Physics 111

Exam #1

January 28, 2013

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg)\times\left(2\frac{m}{s}\right) = 10\frac{kg\cdot m}{s}$
- Give standard SI units with your results.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 3 points and each free-response part is worth 6 points

Problem #1	/27
Problem #2	/24
Problem #3	/24
Total	/75

I affirm that I have carried out my academic endeavors with full academic honesty.

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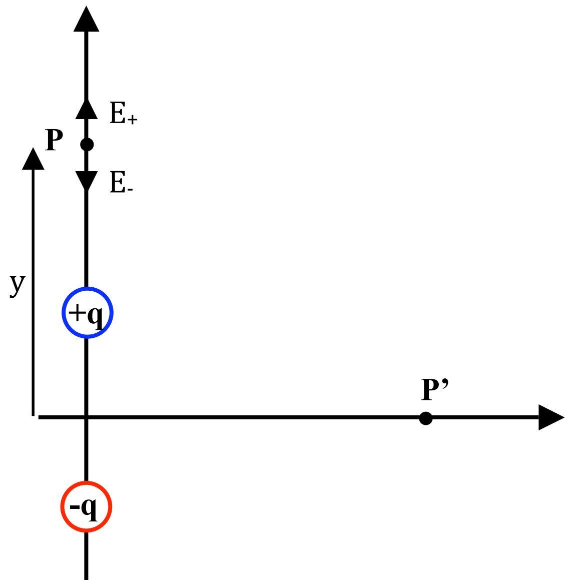
1. Two equal and opposite point charges are situated along the y-axis as shown. The $+q$ charge is at $y = +a$, while the $-q$ charge is a $y = -a$. In the questions below, express your answers in the simplest form possible, reducing fractions as necessary.



a. What is the net electric field at point P located along the y-axis?

$$E_{net} = E_{net,y} = E_+ + E_- = \frac{kq}{r_+^2} - \frac{kq}{r_-^2} = kq\left[\frac{1}{(y-a)^2} - \frac{1}{(y+a)^2}\right]$$

$$= kq\left[\frac{(y+a)^2 - (y-a)^2}{(y+a)^2(y-a)^2}\right] = \frac{4kaqy}{(y+a)^2(y-a)^2}$$

So, the net electric field points in the positive y-direction and has a magnitude of $E_{net} = \frac{4kaqy}{(y+a)^2(y-a)^2}$.

b. What is the net electrostatic force on a charge $-q$ placed at point P?

The net force is given by

$$\vec{F}_{net} = q\vec{E}_{net} = -q \times \left(\frac{4kaqy}{(y+a)^2(y-a)^2}\right) = -\frac{4kaq^2y}{(y+a)^2(y-a)^2}$$

c. What is the electric potential at point P?

The electric potential is given by $V_P = \frac{kq}{r_+} - \frac{kq}{r_-} = kq\left[\frac{1}{y-a} - \frac{1}{y+a}\right] = \frac{2kaq}{y^2 - a^2}$.

d. How much work is done moving the charge $-q$ to point P from a location very far away?

The work done is given by $W = -q\Delta V = -(-q)\left[\frac{2kqa}{y^2 - a^2} - 0\right] = \frac{2kaq^2}{y^2 - a^2}$.

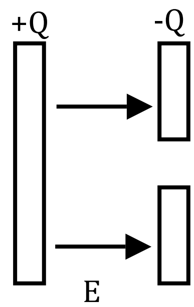
e. Suppose that instead we were interested in the point P' located on the x-axis as shown. Which of the following statements would be true for a charge $-q$ located at the point P'?

1. An external force would be required to move $-q$ from very far away to point P', meaning $W < 0$.
2. The electric field would do work moving $-q$ from very far away to point P', meaning $W > 0$.
3. No work would be done on $-q$, since $-q$ is moving along an equipotential surface.
4. Non-zero work would be done on $-q$ but one cannot determine from the information given if the system or an external force would do the work.

The potential at point P' is given by $V_{P'} = \frac{kq}{r_+} - \frac{kq}{r_-} = kq\left[\frac{1}{r} - \frac{1}{r}\right] = 0$ and therefore the work done moving $-q$ from very far away to this location would be zero.

2. Suppose that you wanted to accelerate a proton to a very high speed using the setup shown below, where the right plate has a hole in it to allow the protons to escape.

   a. Explain how the plates should be charged so that the proton is accelerated to the right? What is the direction of the electric field (assumed constant) between the plates? Explain your choice for the direction of the electric field and label the plates below with your choice and also draw the electric field between the plates.



Charge the left plate with $+Q$ so that the proton feels a repulsive force.

Charge the right plate with $-Q$ so that the proton feels an attractive force.

The electric field E points from the left to the right plate along decreasing values of the electric potential.

   b. If you wanted the protons to be moving with a velocity of $2\times10^{7}\,\frac{m}{s}$ when they emerge from the hole on the right plate, what potential difference would be required to accelerate the proton through? (Hints: The mass of the proton is $1.67\times10^{-27}\,kg$ and assume that the proton initially starts from rest.)

The speed is given from the work done by the electric field accelerating the proton from rest across the gap between the two plates. Thus we have,

$$W=-q\Delta V=-q\left[V_R-V_L\right]=\Delta KE=\tfrac{1}{2}m_pv_f^2-0$$

$$\Delta V=V_L-V_R=\frac{m_pv_f^2}{2q}=\frac{1.67\times10^{-27}\,kg\times\left(2\times10^{7}\,\frac{m}{s}\right)^2}{2\times1.6\times10^{-19}\,C}=2.1\times10^{6}\,V=2.1\,MV$$

   c. In the diagram above, which of the following is true?

      1. The left plate is at a higher electric potential than the right plate.
      2. The right plate is at a higher electric potential than the left plate.
      3. Both plates are at the same potential.
      4. There is no way to determine which plate is at the higher potential.

Since the electric field points along decreasing electric potentials, the left plate is at a higher potential than the right plate.

d. Suppose that the proton (with the velocity given in part b) were directed at an atom of copper ($M_{Cu} = 1.08 \times 10^{-25}\,kg$ and $Q_{Cu} = +29e$) located initially very far away from the proton. What is the distance of closest approach that the proton would get to the nucleus? (Hints: 1) ignore the fact that a copper atom has electrons surrounding it, at these speeds the proton will zip right on past them on towards the nucleus and they will not change the speed of the proton by any appreciable amount and 2) there are no external forces acting on the system so energy is conserved.)

Since there are no external forces present, then energy is conserved and we have the loss of kinetic energy by the proton (due to the repulsive electric force of the proton-Cu atom combination) manifests as a gain in the electric potential energy of the proton-Cu atom combination. Thus, when the proton comes to rest, it will be at a distance r_f from the Cu atom having started very far away from the Cu atom. Therefore, assuming that the Cu atom starts from rest and remains at rest during the interaction (which is not a bad assumption since the Cu atom is much heavier than the proton and we probably have a lattice of Cu atoms which is infinitely heavier than a proton)

$$\Delta KE + \Delta EPE = 0 = \left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right) + \left(\frac{kQq}{r_f} - \frac{kQq}{r_i}\right)$$

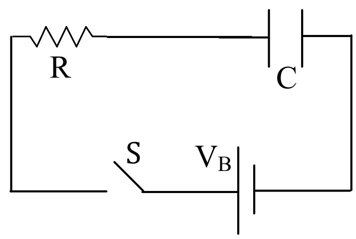
$$\therefore r_f = \frac{2kQq}{m_p v_i^2} = \frac{2 \times 9 \times 10^9 \,\tfrac{Nm^2}{C^2} \times \left(29 \times 1.6 \times 10^{-29}\,C\right) \times 1.6 \times 10^{-29}\,C}{1.67 \times 10^{-27}\,kg \times \left(2 \times 10^7\,\tfrac{m}{s}\right)^2} = 2 \times 10^{-14}\,m$$

e. If the proton was directed instead an atom with a higher mass and the experiment was done exactly the same, (meaning that the proton was accelerated to the same velocity as in part b,)

1. the distance of closest approach would be smaller due to a smaller repulsive force between the proton and the nucleus.
2. the distance of closest approach would be larger due to a smaller repulsive force between the proton and the nucleus.
3. the distance of closest approach would be smaller due to a larger repulsive force between the proton and the nucleus.
4. the distance of closest approach would be larger due to a larger repulsive force between the proton and the nucleus.

Assuming that the charge Q does not move during the interaction and since the distance of closest approach is given by $r_f = \dfrac{2kQq}{m_p v_i^2}$, as Q increases, so to does r_f.

3. Suppose that you are given the circuit below that contains a battery ($V_B = 12V$), a resistor ($R = 10k\Omega$), and an uncharged capacitor ($C = 6\mu F$).



a. As time $t = 0$ the switch S is closed. As time increases, the potential differences across the capacitor and resistor change according to

1. $V_C \uparrow$; $V_R \uparrow$.
2. $V_C \uparrow$; $V_R \downarrow$.
3. $V_C \downarrow$; $V_R \uparrow$.
4. $V_C \downarrow$; $V_R \downarrow$.

At time $t = 0$, $V_C = 0$ and $V_R = V_B$. As time increases, V_C increases and V_R decreases in order for energy to be conserved.

b. What is the time constant associated with this circuit and how much charge could be stored on the capacitor?

The time constant is the product of the resistance and the capacitance and is $\tau = RC = 10000\Omega \times 6 \times 10^{-6} F = 0.06s = 60ms$.

The charge is given by
$Q = CV = 6 \times 10^{-6} F \times 12V = 7.2 \times 10^{-5} C = 72 \times 10^{-6} C = 72\mu C$.

c. How long would it take to accumulate 47% of the total charge on the capacitor?

For a charging capacitor we have,

$$Q_f = 0.47 Q_{\max} = Q_{\max}\left(1 - e^{-\frac{t}{RC}}\right)$$

$$e^{-\frac{t}{RC}} = 1 - 0.47 = 0.53 \rightarrow t = -RC\ln(0.53) = -(0.06s)\ln(0.53) = 0.038s = 38ms$$

d. At what time are the potential differences across the resistor (V_R) and the capacitor (V_C) equal?

Since energy has to be conserved in the circuit at any time, as the potential across the capacitor is increasing the potential across the resistor is decreasing. Thus we have,

$$V_R = V_{\max} e^{-\frac{t}{RC}} = V_C = V_{\max}\left(1 - e^{-\frac{t}{RC}}\right)$$

$$\therefore e^{-\frac{t}{RC}} = \left(1 - e^{-\frac{t}{RC}}\right) \rightarrow t = -RC\ln\left(\tfrac{1}{2}\right) = -(0.06s)\ln\left(\tfrac{1}{2}\right) = 0.042s = 42ms$$

e. Why are the potential differences across the resistor and capacitor changing with time especially since the battery is a source of constant potential?

1. The potential differences across the resistor and capacitor change with time because the time constant changes with the value of the resistance and capacitance.
2. The potential differences across the resistor and capacitor change with time because the resistance is not constant.
3. The potential differences across the resistor and capacitor change with time because the capacitance is not constant.
4. The potential differences across the resistor and capacitor change with time in order to conserve energy.

Physics 111 Equation Sheet

Electric Forces, Fields and Potentials

$$\vec{F} = k\frac{Q_1Q_2}{r^2}\hat{r}$$
$$\vec{E} = \frac{\vec{F}}{q}$$
$$\vec{E}_Q = k\frac{Q}{r^2}\hat{r}$$
$$PE = k\frac{Q_1Q_2}{r}$$
$$V(r) = k\frac{Q}{r}$$
$$E_x = -\frac{\Delta V_{B,A}}{\Delta x}$$
$$W_{A\to B} = q\Delta V_{A\to B} = -q\Delta V_{B\to A}$$

Magnetic Forces and Fields

$$F = qvB\sin\theta$$
$$F = ILB\sin\theta$$
$$\tau = NIAB\sin\theta = \mu B\sin\theta$$
$$PE = -\mu B\cos\theta$$
$$B = \frac{\mu_0 I}{2\pi r}$$
$$\varepsilon_{induced} = -N\frac{\Delta\phi_B}{\Delta t} = -N\frac{\Delta(BA\cos\theta)}{\Delta t}$$

Electric Circuits

$$I = \frac{\Delta Q}{\Delta t}$$
$$V = IR = I\left(\frac{\rho L}{A}\right)$$
$$R_{series} = \sum_{i=1}^{N} R_i$$
$$\frac{1}{R_{parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$$
$$P = IV = I^2R = \frac{V^2}{R}$$
$$Q = CV = \left(\frac{\kappa\varepsilon_0 A}{d}\right)V = (\kappa C_0)V$$
$$PE = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}$$
$$Q_{charge}(t) = Q_{max}\left(1 - e^{-\frac{t}{RC}}\right)$$
$$Q_{discharge}(t) = Q_{max}e^{-\frac{t}{RC}}$$
$$C_{parallel} = \sum_{i=1}^{N} C_i$$
$$\frac{1}{C_{series}} = \sum_{i=1}^{N}\frac{1}{C_i}$$

Light as a Wave

$$c = f\lambda = \frac{1}{\sqrt{\varepsilon_o\mu_o}}$$
$$S(t) = \frac{energy}{time \times area} = c\varepsilon_o E^2(t) = c\frac{B^2(t)}{\mu_0}$$
$$I = S_{avg} = \tfrac{1}{2}c\varepsilon_o E_{\max}^2 = c\frac{B_{\max}^2}{2\mu_0}$$
$$P = \frac{S}{c} = \frac{Force}{Area}$$
$$S = S_o\cos^2\theta$$
$$v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{n}$$
$$\theta_{inc} = \theta_{refl}$$
$$n_1\sin\theta_1 = n_2\sin\theta_2$$
$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$
$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$
$$M_{total} = \prod_{i=1}^{N} M_i$$
$$d\sin\theta = m\lambda \text{ or } (m + \tfrac{1}{2})\lambda$$
$$a\sin\phi = m'\lambda$$

Constants

$$g = 9.8\,\tfrac{m}{s^2}$$
$$G = 6.67\times10^{-11}\,\tfrac{Nm^2}{kg^2}$$
$$1e = 1.6\times10^{-19}C$$
$$k = \frac{1}{4\pi\varepsilon_o} = 9\times10^9\,\tfrac{C^2}{Nm^2}$$
$$\varepsilon_o = 8.85\times10^{-12}\,\tfrac{Nm^2}{C^2}$$
$$1eV = 1.6\times10^{-19}J$$
$$\mu_o = 4\pi\times10^{-7}\,\tfrac{Tm}{A}$$
$$c = 3\times10^8\,\tfrac{m}{s}$$
$$h = 6.63\times10^{-34}Js$$
$$m_e = 9.11\times10^{-31}kg = \frac{0.511MeV}{c^2}$$
$$m_p = 1.67\times10^{-27}kg = \frac{937.1MeV}{c^2}$$
$$m_n = 1.69\times10^{-27}kg = \frac{948.3MeV}{c^2}$$
$$1amu = 1.66\times10^{-27}kg = \frac{931.5MeV}{c^2}$$
$$N_A = 6.02\times10^{23}$$
$$Ax^2 + Bx + C = 0 \to x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

Light as a Particle & Relativity

$$E = hf = \frac{hc}{\lambda} = pc$$
$$KE_{\max} = hf - \phi = eV_{stop}$$
$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\phi)$$
$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$p = \gamma mv$$
$$E_{total} = KE + E_{rest} = \gamma mc^2$$
$$E_{total}^2 = p^2c^2 + m^2c^4$$
$$E_{rest} = mc^2$$
$$KE = (\gamma - 1)mc^2$$

Nuclear Physics

$$E_{binding} = (Zm_p + Nm_n - m_{rest})c^2$$
$$\frac{\Delta N}{\Delta t} = -\lambda N_o \to N(t) = N_o e^{-\lambda t}$$
$$A(t) = A_o e^{-\lambda t}$$
$$m(t) = m_o e^{-\lambda t}$$
$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

Misc. Physics 110 Formulae

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\Delta(m\vec{v})}{\Delta t} = m\vec{a}$$
$$\vec{F} = -k\vec{y}$$
$$\vec{F}_C = m\frac{v^2}{R}\hat{r}$$
$$W = \Delta KE = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right) = -\Delta PE$$
$$PE_{gravity} = mgy$$
$$PE_{spring} = \tfrac{1}{2}ky^2$$
$$x_f = x_i + v_{ix}t + \tfrac{1}{2}a_x t^2$$
$$v_{fx} = v_{ix} + a_x t$$
$$v_{vx}^2 = v_{ix}^2 + 2a_x\Delta x$$

Geometry

$$Circles: \; C = 2\pi r = \pi D \qquad A = \pi r^2$$
$$Triangles: \; A = \tfrac{1}{2}bh$$
$$Spheres: \; A = 4\pi r^2 \qquad V = \tfrac{4}{3}\pi r^3$$