## Physics 111

## Exam \#1

January 24, 2011

Name

| Multiple Choice | $/ 16$ |
| :---: | :---: |
| Problem \#1 | $/ 28$ |
| Problem \#2 | $/ 28$ |
| Problem \#3 | $/ 28$ |
| Total | $/ 100$ |

Part I: Multiple-Choice: Circle the best answer to each question. Any other marks will not be given credit. Each multiple-choice question is worth 4 points for a total of 16 points.

1. When the electric field is zero at all points in any region of space
a. the electric potential must be zero at all points in that region.
b. the electric potential must have a constant value at all points in that region.
c. the electric potential cannot be found unless we know its value at a minimum of two points in the region.
d. the region might be found between the plates of a charged capacitor.
2. A point charge $Q$ is fixed in space. Suppose that a small test charge $-q$ is placed at a distance $r_{A}$ from $Q$. If the test charge is released from rest the total energy of charge distribution
a. increases with increasing distance between $Q$ and $-q$.
b. remains constant.
c. increases with decreasing distance between $Q$ and $-q$.
d. decreases with decreasing distance between $Q$ and $-q$.
3. Plate $a$ (of a parallel-plate air-filled capacitor) is connected to a spring having force constant $k$, while plate $b$ is fixed. If a charge $+Q$ is placed on plate $a$ and $-Q$ is placed on plate $b$, and if the electric field is for a plate of charge is given by $E=\frac{Q}{\varepsilon_{o} A}$, plate $a$ moves

a. $\quad x=\frac{Q}{k \varepsilon_{o} A}$ to the left.
b. $x=\frac{Q}{k \varepsilon_{o} A}$ to the right.
c. $x=\frac{Q^{2}}{k \varepsilon_{o} A}$ to the left.
d. $x=\frac{Q^{2}}{k \varepsilon_{o} A}$ to the right.
4. Suppose that you have two point charges $Q_{1}$ and $Q_{2}$ separated by a distance $r$ so that there exists a force of repulsion with magnitude $F$. If each of the magnitudes of the charges were tripled and the separation between them doubled, the magnitude of the repulsive force would now become
a. $\frac{3}{4} F$.
b. $\frac{3}{2} F$
c. $\frac{9}{2} F$
d. $\frac{9}{4} F$

Part II: Free Response Problems: The three problems below are worth 84 points total and each subpart is worth 7 points each. Please show all work in order to receive partial credit. If your solutions are illegible or illogical no credit will be given. A number with no work shown (even if correct) will be given no credit. Please use the back of the page if necessary, but number the problem you are working on.

1. Given below is a set of data on a collection of point charges along with their locations (in meters) in a standard Cartesian coordinate system.
a. What is the electric field at the origin due to point charges $q_{1}-q_{4}$ ?

|  | Charge $(\mu \mathrm{C})$ | Location $(\mathrm{x}, \mathrm{y})$ |
| :---: | :---: | :---: |
| $q_{1}$ | -6 | $(0,3 / 4)$ |
| $q_{2}$ | 3 | $(1,0)$ |
| $q_{3}$ | -5 | $(0,-1 / 2)$ |
| $q_{4}$ | 1 | $(-1 / 4,0)$ |

$$
\begin{aligned}
& E_{\text {net }, x}=-E_{2}+E_{4}=-\frac{k q_{2}}{r_{2}^{2}}+\frac{k q_{4}}{r_{4}^{2}}=9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{C^{2}}\left[-\frac{3 \times 10^{-6} \mathrm{C}}{(1 \mathrm{~m})^{2}}+\frac{1 \times 10^{-6} \mathrm{C}}{(0.25 \mathrm{~m})^{2}}\right]=1.2 \times 10^{5} \frac{\mathrm{~N}}{\mathrm{C}} \\
& E_{\text {net, },}=-E_{3}+E_{1}=-\frac{k q_{3}}{r_{3}^{2}}+\frac{k q_{1}}{r_{1}^{2}}=9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{\mathrm{C}^{2}}\left[-\frac{5 \times 10^{-6} \mathrm{C}}{(0.5 \mathrm{~m})^{2}}+\frac{6 \times 10^{-6} \mathrm{C}}{(0.75 \mathrm{~m})^{2}}\right]=-8.4 \times 10^{4} \frac{\mathrm{~N}}{\mathrm{C}} \\
& E_{\text {net }}=\sqrt{E_{\text {net }, x}^{2}+E_{\text {net,y }, ~}^{2}} @ \theta=\tan ^{-1}\left(\frac{E_{\text {net,y,y}}}{E_{\text {net }, x}}\right)=1.44 \times 10^{5} \frac{\mathrm{~N}}{\mathrm{C}} @ \theta=35^{\circ} \text { below the }+\mathrm{x}-\text { axis }
\end{aligned}
$$

b. What is the electrostatic force on a $-1 / 2 \mu C$ point charge placed at the origin?

$$
\vec{F}_{n e t}=q \vec{E}_{n e t}=0.5 \times 10^{-6} \mathrm{C} \times 1.44 \times 10^{5} \frac{N}{C}=0.072 N @ \theta=145^{\circ} \text { above the }+\mathrm{x}-\mathrm{axis}
$$

c. What is the electric potential at the origin due to point charges $q_{1}$ through $q_{4}$ ?

$$
\begin{aligned}
& V_{P}=V_{1}+V_{2}+V_{3}+V_{4}=\frac{k q_{1}}{r_{1}}+\frac{k q_{2}}{r_{2}}+\frac{k q_{3}}{r_{3}}+\frac{k q_{4}}{r_{4}} \\
& V_{P}=9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{\mathrm{C}^{2}}\left[-\frac{6 \times 10^{-6} \mathrm{C}}{0.75 \mathrm{~m}}+\frac{3 \times 10^{-6} \mathrm{C}}{1.0 \mathrm{~m}}-\frac{5 \times 10^{-6} \mathrm{C}}{0.5 \mathrm{~m}}+\frac{1 \times 10^{-6} \mathrm{C}}{0.25 \mathrm{~m}}\right]=-9.9 \times 10^{4} \mathrm{~V}
\end{aligned}
$$

d. How much work was done on the $-1 / 2 \mu C$ point charge if it was brought in from infinity to the origin?

$$
W=q \Delta V=\left(-0.5 \times 10^{-6} \mathrm{C}\right)[0 \mathrm{~V}-(-9.9 \times 104 \mathrm{~V})]=-0.050 \mathrm{~J}=-50 \mathrm{~mJ}
$$

2. Suppose that you charge a $0.19 n F$ capacitor to full charge using a 10.0 V battery. The capacitor plates are circular each with a diameter of 20 cm .
a. Suppose you wanted to build this capacitor out of the two metal plates (with dimensions above and rated capacitance) and further you wanted to insulate the plates from one another with a piece of rubber $(\kappa=7)$, how thick would the insulating layer need to be to create this capacitor?
$C=\frac{\kappa \varepsilon_{o} A}{d} \rightarrow d=\frac{\kappa \varepsilon_{o} A}{C}=\frac{7 \times 8.85 \times 10^{-12} \frac{C^{2}}{\mathrm{Nm}^{2}} \times \pi(0.1 \mathrm{~m})^{2}}{0.19 \times 10^{-9} \mathrm{~F}}=0.010 \mathrm{~m}=10 \mathrm{~mm}$
b. How much charge is initially stored on the capacitor when connected to the battery?

$$
Q_{\max }=C V=0.19 \times 10^{-9} \mathrm{~F} \times 10 \mathrm{~V}=1.9 \times 10^{-9} \mathrm{C}=1.9 \mathrm{nC}
$$

c. Suppose that you remove the battery and decide to discharge the capacitor through a $1000 \Omega$ resistor. How long will it take for the capacitor to dissipate $63 \%$ of its initial stored charge?

$$
\begin{aligned}
& Q(t)=Q_{\max } e^{-\frac{t}{R C}} \rightarrow Q(t=R C)=0.37 Q_{\max }=Q_{\max } e^{-\frac{t}{R C}} \\
& \rightarrow t=-R C \ln (0.37)=-1000 \Omega \times 0.19 \times 10^{-9} \mathrm{~F} \times \ln (0.37)=1.9 \times 10^{-7} \mathrm{~s}
\end{aligned}
$$

d. How much energy was initially stored in the capacitor and how much energy will remain after one time constant has passed?

$$
\begin{aligned}
& E_{\max }=\frac{1}{2} C V^{2}=\frac{1}{2} \times 0.19 \times 10^{-9} F(10 \mathrm{~V})^{2}=9.5 \times 10^{-9} \mathrm{~J}=9.5 \mathrm{~nJ} \\
& E(t)=\frac{1}{2} C V^{2}(t)=\frac{1}{2} C\left[V_{\max } e^{-\frac{t}{R C}}\right]^{2}=\frac{1}{2} C V_{\max }^{2} e^{-\frac{2 t}{R C}} \\
& E(t=R C)=E_{\max } e^{-\frac{2 R C}{R C}}=\frac{E_{\max }}{e^{2}}=\frac{9.5 \times 10^{-9} \mathrm{~J}}{7.39}=1.3 \times 10^{-9} \mathrm{~J}=1.3 \mathrm{~nJ}
\end{aligned}
$$

3. Suppose that you have a beam of electrons that you want to turn a $90^{\circ}$ corner as shown in the figure below, by using a parallel plate capacitor. Assume that the electron has a kinetic energy $K E=3 \times 10^{-17} \mathrm{~J}$ and moves up through a small hole in the bottom plate of the capacitor.
a. Should the bottom plate be charged positively or negatively (relative to the top plate) if you wan the electron beam to turn right? Explain you choice fully.

To repel the electron from the top plate, it should have a negative charge and thus the bottom plate should be charged positively, which will also attract the electron toward the exit hole.
b. What is the magnitude of the velocity of the electron?

$$
K E=\frac{1}{2} m_{e} v_{e}^{2} \rightarrow v_{e}=\sqrt{\frac{2 \times K E}{m_{e}}}=\sqrt{\frac{2 \times 3 \times 10^{-17 J}}{9.11 \times 10^{-31} \mathrm{~kg}}}=8.1 \times 10^{6} \frac{\mathrm{~m}}{\mathrm{~s}}
$$

c. Using the coordinate system on the figure, what constant magnitude electric field would be needed if you wanted the electron to emerge 1.0 cm from where it enters traveling at a right angle to its original direction? Draw the electric field's direction on the diagram above. (Hint: Assume that the electric field bisects the right angle in the figure above.)

$$
\begin{aligned}
& v_{f y}=0=v_{i y}-a_{y} t_{\text {rise }}=v_{i y}-\frac{F}{m_{e}} \times\left(\frac{x_{f}}{2 v_{i x}}\right)=v_{i} \sin \theta-\left(\frac{e E}{m_{e}}\right) \times\left(\frac{x_{f}}{2 v_{i} \cos \theta}\right) \\
& E=\frac{2 m_{e} v_{i}^{2} \sin \theta \cos \theta}{e x_{f}}=\frac{2 \times 9.11 \times 10^{-31} \mathrm{~kg}\left(8.1 \times 10^{6} \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2} \sin 45 \cos 45}{1.6 \times 10^{-19} \mathrm{C} \times 0.01 \mathrm{~m}}=3.7 \times 10^{4} \frac{\mathrm{~N}}{\mathrm{C}}
\end{aligned}
$$

## directed from the positive to the negative plate.

d. What minimum separation, $d_{\min }$, would the capacitor plates need to have?

$$
\begin{aligned}
& d_{\min }=y_{f}=y_{i}+v_{i y} t_{\text {rise }}-\frac{1}{2} a_{y} t_{\text {rise }}^{2}=v_{i} \sin \theta \times\left(\frac{x_{f}}{2 v_{i} \cos \theta}\right)-\frac{1}{2}\left(\frac{e \times E}{m_{e}}\right) \times\left(\frac{x_{f}}{2 v_{i} \cos \theta}\right)^{2} \\
& d_{\min }=\frac{x_{f}}{2} \tan \theta-\frac{e E x_{f}^{2}}{4 m_{e} v_{i}^{2} \cos ^{2} \theta} \\
& d_{\min }=\frac{0.01 \mathrm{~m}}{2} \tan 45-\frac{1}{2}\left(\frac{1.6 \times 10^{-19} \mathrm{C} \times 3.7 \times 10^{4} \frac{\mathrm{~N}}{\mathrm{C}} \times(0.01 \mathrm{~m})^{2}}{4 \times 9.11 \times 10^{-31} \mathrm{~kg} \times\left(8.1 \times 10^{6} \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2} \cos ^{2} 45}\right)=0.0028 \mathrm{~m}=2.8 \mathrm{~mm}
\end{aligned}
$$

## Physics 111 Equation Sheet

Electric Forces, Fields and Potentials

## Electric Circuits

$$
\begin{aligned}
& \vec{F}=k \frac{Q_{1} Q_{2}}{r^{2}} \\
& \vec{E}=\frac{\vec{F}}{q} \\
& \vec{E}_{Q}=k \frac{Q}{r^{r}} \hat{r} \\
& P E=k \frac{Q_{1} Q_{2}}{r} \\
& V(r)=k \frac{Q}{r} \\
& E_{x}=-\frac{\Delta V_{B, A}}{\Delta x} \\
& W_{A, B}=q \Delta V_{A, B}
\end{aligned}
$$

Magnetic Forces and Fields

$$
\begin{aligned}
& F=q \nu B \sin \theta \\
& F=I l B \sin \theta \\
& \tau=N I A B \sin \theta=\mu B \sin \theta \\
& P E=-\mu B \cos \theta \\
& B=\frac{\mu_{0} I}{2 \pi r} \\
& \varepsilon_{\text {induced }}=-N \frac{\Delta \phi_{B}}{\Delta t}=-N \frac{\Delta(B A \cos \theta)}{\Delta t}
\end{aligned}
$$

$$
\begin{aligned}
& I=\frac{\Delta Q}{\Delta t} \\
& V=I R=I\left(\frac{\rho L}{A}\right) \\
& R_{\text {series }}=\sum_{i=1}^{N} R_{i} \\
& \frac{1}{R_{\text {parallel }}}=\sum_{i=1}^{N} \frac{1}{R_{i}} \\
& P=I V=I^{2} R=\frac{V^{2}}{R} \\
& Q=C V=\left(\frac{\kappa \varepsilon_{0} A}{d}\right) V=\left(\kappa C_{0}\right) V \\
& P E=\frac{1}{2} Q V=\frac{1}{2} C V^{2}=\frac{Q^{2}}{2 C} \\
& Q_{\text {charge }}(t)=Q_{\max }\left(1-e^{-\frac{t}{R C}}\right) \\
& Q_{\text {discharge }}(t)=Q_{\max } e^{-\frac{t}{R C}} \\
& C_{\text {parallel }}=\sum_{i=1}^{N} C_{i} \\
& \frac{1}{C_{\text {series }}}=\sum_{i=1}^{N} \frac{1}{C_{i}}
\end{aligned}
$$

## Light as a Wave

$$
\begin{aligned}
& c=f \lambda=\frac{1}{\sqrt{\varepsilon_{o} \mu_{o}}} \\
& S(t)=\frac{\text { energy }}{\text { time } \times \text { area }}=c \varepsilon_{o} E^{2}(t)=c \frac{B^{2}(t)}{\mu_{0}} \\
& I=S_{\text {avg }}=\frac{1}{2} c \varepsilon_{o} E_{\max }^{2}=c \frac{B_{\max }^{2}}{2 \mu_{0}} \\
& P=\frac{S}{c}=\frac{F o r c e}{\text { Area }} \\
& S=S_{o} \cos ^{2} \theta \\
& v=\frac{1}{\sqrt{\varepsilon \mu}}=\frac{c}{n} \\
& \theta_{\text {inc }}=\theta_{\text {ref } l} \\
& n_{1} \sin \theta_{1}=n_{2} \sin \theta_{2} \\
& \frac{1}{f}=\frac{1}{d_{o}}+\frac{1}{d_{i}} \\
& M=\frac{h_{i}}{h_{o}}=-\frac{d_{i}}{d_{o}} \\
& M_{\text {total }}=\prod_{i=1}^{N} M_{i} \\
& d \sin \theta=m \lambda \text { or }\left(m+\frac{1}{2}\right) \lambda \\
& a \sin \phi=m^{\prime} \lambda
\end{aligned}
$$

## Constants

$g=9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$
$G=6.67 \times 10^{-11} \frac{\mathrm{Nm}^{2}}{\mathrm{~kg}^{2}}$
$1 e=1.6 \times 10^{-19} \mathrm{C}$
$k=\frac{1}{4 \pi \varepsilon_{o}}=9 \times 10^{9} \frac{\mathrm{C}^{2}}{\mathrm{Nm} m^{2}}$
$\varepsilon_{o}=8.85 \times 10^{-12} \frac{\mathrm{Nm}{ }^{2}}{\mathrm{c}^{2}}$
$1 \mathrm{eV}=1.6 \times 10^{-19} \mathrm{~J}$
$\mu_{o}=4 \pi \times 10^{-7} \frac{\mathrm{Tm}}{\mathrm{A}}$
$c=3 \times 10^{8} \frac{\mathrm{~m}}{\mathrm{~s}}$
$h=6.63 \times 10^{-34} \mathrm{JS}$
$m_{e}=9.11 \times 10^{-3 \mathrm{l}} \mathrm{kg}=\frac{0.511 \mathrm{MeV}}{c^{2}}$
$m_{p}=1.67 \times 10^{-27} \mathrm{~kg}=\frac{937.1 \mathrm{MeV}}{c^{2}}$
$m_{n}=1.69 \times 10^{-27} \mathrm{~kg}=\frac{948.3 \mathrm{MeV}}{c^{2}}$
$1 \mathrm{amu}=1.66 \times 10^{-27} \mathrm{~kg}=\frac{931.5 \mathrm{MeV}}{c^{2}}$
$N_{A}=6.02 \times 10^{23}$
$A x^{2}+B x+C=0 \rightarrow x=\frac{-B \pm \sqrt{B^{2}-4 A C}}{2 A}$

Light as a Particle \& Relativity Nuclear Physics

$$
\begin{array}{ll}
E=h f=\frac{h c}{\lambda}=p c & E_{\text {binding }}=\left(Z m_{p}+N m_{n}-m_{\text {rest }}\right) c^{2} \\
K E_{\max }=h f-\phi=e V_{\text {stop }} & \frac{\Delta N}{\Delta t}=-\lambda N_{o} \rightarrow N(t)=N_{o} e^{-\lambda t} \\
\Delta \lambda=\frac{h}{m_{e} c}(1-\cos \phi) & A(t)=A_{o} e^{-\lambda t} \\
\gamma=\frac{1}{\sqrt{v^{2}}} & m(t)=m_{o} e^{-\lambda t} \\
& t_{\frac{1}{2}}=\frac{\ln 2}{\lambda}
\end{array}
$$

$$
\gamma=\frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}}
$$

Misc. Physics 110

$$
p=\gamma m v
$$

Formulae

$$
E_{\text {total }}=K E+E_{\text {rest }}=\gamma m c^{2}
$$

$\vec{F}=\frac{\Delta \vec{p}}{\Delta t}=\frac{\Delta(m v)}{\Delta t}=m \vec{a}$
$E_{\text {total }}^{2}=p^{2} c^{2}+m^{2} c^{4}$
$\vec{F}=-k \vec{y}$
$E_{\text {rest }}=m c^{2}$
$\vec{F}_{C}=m \frac{v^{2}}{R} \hat{r}$
$K E=(\gamma-1) m c^{2}$
$W=\Delta K E=\frac{1}{2} m\left(v_{f}^{2}-v_{i}^{2}\right)=-\Delta P E$
Geometry
Circles: $C=2 \pi r=\pi D \quad A=\pi r^{2}$
$P E_{\text {gravity }}=m g y$

Triangles: $A=\frac{1}{2} b h$
$P E_{\text {spring }}=\frac{1}{2} k y^{2}$

Spheres: $A=4 \pi r^{2} \quad V=\frac{4}{3} \pi r^{3}$
$x_{f}=x_{i}+v_{i x} t+\frac{1}{2} a_{x} t^{2}$
$v_{f x}=v_{i x}+a_{x} t$
$v_{v x}^{2}=v_{i x}^{2}+2 a_{x} \Delta x$

