Physics 111

Exam #1

January 31, 2024

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So, erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $\left|\vec{p}\right| \approx m\left|\vec{v}\right| = (5kg)\times\left(2\frac{m}{s}\right) = 10\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 6 points.

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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1. Three point-charges are assembled along the x-axis with point-charge $q_1 = +9\mu C$ located at $(x_1, y_1) = (-5{,}0)cm$, point-charge $q_2 = -3\mu C$ at $(x_2, y_2) = (0{,}0)cm$, and point-charge $q_3 = +2\mu C$ located at $(x_3, y_3) = (+7{,}0)cm$.

   a. Assuming that each charge is brought in one at a time from very far away and placed in their final respective positions, how much work did it take to assemble the collection of point charges?

   To place q_1: $W_1 = 0$

   To place q_2: $W_2 = -q_2 \Delta V_1 = -q_2 \left[\frac{kq_1}{r_{12f}} - \frac{kq_1}{r_{12i}}\right]$

   $$W_2 = -(-3 \times 10^{-6} C)\left[\frac{9\times10^9 \frac{Nm^2}{c^2} \times 9\times10^{-6} C}{0.05m} - 0\right] = 4.86J$$

   To place q_3: $W_3 = -q_3 \Delta V_1 - q_3 \Delta V_2 = -q_3 \left[\frac{kq_1}{r_{31f}} - \frac{kq_1}{r_{31i}}\right] - q_3 \left[\frac{kq_2}{r_{32f}} - \frac{kq_2}{r_{32i}}\right]$

   $$W_3 = -(2 \times 10^{-6} C)\left[\frac{9 \times 10^9 \frac{Nm^2}{c^2} \times 9 \times 10^{-6} C}{0.12m} - 0\right] - (2 \times 10^{-6} C)\left[\frac{9 \times 10^9 \frac{Nm^2}{c^2} \times (-3 \times 10^{-6} C)}{0.07m} - 0\right] = -0.58J$$

   The net work: $W_{net} = W_1 + W_2 + W_3 = 0J + 4.86J - 0.58J = 4.28J$

   b. What is the net electric force on point-charge q_1 due to point-charges q_2 and q_3?

   In the x-direction:

   $$F_{net,1} = -F_{13} + F_{12} = \frac{kq_1q_3}{r_{13}^2} - \frac{kq_1q_2}{r_{12}^2} =$$

   $$F_{net,2} = 9 \times 10^9 \frac{Nm^2}{c^2} \times 9 \times 10^{-6} C\left[-\frac{2 \times 10^{-6} C}{(0.12m)^2} + \frac{3 \times 10^{-6} C}{(0.05m)^2}\right]$$

   $F_{net,2} = 85.95N$ or $86N$ in the positive x-direction.

c. What is the net electric field at the origin, $(x, y) = (0,0)$, due to q_1 and q_3?

$$E_P = E_{P,q_1} - E_{P,q_2} = \frac{kq_1}{r_{1P}^2} - \frac{kq_2}{r_{2P}^2} = 9 \times 10^9 \frac{Nm^2}{c^2}\left[\frac{9 \times 10^{-6}C}{(0.05m)^2} - \frac{2 \times 10^{-6}C}{(0.07m)^2}\right]$$

$E_p = 2.9 \times 10^7 \frac{N}{C}$ or $2.9 \times 10^7 \frac{N}{C}$ in the positive x-direction.

Or

$$F_{net,2} = F_{23} - F_{21} = \frac{kq_2q_3}{r_{23}^2} - \frac{kq_2q_1}{r_{21}^2} =$$

$$F_{net,2} = 9 \times 10^9 \frac{Nm^2}{c^2} \times 3 \times 10^{-6}C\left[\frac{2 \times 10^{-6}C}{(0.07m)^2} - \frac{9 \times 10^{-6}C}{(0.05m)^2}\right]$$

$F_{net,2} = -86.2N$ or $86.2N$ in the negative x-direction. And, since $\vec{F}_{net,2} = q_2\vec{E}_{net,2} \rightarrow \vec{E}_{net,2} = \frac{\vec{F}_{net,2}}{q_2} \rightarrow \vec{E}_{net,2} = \frac{-86.2N}{-3\times10^{-6}C} = 2.9 \times 10^7\frac{N}{C}$ or $2.9 \times 10^7\frac{N}{C}$ in the positive x-direction.

d. Suppose that point-charge q_2 were released from rest. What will the speed of point-charge q_2 be when it is at a distance of $2cm$ from point-charge q_2? Assume that the mass of point-charge q_2 is $0.5kg$.

$$W = -q_2[\Delta V_1 + \Delta V_3] = \Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}mv_f^2$$

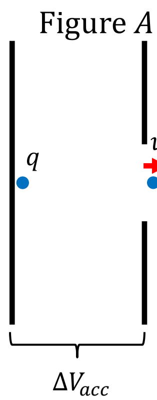
$$W = -q_2\left[\left(\frac{kq_1}{r_{21f}} - \frac{kq_1}{r_{21i}}\right) + \left(\frac{kq_3}{r_{32f}} - \frac{kq_3}{r_{32i}}\right)\right] = \tfrac{1}{2}mv_f^2$$

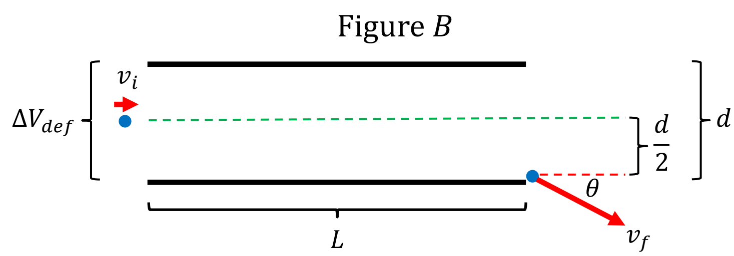
$$W = -9 \times 10^9\frac{Nm^2}{c^2} \times (-3 \times 10^{-6}C)\left[\left(\frac{9 \times 10^{-6}C}{0.02m} - \frac{9 \times 10^{-6}C}{0.05m}\right) + \left(\frac{2 \times 10^{-6}C}{0.10m} - \frac{2 \times 10^{-6}C}{0.07m}\right)\right]$$

$$W = 7.39 - 0.23J = 7.06J = \tfrac{1}{2}mv_f^2 \rightarrow v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2 \times 7.06J}{0.5kg}} = 5.3\frac{m}{s}$$

2. As shown in Figure A below, a particle accelerator is used to accelerate a positive point-charge $q = 12e$ and mass $m = 1.66 \times 10^{-26} kg$ from rest near the left plate of the accelerator, through a potential difference ΔV_{acc}. The point-charge exits through a hole in the right plate with a speed v_i. The positive point-charge is then incident at the midpoint between two horizontal plates separated by a distance of $d = 2.4m$ as shown in Figure B. The positive point-charge travels a horizontal distance L between the plates and exits the system with a speed $v_f = 5 \times 10^{6} \frac{m}{s}$ directed at an angle $\theta = 30^0$ below the horizontal.

   Note: There was a typo in the problem. The final speed should have been $5 \times 10^{6} \frac{m}{s}$, not $5 \times 10^{-6} \frac{m}{s}$. This didn't affect anyone's score on the problem. I graded the problem using $5 \times 10^{-6} \frac{m}{s}$.





   a. If the potential difference across the horizontal plates is $\Delta V_{def} = 56000V$, what is the magnitude and direction of the electric field between the horizontal plates and explain which plate (upper or lower) is at the higher electric potential?

      The positive charge accelerates down (in the negative y-direction); thus, the electric field must point vertically down since positive charges accelerate in the direction of the electric field. And, electric fields point along decreasing electric potentials, so the upper plate must be at the higher electric potential.

$$E = -\frac{\Delta V_{def}}{\Delta y} = -\left[\frac{(0V - 56000V)}{(0m - 2.8m)}\right] = -20000\frac{V}{m}$$

      Or the electric field has magnitude $20000\frac{V}{m}$ vertically down.

b. If the point-charge was accelerated over a vertical distance of $\frac{d}{2}$, how long was the point-charge interacting with the electric field?

$$v_{fy} = -v_f \sin\theta = -5 \times 10^{-6}\frac{m}{s} \sin 30 = -2.5 \times 10^{6}\frac{m}{s}$$

$$v_{fy} = v_{iy} + a_y t \rightarrow t = \frac{v_{fy}}{a_y} = \frac{mv_{fy}}{qE} = \frac{1.66 \times 10^{-26}kg \times \left(-2.5 \times 10^{6}\frac{m}{s}\right)}{12 \times 1.6 \times 10^{-19}C \times \left(-20000\frac{N}{C}\right)}$$

$$t = 1.1 \times 10^{-6}s$$

or

$$y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \rightarrow t = \sqrt{\frac{2y_f}{a_y}} = \sqrt{-\frac{2m\frac{d}{2}}{qE}}$$

$$t = \sqrt{-\frac{1.66 \times 10^{-26}kg \times (-2.8m)}{12 \times 1.6 \times 10^{-19}C \times \left(-20000\frac{N}{C}\right)}} = 1.1 \times 10^{-6}s$$

c. What is the length L of the horizontal plates?

$$v_{fx} = v_f \cos\theta = 5 \times 10^{-6}\frac{m}{s} \cos 30 = 4.33 \times 10^{6}\frac{m}{s}$$

$$x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 \rightarrow L = v_{ix}t = 4.33 \times 10^{6}\frac{m}{s} \times 1.1 \times 10^{-6}s$$

$$L = 4.76m$$

d. Through what potential difference ΔV_{acc} was the point-charge initially accelerated?

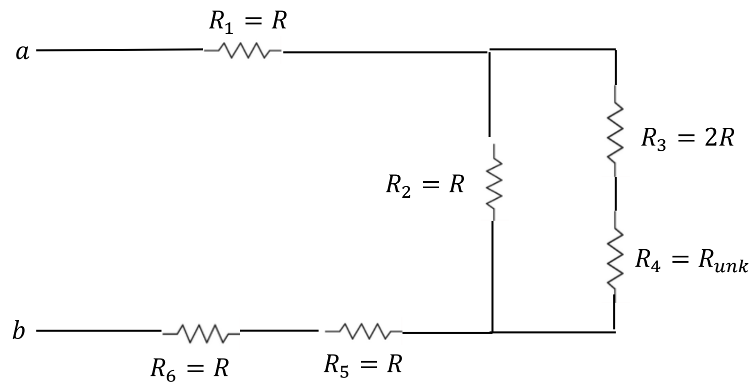
$$v_{fx} = v_{ix} + a_x t \rightarrow v_{fx} = v_{ix} = 4.33 \times 10^{6}\frac{m}{s}$$

$$W = -q\Delta V_{acc} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 \rightarrow \Delta V_{acc} = -\frac{mv_f^2}{2q}$$

$$\Delta V_{acc} = -\frac{1.66 \times 10^{-26}kg\left(4.33 \times 10^{6}\frac{m}{s}\right)^2}{2 \times 12 \times 1.6 \times 10^{-19}C} = -8.1 \times 10^{4}V$$

3. Consider the following part of a circuit shown in Figure C where several resistors are wired together between points a and b. Each resistor has a resistance R except for the unknown resistance R_{unk}.

Figure C



a. If the equivalent resistance between points a and b needs to be $R_{eq} = \frac{19}{5}R$, what is the value of R_{unk} in terms of R? Note, do not use any numerical value of any resistor given in the remainder of the problem to answer this part. Your answer should be of the form $R_{unk} = CR$, where the constant C is either a fraction (number less than 1) or a multiple (number greater than 1) of R and the answer to this question is not required to complete the remainder of the problem.

R_3 and R_4 are in series: $R_{34} = R_3 + R_4 = 2R + R_{unk} = (2 + C)R$

R_2 and R_{34} are in parallel: $\frac{1}{R_{234}} = \frac{1}{R_2} + \frac{1}{R_{34}} = \frac{1}{R} + \frac{1}{(2+C)R} = \frac{(2+C)R+R}{R(2+C)R}$

$$R_{234} = \frac{(2 + C)R^2}{3R + CR} = \frac{(2 + C)R}{3 + C}$$

R_1, R_{234}, R_5 and R_6 are in series: $R_{eq} = R_{123456} = R_1 + R_{234} + R_5 + R_6$

$$R_{eq} = \frac{19}{5}R = R + \frac{(2 + C)R}{3 + C} + R + R = 3R + \frac{(2 + C)R}{3 + C}$$

$$\rightarrow \frac{19}{5}R - 3R = \frac{4}{5}R = \frac{(2 + C)R}{3 + C}$$

$$\rightarrow 4(3 + C) = 5(2 + C) \rightarrow 12 + 4C = 10 + 5C \rightarrow C = 2$$

$R_{unk} = 2R$

b. Suppose this network of resistors were wired to an uncharged $30{,}000\mu F$ capacitor and a $12V$ battery. At what time t does the potential difference across the capacitor V_C equal $8V$ if $R = 10{,}000\Omega$?

$$V = V_{max}\left(1 - e^{-\frac{t}{R_{eq}C}}\right) \rightarrow t = -R_{eq}C\ln\left(1 - \frac{V}{V_{max}}\right) = -\frac{19}{5}RC\ln\left(1 - \frac{V}{V_{max}}\right)$$

$$t = -\frac{19}{5} \times 10000\Omega \times 30000 \times 10^{-6}F\ln\left(1 - \frac{8V}{12V}\right) = 1252.4s$$

c. At the time t determined in part b, what is the current I that is flowing in the circuit and what is the potential difference across the equivalent resistor, V_R?

By conservation of energy, $V = V_C + V_R \rightarrow V_R = V - V_C = 12V - 8V = 4V$

$$V_{R_{eq}} = IR_{eq} \rightarrow I = \frac{V_{R_{eq}}}{R_{eq}} = \frac{4V}{\frac{19}{5} \times 10000\Omega} = 1.1 \times 10^{-4}A = 0.11mA$$

Or, $I = I_{max}e^{-\frac{t}{R_{eq}C}} = \frac{Q_{max}}{R_{eq}C}e^{-\frac{t}{R_{eq}C}} = \frac{CV_{max}}{R_{eq}C}e^{-\frac{t}{R_{eq}C}} = \frac{V_{max}}{R_{eq}}e^{-\frac{t}{R_{eq}C}}$

$$I = \frac{12V}{\frac{19}{5}\times 10000\Omega}e^{-\frac{1252.4s}{\frac{19}{5}\times 10000\Omega\times 30000\times 10^{-6}F}} = 1.1 \times 10^{-4}A = 0.11mA$$

Then, $V_R = IR = 1.1 \times 10^{-4}A \times \frac{19}{5} \times 10000\Omega = 4\text{V}$

d. For the current I found in part c, what is the drift velocity of the charge carriers in the wire? Assume that the wires in the circuit are made from tungsten (W) with a density $\rho_W = 19250\frac{kg}{m^3}$, molecular mass $M_W = 181\frac{g}{mol}$, have a radius $r = 1mm$, and that tungsten donates 2 charge carriers per tungsten atom.

$$I = neAv_d \rightarrow v_d = \frac{I}{neA}$$

$$v_d = \frac{1.1 \times 10^{-4}A}{1.26 \times 10^{29}m^{-3} \times 1.6 \times 10^{-19}C \times \pi(1 \times 10^{-3}m)^3} = 1.7 \times 10^{-9}A$$

$$v_d = 1.7nA$$

where,

$$n = \left(\frac{\rho_W N_A}{M_W}\right) \times \frac{charge\ carriers}{atom} = \frac{19250\frac{kg}{m^3} \times 6.02 \times 10^{23}\frac{Watoms}{mol}}{0.181\frac{kg}{mol}} \times 2atom^{-1}$$

$$n = 1.28 \times 10^{29}m^{-3}$$

Physics 111 Formula Sheet

Electrostatics

$F = k\frac{q_1 q_2}{r^2}$

$\vec{F} = q\vec{E}; \quad E_{pc} = k\frac{q}{r^2}; \quad E_{plate} = \frac{q}{\epsilon_0 A}$

$E = -\frac{\Delta V}{\Delta x}$

$V_{pc} = k\frac{q}{r}$

$U_e = k\frac{q_1 q_2}{r} = qV$

$W = -q\Delta V = -\Delta U_e = \Delta K$

Magnetism

$\vec{F} = q\vec{v} \times \vec{B} \rightarrow F = qvB\sin\theta$

$\vec{F} = I\vec{L} \times \vec{B} \rightarrow F = ILB\sin\theta$

$V_{Hall} = wv_d B$

$B = \frac{\mu_0 I}{2\pi r}$

$\varepsilon = \Delta V = -N\frac{\Delta\phi_B}{\Delta t}$

$\phi_B = BA\cos\theta$

Electric Circuits - Capacitors

$Q = CV; \quad C = \frac{\kappa\epsilon_0 A}{d}$

$C_{parallel} = \sum_{i=1}^{N} C_i$

$\frac{1}{C_{series}} = \sum_{i=1}^{N}\frac{1}{C_i}$

$Q_{charging}(t) = Q_{max}\left(1 - e^{-\frac{t}{\tau}}\right)$

$Q_{discharging}(t) = Q_{max}e^{-\frac{t}{\tau}}$

$I(t) = I_{max}e^{-\frac{t}{\tau}} = \frac{Q_{max}}{\tau}e^{-\frac{t}{\tau}}$

$\tau = RC$

$U_C = \frac{1}{2}qV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$

Electric Circuits - Resistors

$I = \frac{\Delta Q}{\Delta t}$

$I = neAv_d; \quad n = \frac{\rho N_A}{m}$

$V = IR$

$R = \frac{\rho L}{A}$

$R_{series} = \sum_{i=1}^{N} R_i$

$\frac{1}{R_{parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$

$P = \frac{\Delta E}{\Delta t} = IV = I^2 R = \frac{V^2}{R}$

Light as a Wave

$c = f\lambda$

$S(t) = \frac{\text{Energy}}{\text{time}\times\text{Area}} = c\epsilon_0 E^2(t) = c\frac{B^2(t)}{\mu_0}$

$I = S_{avg} = \frac{1}{2}c\epsilon_0 E_{max}^2 = c\frac{B_{max}^2}{2\mu_0}$

$P = \begin{cases} \frac{S}{c}; & \text{absorbed} \\ \frac{2S}{c}; & \text{reflected} \end{cases}$

$S = S_0\cos^2\theta$

$v = \frac{c}{n}$

$\theta_{\text{incident}} = \theta_{\text{reflected}}$

$n_1\sin\theta_1 = n_2\sin\theta_2$

$P = \frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i}$

$M = \frac{d_i}{d_0}; \quad |M| = \frac{h_i}{h_0}$

Light as a Particle/Relativity

$E = hf = \frac{hc}{\lambda}$

$K_{max} = hf - \phi$

$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$

$\frac{1}{E'} = \frac{1}{E} + \frac{(1-\cos\phi)}{E_{rest}}; \quad E_{rest} = mc^2$

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$p = \gamma mv$

$E_{total} = E_{rest} + K = \gamma mc^2$

$K = (\gamma - 1)mc^2$

$E_{total}^2 = p^2c^2 + m^2c^4$

Nuclear Physics

$N = N_0 e^{-\lambda t}$

$m = m_0 e^{-\lambda t}$

$A = A_0 e^{-\lambda t}$

$A = \lambda N$

$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

Constants

$g = 9.8\frac{m}{s^2}$

$1e = 1.6 \times 10^{-19} C$

$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$

$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$

$1eV = 1.6 \times 10^{-19} J$

$\mu_0 = 4\pi \times 10^{-7} \frac{Tm}{A}$

$c = 3 \times 10^8 \frac{m}{s}$

$h = 6.63 \times 10^{-34} Js = 4.14 \times 10^{-15} eVs$

$N_A = 6.02 \times 10^{23}$

$1u = 1.66 \times 10^{-27} kg = 931.5 \frac{MeV}{c^2}$

$m_p = 1.67 \times 10^{-27} kg = 937.1 \frac{MeV}{c^2}$

$m_n = 1.69 \times 10^{-27} kg = 948.3 \frac{MeV}{c^2}$

$m_e = 9.11 \times 10^{-31} kg = 0.511 \frac{MeV}{c^2}$

Physics 110 Formulas

$\vec{F} = m\vec{a}; \quad F_G = \frac{GM_1 m_2}{r^2}; \quad F_s = -ky; \quad a_c = \frac{v^2}{r}$

$W = -\Delta U_g - \Delta U_s = \Delta K$

$U_g = mgy$

$U_s = \frac{1}{2}ky^2$

$K = \frac{1}{2}mv^2$

$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$

$\vec{v}_f = \vec{v}_i + \vec{a}t$

$v_f^2 = v_i^2 + 2a_r \Delta r$

Common Metric Units

$nano\ (n) = 10^{-9}$

$micro\ (\mu) = 10^{-6}$

$milli\ (m) = 10^{-3}$

$centi\ (c) = 10^{-2}$

$kilo\ (k) = 10^{3}$

$mega\ (M) = 10^{6}$

Geometry/Algebra

Circles: $A = \pi r^2$ $\quad C = 2\pi r = \pi$

Spheres: $A = 4\pi r^2$ $\quad V = \frac{4}{3}\pi r^3$

Triangles: $A = \frac{1}{2}bh$

Quadratics: $ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$