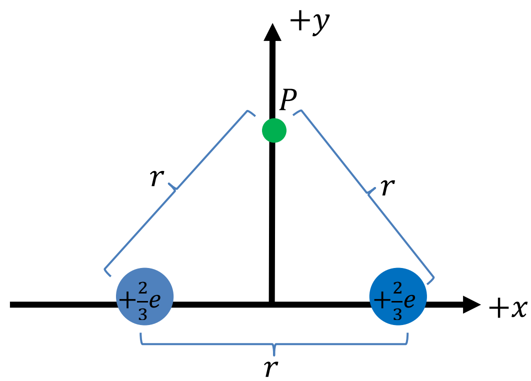
1. A proton is a composite particle made up out of three quarks. There are two "up" quarks each with charge $+\frac{2}{3}e$ and one "down" quark with charge $-\frac{1}{3}e$.



a. Consider the two "up" quarks placed on the x-axis shown below, where the two "up" quarks a are separated by a distance $r = 2 \times 10^{-15}m$. What is the net electric field at a point P, located equidistant from both "up" quarks shown on the diagram by an amount $r = 2 \times 10^{-15}m$?

$$E_x = E_{Lx} - E_{Rx} = E_L \cos\theta - E_R \cos\theta = \frac{kQ_L}{r^2}\cos\theta - \frac{kQ_R}{r^2}\cos\theta = 0$$

$$E_y = E_{Ly} + E_{Ry} = E_L \sin\theta + E_R \sin\theta = \frac{kQ_L}{r^2}\sin\theta + \frac{kQ_R}{r^2}\sin\theta$$

$$E_y = 2\frac{kQ_{up}}{r^2}\sin\theta = \frac{2 \times 9 \times 10^9 \frac{Nm^2}{C^2} \times \frac{2}{3} \times 1.6 \times 10^{-19}C}{(2 \times 10^{-15}m)^2}\sin 60$$

$$E_y = 4.2 \times 10^{20}\frac{N}{C}$$

$E_{net} = \sqrt{E_x^2 + E_y^2}$ @ $\phi = \tan^{-1}\left(\frac{E_y}{E_x}\right) \rightarrow E_{net} = 4.2 \times 10^{20}\frac{N}{C}$ @ $\phi = 90^0$ or in the positive y-direction.

b. Suppose that the "down" quark were place at point P, what would be the net force on the "down" quark?

$F_{up} = Q_{down}E_{net} = \frac{1}{3}e \times E_{net} = \frac{1}{3} \times 1.6 \times 10^{-19}C \times 4.2 \times 10^{20}\frac{N}{C} = 22.2N$ in magnitude and points vertically down in the negative y-direction opposite to the direction of the electric field since the up quark has a negative charge.

c. What is the electrostatic potential energy of the assembled three-quark system?

$$\Delta U_e = \Delta U_{e_{uu}} + \Delta U_{e_{ud}} + \Delta U_{e_{du}} = \frac{kQ_dQ_d}{r} + \frac{kQ_uQ_d}{r} + \frac{kQ_uQ_d}{r}$$
$$\Delta U_e = \frac{ke^2}{r}\left[\left(\frac{2}{3}\times\frac{2}{3}\right) + \left(\frac{2}{3}\times -\frac{1}{3}\right) + \left(-\frac{1}{3}\times\frac{2}{3}\right)\right] = 0$$

Or from the work done to assemble the distribution assuming you could bring in each from very far away and place at their final locations:

$$W_1 = W_u = 0$$
$$W_2 = W_{uu} = -Q_d\left[\frac{kQ_d}{r} - 0\right] = -\frac{4ke^2}{9r}$$
$$W_3 = W_{ud} + W_{ud} = 2W_{ud} = -2Q_u\left[\frac{kQ_d}{r} - 0\right] = \frac{4ke^2}{9r}$$
$$W_{net} = W_1 + W_2 + W_3 = 0 + \left(-\frac{4ke^2}{9r}\right) + \frac{4ke^2}{9r} = 0$$
$$W = -\Delta U_e \rightarrow \Delta U_e = 0$$

d. Instead of the proton (and the three quarks) consider the following situation. A small charge $-q$ is initially located very far away from a large charge $-Q$. The small charge is then thrown at the large charge with a speed v_i. Of the following, which is/are allowed, and which is/are not allowed. Be sure to explain your answer in as much detail as possible.

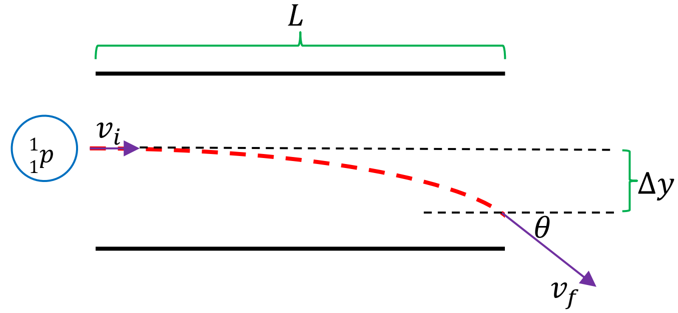
1. $\Delta U_e > 0$ and $\Delta K > 0$. This cannot happen as it would violate conservation of energy. Both the electric potential energy and kinetic energy cannot both increase.

2. $\Delta U_e < 0$ and $\Delta K > 0$. This is possible as the $-q$ moves away from the $-Q$ charge after coming to rest (see #3). $W = -(-q)\left[\frac{k(-Q)}{r_f} - \frac{k(-Q)}{r_i}\right] = +\frac{kqQ}{r_f}$. Since the work done is positive, the change in kinetic energy is positive ($W = \Delta K$) and the change in electric potential energy ($W = -\Delta U_e$) is negative.

3. $\Delta U_e > 0$ and $\Delta K < 0$. This is possible as the $-q$ charge approaches the $-Q$ charge. $W = -(-q)\left[\frac{k(-Q)}{r_f} - \frac{k(-Q)}{r_i}\right] = -\frac{kqQ}{r_f}$. Since the work done is negative, the change in kinetic energy is negative ($W = \Delta K$) and the change in electric potential energy ($W = -\Delta U_e$) is positive.

4. $\Delta U_e < 0$ and $\Delta K < 0$. This cannot happen as it would violate conservation of energy. Both the electric potential energy and kinetic energy cannot both decrease.

2. Two horizontal parallel metal plates $L = 4m$ long are charged using a battery (not shown). A proton (^1_1p) is incident from the left traveling to the right at an initial speed $v_i = 1.5 \times 10^7 \frac{m}{s}$. As the proton passes through the plates, it is deflected downward following the red path as shown and exits at an angle θ below the horizontal with a speed v_f. During the proton's travel through the plates, the proton undergoes a vertical displacement, $\Delta y = -30cm$.



a. How much time does the proton spend traveling between the plates?

$$x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 = v_{ix}t \rightarrow t = \frac{x_f}{v_{ix}} = \frac{4m}{1.5 \times 10^7 \frac{m}{s}} = 2.7 \times 10^{-7} s$$

b. What is the magnitude and direction of the assumed constant electric field needed to produce this motion of the proton?

$$y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 = \frac{1}{2}\left(\frac{F_e}{m}\right)t^2 = \frac{1}{2}\left(\frac{eE}{m}\right)t^2 \rightarrow E = \frac{2my_f}{et^2}$$

$$E = \frac{2 \times 1.67 \times 10^{-27} kg \times (-0.3m)}{1.6 \times 10^{-19} C \times (2.7 \times 10^{-7} s)^2} = -8.6 \times 10^4 \frac{N}{C}$$

The magnitude of the electric field is $8.6 \times 10^4 \frac{N}{C}$ and the direction is negative y.

c. What is the magnitude (v_f) and direction (θ) of the exit velocity of the proton?

$$v_{fx} = v_{ix} + a_x t = v_{ix} = 1.5 \times 10^7 \frac{m}{s}$$

$$v_{fy} = v_{iy} + a_y t = -\frac{eE}{m} t = -\frac{1.6 \times 10^{-19} C \times 8.6 \times 10^4 \frac{N}{C}}{1.67 \times 10^{-27} kg} \times 2.7 \times 10^{-7} s$$

$$v_{fy} = -2.2 \times 10^6 \frac{m}{s}$$

$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{\left(1.5 \times 10^7 \frac{m}{s}\right)^2 + \left(-2.2 \times 10^6 \frac{m}{s}\right)^2} = 1.52 \times 10^7 \frac{m}{s}$$

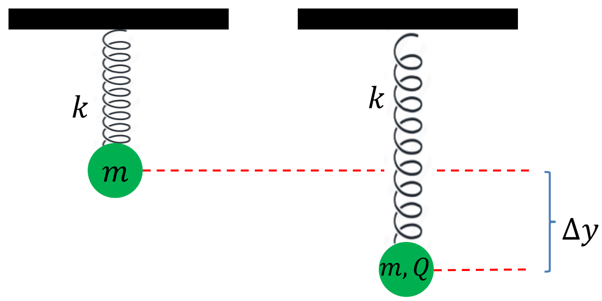
$\phi = \tan^{-1}\left(\frac{v_{fy}}{v_{fx}}\right) = \tan^{-1}\left(\frac{-2.2 \times 10^6 \frac{m}{s}}{1.5 \times 10^7 \frac{m}{s}}\right) = -8.3^0$ or 8.3^0 below the x-axis.

d. In order for the proton to enter the plates from the left at $v_i = 1.5 \times 10^7 \frac{m}{s}$, it must have been accelerated. Through what potential difference was the proton accelerated to acquire the speed $v_i = 1.5 \times 10^7 \frac{m}{s}$? Assume the proton started from rest.

$$W = -q\Delta V = \Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \frac{1}{2} m v_f^2 \rightarrow \Delta V = -\frac{m v_f^2}{2q}$$

$$\Delta V = -\frac{1.67 \times 10^{-27} kg \times \left(1.5 \times 10^7 \frac{m}{s}\right)^2}{2 \times 1.6 \times 10^{-19} C} = -1.2 \times 10^6 V = -1.2 MV$$

3. Suppose that we have a $m = 250g$ mass attached to the spring of stiffness $k = 15\frac{N}{m}$. According to Hooke's law, the spring tries to return to its original unstretched state to oppose the added weight. This gives rise to a force in the spring given by $F = -k\Delta y$ and is produced in the direction opposite to the stretch of the spring. In the figure below on the left, the spring hangs motionless in equilibrium with the mass attached. A charge of magnitude $|Q|$ placed is then placed on the mass. We do not know whether the charge is positive or negative at this moment in time. A constant vertical external electric field E is turned on and the interaction of the charge with the external electric field is observed to stretch the spring from equilibrium by an amount $\Delta y = -30cm$ as shown below in the right figure.



a. If the magnitude of the point charge was $|Q| = 13\mu C$, what is the magnitude of the constant vertical electric field E?

$$F_s - F_W - F_E = ma_y = 0 \rightarrow F_E = F_s - F_W \rightarrow QE = k\Delta y - mg$$

$$E = \frac{k\Delta y - mg}{Q}$$

$$E = \frac{\left(15\frac{N}{m} \times 0.3m\right) - \left(0.25kg \times 9.8\frac{m}{s^2}\right)}{13 \times 10^{-6}C} = 1.58 \times 10^5\frac{N}{C}$$

b. Suppose that by some other experimental method, it was found that the charge was actually $Q = -13\mu C$. What would be the direction of the constant vertical electric field E? Be sure to fully explain your answer. Simply putting a direction with no explanation may not earn you much credit.

Since Q is negative, Q feels a force opposite to the direction of the electric field E. Thus, since Q fell vertically, the electric force is in the negative y-direction, and this makes the electric field point in the positive y-direction. The direction of the constant vertical electric field is up or in the positive y-direction.

c. Suppose that the constant electric field E were generated by a set of parallel square metal plates with sides of length $L = 20cm$ separated by a distance $60cm$. How much charge was placed on a plate of the capacitor? Assume that the capacitor is air-filled. *Note the separation between the plates on the original exam was $25cm$. If the spring stretches by $30cm$, the spacing between the plates cannot be $25cm$. I've changed it to something more reasonable and this did not affect the outcome of the exam.

$$E_{plate} = \frac{\kappa Q}{\varepsilon_0 A} \rightarrow Q = \frac{\varepsilon_0 A E_{plate}}{\kappa}$$

$$Q = \frac{8.85 \times 10^{-12} \frac{C^2}{Nm^2} \times (0.2m \times 0.2m) \times 1.58 \times 10^5 \frac{N}{C}}{1}$$

$$Q = 5.6 \times 10^{-8} C = 56nC = 0.056\mu C$$

d. What was the voltage V of the battery that was used to charge the capacitor plates used to generate the vertical electric field E and what plate, upper or lower, was at the higher potential (voltage)? Be sure to explain your choice for which plate is at the higher voltage.

Since the electric field is vertically upward and points from the positive plate to negative plate along decreasing electric potentials, the lower plate has the positive charge and corresponds to the higher voltage while the upper plate is negatively charged and corresponds to the lower voltage.

$$E = -\frac{\Delta V}{\Delta y} \rightarrow |\Delta V| = E\Delta y = \left(1.58 \times 10^5 \frac{N}{C}\right) \times 0.60m = 9.3 \times 10^4 V$$

Or,

$$Q = CV \rightarrow V = \frac{Q}{C} = \frac{Q}{\kappa\varepsilon_0 A / d} = \frac{Qd}{\kappa\varepsilon_0 A} = \frac{5.6 \times 10^{-8} C \times 0.6m}{1 \times 8.85 \times 10^{-12} \frac{C^2}{Nm^2} \times (0.2m \times 0.2m)}$$

$$V = 9.4 \times 10^4 V$$

Physics 111 Formula Sheet

Electrostatics

$F = k\frac{q_1 q_2}{r^2}$

$\vec{F} = q\vec{E}; \quad E_{pc} = k\frac{q}{r^2}; \quad E_{plate} = \frac{q}{\epsilon_0 A}$

$E = -\frac{\Delta V}{\Delta x}$

$V_{pc} = k\frac{q}{r}$

$U_e = k\frac{q_1 q_2}{r} = qV$

$W = -q\Delta V = -\Delta U_e = \Delta K$

Electric Circuits - Capacitors

$Q = CV; \quad C = \frac{\kappa\epsilon_0 A}{d}$

$C_{parallel} = \sum_{i=1}^{N} C_i$

$\frac{1}{C_{series}} = \sum_{i=1}^{N}\frac{1}{C_i}$

$Q_{charging}(t) = Q_{max}\left(1 - e^{-\frac{t}{\tau}}\right)$

$Q_{discharging}(t) = Q_{max}e^{-\frac{t}{\tau}}$

$I(t) = I_{max}e^{-\frac{t}{\tau}} = \frac{Q_{max}}{\tau}e^{-\frac{t}{\tau}}$

$\tau = RC$

$U_C = \frac{1}{2}qV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$

Light as a Wave

$c = f\lambda$

$S(t) = \frac{\text{Energy}}{\text{time}\times\text{Area}} = c\epsilon_0 E^2(t) = c\frac{B^2(t)}{\mu_0}$

$I = S_{avg} = \frac{1}{2}c\epsilon_0 E_{max}^2 = c\frac{B_{max}^2}{2\mu_0}$

$P = \begin{cases} \frac{S}{c}; & \text{absorbed} \\ \frac{2S}{c}; & \text{reflected} \end{cases}$

$S = S_0\cos^2\ \theta$

$v = \frac{c}{n}$

$\theta_{\text{incident}} = \theta_{\text{reflected}}$

$n_1\sin\theta_1 = n_2\sin\theta_2$

$P = \frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i}$

$M = \frac{d_i}{d_0}; \quad |M| = \frac{h_i}{h_0}$

Magnetism

$\vec{F} = q\vec{v}\times\vec{B} \rightarrow F = qvB\sin\theta$

$\vec{F} = I\vec{L}\times\vec{B} \rightarrow F = ILB\sin\theta$

$V_{Hall} = wv_dB$

$B = \frac{\mu_0 I}{2\pi r}$

$\varepsilon = \Delta V = -N\frac{\Delta\phi_B}{\Delta t}$

$\phi_B = BA\cos\theta$

Electric Circuits - Resistors

$I = \frac{\Delta Q}{\Delta t}$

$I = neAv_d; \quad n = \frac{\rho N_A}{m}$

$V = IR$

$R = \frac{\rho L}{A}$

$R_{series} = \sum_{i=1}^{N} R_i$

$\frac{1}{R_{parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$

$P = \frac{\Delta E}{\Delta t} = IV = I^2R = \frac{V^2}{R}$

Light as a Particle/Relativity

$E = hf = \frac{hc}{\lambda}$

$K_{max} = hf - \phi$

$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$

$\frac{1}{E'} = \frac{1}{E} + \frac{(1-\cos\phi)}{E_{rest}}; \quad E_{rest} = mc^2$

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$p = \gamma mv$

$E_{total} = E_{rest} + K = \gamma mc^2$

$K = (\gamma - 1)mc^2$

$E_{total}^2 = p^2c^2 + m^2c^4$

Nuclear Physics

$N = N_0 e^{-\lambda t}$

$m = m_0 e^{-\lambda t}$

$A = A_0 e^{-\lambda t}$

$A = \lambda N$

$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

Constants

$g = 9.8\frac{m}{s^2}$

$1e = 1.6 \times 10^{-19} C$

$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$

$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$

$1eV = 1.6 \times 10^{-19} J$

$\mu_0 = 4\pi \times 10^{-7} \frac{Tm}{A}$

$c = 3 \times 10^8 \frac{m}{s}$

$h = 6.63 \times 10^{-34} Js = 4.14 \times 10^{-15} eVs$

$N_A = 6.02 \times 10^{23}$

$1u = 1.66 \times 10^{-27} kg = 931.5 \frac{MeV}{c^2}$

$m_p = 1.67 \times 10^{-27} kg = 937.1 \frac{MeV}{c^2}$

$m_n = 1.69 \times 10^{-27} kg = 948.3 \frac{MeV}{c^2}$

$m_e = 9.11 \times 10^{-31} kg = 0.511 \frac{MeV}{c^2}$

Physics 110 Formulas

$\vec{F} = m\vec{a}; \quad F_G = \frac{GM_1 m_2}{r^2}; \quad F_s = -ky; \quad a_c = \frac{v^2}{r}$

$W = -\Delta U_g - \Delta U_s = \Delta K$

$U_g = mgy$

$U_s = \frac{1}{2}ky^2$

$K = \frac{1}{2}mv^2$

$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$

$\vec{v}_f = \vec{v}_i + \vec{a}t$

$v_f^2 = v_i^2 + 2a_r \Delta r$

Common Metric Units

$nano\ (n) = 10^{-9}$

$micro\ (\mu) = 10^{-6}$

$milli\ (m) = 10^{-3}$

$centi\ (c) = 10^{-2}$

$kilo\ (k) = 10^3$

$mega\ (M) = 10^6$

Geometry/Algebra

Circles: $A = \pi r^2$ $\quad C = 2\pi r = \pi$

Spheres: $A = 4\pi r^2$ $\quad V = \frac{4}{3}\pi r^3$

Triangles: $A = \frac{1}{2}bh$

Quadratics: $ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$