Physics 111

Exam #1

January 23, 2026

Name____________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So, erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}| = (5kg)\times\left(2\,\tfrac{m}{s}\right) = 10\,\tfrac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 6 points.

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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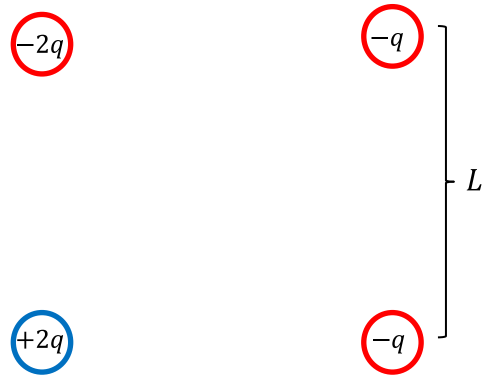
1. Consider the following arrangement of four point-charges arranged on the corners of a square. Assume that the square has sides of length L.



   a. How much work was done to assemble this collection of point-charges. Assume each point-charge is brought in one at a time from very far away and put in their final locations.

   Starting with the $+2q$ point-charge and proceeding counterclockwise we have:

$$W_{2q} = 0$$

$$W_{-q} = -(-q)\left[\frac{k2q}{L} - 0\right] = \frac{2kq^2}{L}$$

$$W_{-q} = -(-q)\left[\frac{k(-q)}{L} - 0\right] - (-q)\left[\frac{k2q}{\sqrt{2}L} - 0\right] = -\frac{kq^2}{L} + \frac{\sqrt{2}}{2}\frac{kq^2}{L}$$

$$W_{-q} = \left(\frac{\sqrt{2}}{2} - 1\right)\frac{kq^2}{L}$$

$$W_{-2q} = -(-2q)\left[\frac{k(-q)}{L} - 0\right] - (-2q)\left[\frac{k(-q)}{\sqrt{2}L} - 0\right] - (-2q)\left[\frac{k(2q)}{L} - 0\right]$$

$$W_{-2q} = -\frac{2kq^2}{L} - \frac{2}{\sqrt{2}}\frac{kq^2}{L} + \frac{4kq^2}{L} = \left(2 - \frac{\sqrt{2}}{2}\right)\frac{kq^2}{L}$$

$$W_{net} = W_{2q} + W_{-q} + W_{-q} + W_{-2q}$$

$$W_{net} = 0 + \frac{2kq^2}{L} + \left(\frac{\sqrt{2}}{2} - 1\right)\frac{kq^2}{L} + \left(2 - \frac{\sqrt{2}}{2}\right)\frac{kq^2}{L}$$

$$W_{net} = \left(2 + \frac{\sqrt{2}}{2} - 1 + 2 - \frac{\sqrt{2}}{2}\right)\frac{kq^2}{L} = 3\frac{kq^2}{L}$$

   b. What is the electric potential energy in this collection of point-charges? Does the sign of the potential energy make sense? Explain in a sentence or two.

$$W = -\Delta U_e = -\left(U_{ef} - U_{ei}\right) = -U_{ef} \rightarrow U_{ef} = -W = -3\frac{kq^2}{L}$$

c. What is the net electric field at the center of the square due to the four point-charges on the corners?

$$E_{net,x} = E_{-q,x} + E_{+2qx} - E_{-2qx} + E_{-qx} = 2E_{-qx} = 2\left(\frac{kq}{L^2/2}\right)\cos 45 = 2\sqrt{2}\frac{kq}{L^2}$$

$$E_{net,y} = E_{-q,y} + E_{+2qy} + E_{-2qy} - E_{-qy} = 2E_{-2qy} = 2\left(\frac{kq}{L^2/2}\right)\sin 45 = 4\sqrt{2}\frac{kq}{L^2}$$

$$E_{net} = \sqrt{E_{net,x}^2 + E_{net,y}^2} = \frac{kq}{L^2}\sqrt{\left(2\sqrt{2}\right)^2 + \left(4\sqrt{2}\right)^2} = \sqrt{40}\frac{kq}{L^2} = 6.3\frac{kq}{L^2}$$

$\tan\phi = \frac{E_{net,y}}{E_{net,x}} = \frac{4\sqrt{2}\frac{kq}{L^2}}{2\sqrt{2}\frac{kq}{L^2}} = 2 \rightarrow \phi = 63^0$ above the positive x-axis.

d. Suppose that a charge $Q = -4q$ were placed at the center of the square. What net force would Q feel?

$F_{net} = qE_{net} = 4q \cdot \frac{6.3kq}{L^2} = 25.2\frac{kq^2}{L^2}$ in magnitude @ $\theta = 63^0 + 180^0 = 243^0$ measured with respect to the positive x-axis (or 63^0 below the negative x-axis.)

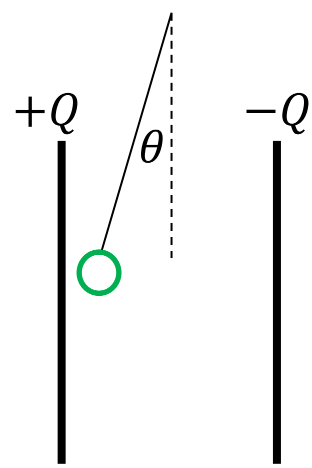
2. A capacitor is constructed out of two circular parallel plates of diameter $10cm$ separated by an unknown distance d. The initially uncharged capacitor is connected to a resistor and a $1000V$ battery and charged to its maximum value Q_{max} where $Q_{max} = 0.7nC$.

   a. If the capacitor is air filled, what is the separation between the plates of this capacitor?

$$Q = CV = \left(\frac{\kappa \epsilon_0 A}{d}\right) V \rightarrow d = \frac{\kappa \epsilon_0 A V}{Q}$$

$$d = \frac{1 \cdot 8.85 \times 10^{-12} \frac{C^2}{Nm^2} \cdot \pi (0.05m)^2 \cdot 1000V}{0.7 \times 10^{-9} C} = 0.099m = 0.1m = 10cm$$

   b. Suppose the capacitor is oriented as shown on the right. At the midpoint between the plates a point-charge q (the green circle) is suspended from an insulating string of length $L = 75cm$. The point-charge comes into equilibrium when the string makes an angle $\theta = 20^0$ measured with respect to the vertical. What is the magnitude of the electric field between the plates?



$$E = \frac{Q}{\epsilon_0 A} = \frac{0.7 \times 10^{-9} C}{8.85 \times 10^{-12} \frac{C^2}{Nm^2} \cdot \pi (0.05m)^2} = 1 \times 10^4 \frac{N}{C}$$

Or,

$$E = -\frac{\Delta V}{\Delta x} = -\left(\frac{0V - 1000V}{0.099m - 0m}\right) = 1 \times 10^4 \frac{V}{m}$$

The direction: From the positive to the negative plate.

c. What is the sign of the point-charge q? Explain your choice fully to earn full credit. Simply stating positive or negative will earn minimal credit.

Since the electric field points from the positive to the negative plate and the sphere moves to the left in the direction opposite the electric field, the charge on the sphere must be negative.

d. What is the magnitude of the point-charge q if $m = 250g$?

$$F_{net,x} = -F_e + F_{Tx} = -F_e + F_T \sin\theta = ma_x = 0$$

$$F_{net,y} = F_{Ty} - F_W = F_T \cos\theta - mg = ma_y = 0 \rightarrow F_T = \frac{mg}{\cos\theta}$$

$$\rightarrow qE = F_T \sin\theta = \left(\frac{mg}{\cos\theta}\right)\sin\theta = mg\tan\theta \rightarrow q = \frac{mg}{E}\tan\theta = \frac{0.25kg \cdot 9.8\frac{m}{s^2}}{1 \times 10^4 \frac{N}{C}}$$

$$q = 8.9 \times 10^{-5}C = 89\mu C$$

3. Bismuth ($^{209}_{83}Bi$) has a nuclear radius r_N given by $r_N = 1.2 \times 10^{-15} m \cdot A^{\frac{1}{3}}$, where A is the atomic mass in unified (or atomic) mass units. A proton was accelerated from rest when it is initially very far away from a bismuth and acquired a speed v_i. The proton approaches the bismuth nucleus head-on and from this interaction the proton is brought momentarily to rest at a distance of $3r_N$.

a. What was the initial speed v_i of the proton when it was very far away from the bismuth nucleus?

$$W = -\Delta U_e = -q\Delta V = \Delta K \rightarrow -q\Delta V = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -\frac{1}{2}mv_i^2$$

$$-e\left[\frac{kq_{bi}}{r_r} - \frac{kq_{Bi}}{r_i}\right] = -e\left(\frac{k(83e)}{3r_n}\right) = -\frac{1}{2}mv_i^2$$

$$v_i = \sqrt{\frac{166ke^2}{2mr_N}} = \sqrt{\frac{166 \cdot 9 \times 10^9 \frac{Nm^2}{C^2} \cdot (1.6 \times 10^{-19} C)^2}{3 \times 1.67 \times 10^{-27} kg \cdot 1.2 \times 10^{-15} m \cdot (209)^{\frac{1}{3}}}}$$

$$v_i = 3.3 \times 10^7 \frac{m}{s}$$

b. Through what potential difference was the proton accelerated to give it the speed in part a? Assume the proton started from rest.

$$W = -\Delta U_e = -q\Delta V = \Delta K \rightarrow -q\Delta V = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2$$

$$\Delta V = -\left(\frac{m}{2q}\right)v_f^2 = -\left(\frac{1.67 \times 10^{-27} kg}{2 \cdot 1.6 \times 10^{-19} C}\right)\left(3.3 \times 10^7 \frac{m}{s}\right)^2 = -5.6 \times 10^6 V$$

$$\Delta V = -5.6 MV$$

c. Particle accelerators, like those used to accelerate the proton in part b, are modeled as a capacitor. Supose the maximum charge that was stored on the capacitor used in this particle accelerator when fully charged was $Q_{max} = 42.7\mu C$. If the initially uncharged apacitor was charged through a $R = 120G\Omega$ resistor, what is the time constant for the charging circuit?

$$Q = CV \rightarrow C = \frac{Q}{V} = \frac{42.7 \times 10^{-6}C}{5.6 \times 10^{6}V} = 7.6 \times 10^{-12}F$$

$$\tau = RC = 120 \times 10^{9}\Omega \cdot 7.6 \times 10^{-12}F = 0.92s$$

d. Assuming that the capacitor in part c was initially uncharged, how long does it take to store 84% of the total potential energy in the system?

$$U_f = \frac{1}{2}CV^2 = \frac{1}{2}CV_{max}^2\left(1 - e^{-\frac{t}{\tau}}\right)^2 = U_i\left(1 - e^{-\frac{t}{\tau}}\right)^2$$

$$0.84U_i = U_i\left(1 - e^{-\frac{t}{\tau}}\right)^2$$

$$1 - e^{-\frac{t}{\tau}} = \sqrt{0.84} = 0.92$$

$$e^{-\frac{t}{\tau}} = 0.08 \rightarrow t = -0.92\tau \ln 0.08 = -0.92 \cdot 0.92 \ln 0.08 = 2.1s$$

Physics 111 Formula Sheet

Electrostatics

$F = k\frac{q_1 q_2}{r^2}$

$\vec{F} = q\vec{E}; \quad E_{pc} = k\frac{q}{r^2}; \quad E_{plate} = \frac{q}{\epsilon_0 A}$

$E = -\frac{\Delta V}{\Delta x}$

$V_{pc} = k\frac{q}{r}$

$U_e = k\frac{q_1 q_2}{r} = qV$

$W = -q\Delta V = -\Delta U_e = \Delta K$

Magnetism

$\vec{F} = q\vec{v} \times \vec{B} \rightarrow F = qvB\sin\theta$

$\vec{F} = I\vec{L} \times \vec{B} \rightarrow F = ILB\sin\theta$

$V_{Hall} = wv_d B$

$B = \frac{\mu_0 I}{2\pi r}$

$\varepsilon = \Delta V = -N\frac{\Delta\phi_B}{\Delta t}$

$\phi_B = BA\cos\theta$

Electric Circuits - Capacitors

$Q = CV; \quad C = \frac{\kappa\epsilon_0 A}{d}$

$C_{parallel} = \sum_{i=1}^{N} C_i$

$\frac{1}{C_{series}} = \sum_{i=1}^{N}\frac{1}{C_i}$

$Q_{charging}(t) = Q_{max}\left(1 - e^{-\frac{t}{\tau}}\right)$

$Q_{discharging}(t) = Q_{max}e^{-\frac{t}{\tau}}$

$I(t) = I_{max}e^{-\frac{t}{\tau}} = \frac{Q_{max}}{\tau}e^{-\frac{t}{\tau}}$

$\tau = RC$

$U_C = \frac{1}{2}qV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$

Electric Circuits - Resistors

$I = \frac{\Delta Q}{\Delta t}$

$I = neAv_d; \quad n = \frac{\rho N_A}{m}$

$V = IR$

$R = \frac{\rho L}{A}$

$R_{series} = \sum_{i=1}^{N} R_i$

$\frac{1}{R_{parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$

$P = \frac{\Delta E}{\Delta t} = IV = I^2 R = \frac{V^2}{R}$

Light as a Wave

$c = f\lambda$

$S(t) = \frac{\text{Energy}}{\text{time}\times\text{Area}} = c\epsilon_0 E^2(t) = c\frac{B^2(t)}{\mu_0}$

$I = S_{avg} = \frac{1}{2}c\epsilon_0 E_{max}^2 = c\frac{B_{max}^2}{2\mu_0}$

$P = \begin{cases} \frac{S}{c}; & \text{absorbed} \\ \frac{2S}{c}; & \text{reflected} \end{cases}$

$S = S_0\cos^2\ \theta$

$v = \frac{c}{n}$

$\theta_{\text{incident}} = \theta_{\text{reflected}}$

$n_1\sin\theta_1 = n_2\sin\theta_2$

$P = \frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i}$

$M = \frac{d_i}{d_0}; \quad |M| = \frac{h_i}{h_0}$

Light as a Particle/Relativity

$E = hf = \frac{hc}{\lambda}$

$K_{max} = hf - \phi$

$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$

$\frac{1}{E'} = \frac{1}{E} + \frac{(1-\cos\phi)}{E_{rest}}; \quad E_{rest} = mc^2$

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$p = \gamma mv$

$E_{total} = E_{rest} + K = \gamma mc^2$

$K = (\gamma - 1)mc^2$

$E_{total}^2 = p^2c^2 + m^2c^4$

Nuclear Physics

$N = N_0 e^{-\lambda t}$

$m = m_0 e^{-\lambda t}$

$A = A_0 e^{-\lambda t}$

$A = \lambda N$

$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

Constants

$g = 9.8\frac{m}{s^2}$

$1e = 1.6 \times 10^{-19} C$

$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$

$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$

$1eV = 1.6 \times 10^{-19} J$

$\mu_0 = 4\pi \times 10^{-7} \frac{Tm}{A}$

$c = 3 \times 10^8 \frac{m}{s}$

$h = 6.63 \times 10^{-34} Js = 4.14 \times 10^{-15} eVs$

$N_A = 6.02 \times 10^{23}$

$1u = 1.66 \times 10^{-27} kg = 931.5 \frac{MeV}{c^2}$

$m_p = 1.67 \times 10^{-27} kg = 937.1 \frac{MeV}{c^2}$

$m_n = 1.69 \times 10^{-27} kg = 948.3 \frac{MeV}{c^2}$

$m_e = 9.11 \times 10^{-31} kg = 0.511 \frac{MeV}{c^2}$

Physics 110 Formulas

$\vec{F} = m\vec{a};\quad F_G = \frac{GM_1 m_2}{r^2};\quad F_s = -ky;\quad a_c = \frac{v^2}{r}$

$W = -\Delta U_g - \Delta U_s = \Delta K$

$U_g = mgy$

$U_s = \frac{1}{2} k y^2$

$K = \frac{1}{2} m v^2$

$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a} t^2$

$\vec{v}_f = \vec{v}_i + \vec{a} t$

$v_f^2 = v_i^2 + 2 a_r \Delta r$

Common Metric Units

$nano\ (n) = 10^{-9}$

$micro\ (\mu) = 10^{-6}$

$milli\ (m) = 10^{-3}$

$centi\ (c) = 10^{-2}$

$kilo\ (k) = 10^{3}$

$mega\ (M) = 10^{6}$

Geometry/Algebra

Circles: $A = \pi r^2$ $C = 2\pi r = \pi$

Spheres: $A = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$

Triangles: $A = \frac{1}{2} bh$

Quadratics: $ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$