Physics 111

Exam #2

October 31, 2014

Name____________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg)\times\left(2\,\tfrac{m}{s}\right) = 10\,\tfrac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 3 points and each free-response part is worth 8 points

Problem #1	/22
Problem #2	/27
Problem #3	/22
Total	/71

I affirm that I have carried out my academic endeavors with full academic honesty.

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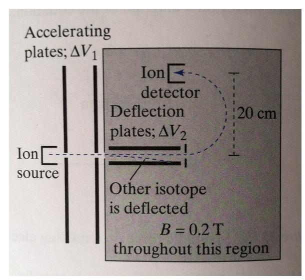
1. In organic material the ratio of ${}^{14}C$ to ${}^{12}C$ depends on how old the material is, which is the basis for "carbon-14 dating." ${}^{14}C$ is continually produced in the upper atmosphere by nuclear reactions caused by cosmic rays (mostly protons from the sun) and is radioactive with a half-life of $5730\,yrs$. When a cotton plant is growing, some of the CO_2 it extracts from the air to build tissue contains ${}^{14}C$ that has diffused down from the upper atmosphere. After the cotton plant is harvested, and say made into a cotton cloth, there is no further uptake of ${}^{14}C$ from the air, and the cosmic rays that create ${}^{14}C$ in the atmosphere cannot penetrate to the surface and reach the cloth. The amount of ${}^{14}C$ in a cotton cloth decreases with time while the amount of ${}^{12}C$ remains constant. A mass spectrometer shown below is used to ionize a sample of cotton cloth. Carbon from the sample starts out in the ion source and is accelerated through a region of space across which a potential difference ΔV_1 exits. The accelerated ions (${}^{14}C^{+}$ and ${}^{12}C^{+}$) then pass through a region of space where there exists crossed electric and magnetic fields. The magnetic field is fixed at a value of $2T$ while varying the potential difference ΔV_2 across the deflection plates (separated by $1cm$) can vary the electric field in this region of space.

   a. In terms of the quantities given in the problem, what are the expression for ΔV_1 and ΔV_2?



There are three regions:

Accelerating: $q\Delta V_1 = \frac{1}{2}mv^2 \rightarrow \Delta V_1 = \dfrac{mv^2}{2q} = \dfrac{qB^2R}{2m}$

Deflecting:

$$F_B - F_E = ma_y = 0 \rightarrow qvB = qE \rightarrow vB = E = \frac{\Delta V_2}{d} \rightarrow \Delta V_2 = vBd = \frac{qB^2Rd}{m}$$

Magnetic field only: $F_B = qvB = m\dfrac{v^2}{R} \rightarrow v\dfrac{qBR}{m}$

   b. Using the data supplied in the problem, what are two approximate values for ΔV_1 and ΔV_2 for ${}^{14}C^{+}$?

$$\Delta V_1 = \frac{qB^2R}{2m} = \frac{1.6\times10^{-19}C\times(2T)^2\times0.2m}{2\left(14\times1.67\times10^{-27}kg\right)} = 2.7\times10^{6}V = 2.7MV$$

$$\Delta V_2 = \frac{qB^2Rd}{m} = \frac{1.6\times10^{-19}C\times(2T)^2\times0.2m\times0.01m}{14\times1.67\times10^{-27}kg} = 5.5\times10^{4}V = 55kV$$

c. For $^{12}C^{+}$ what would happen to the values of ΔV_1 and ΔV_2 assuming that the detector location, deflector plate spacing, and magnetic field remain unchanged?

1. $\Delta V_1 \uparrow$ and $\Delta V_2 \uparrow$.
2. $\Delta V_1 \uparrow$ and $\Delta V_2 \downarrow$.
3. $\Delta V_1 \downarrow$ and $\Delta V_2 \uparrow$.
4. $\Delta V_1 \downarrow$ and $\Delta V_2 \downarrow$.
5. The values of ΔV_1 and ΔV_2 would remain constant and not change with ion species.

d. Suppose that instead of accelerating carbon ions out of the ion source Cl^{-} were instead accelerated out of the ion source. In this situation, what change(s) to the mass spectrometer would have to happen to detect the ions at the detector located in the same position? Assume that the magnetic field is constant in magnitude and direction.

1. The magnitude of the accelerating potential difference would have to be altered since the chlorine ions have different masses compared to the carbon ions.
2. The directions of the electric fields in the accelerating and deflecting regions would have to change direction.
3. The direction of the electric field in the accelerating region would change direction but the electric field in the deflection region would not change direction.
4. The direction of the electric field in the deflection region would change direction but the electric field in the accelerating region would not change direction.
5. The magnetic field would have to change direction.
6. There are no changes that would need to be made to the mass spectrometer.
7. This was not a choice on the exam, but the detector would in fact have to be moved also to the same numerical position but below the exit of the deflecting region rather than above.

2. Suppose that you have two wires that lie in the plane of the page. Each wire has a current of $I = 2A$ flowing, but in the leftmost wire the current is flowing up the page and in the rightmost wire the current is flowing down the page.

   a. What is the net magnetic field produced at the midpoint between the two wires if the wires are separated by $d = 1m$? (Hint: Be sure to specify a direction for your magnetic field vectors and assume that the currents in each wire are constant.)

   By the right hand rule, the magnetic field due to the current flow in each wire points into the page at the midpoint between the wires. Therefore, assuming into the page as the positive direction for the magnetic field vector we have,

   $$B_{net} = B_L + B_R = \frac{\mu_0 I_L}{2\pi r_L} + \frac{\mu_0 I_R}{2\pi r_R} = 2\left(\frac{\mu_0 I}{2\pi r}\right) = \frac{4\times10^{-7}\,\frac{Tm}{A}\times 2A}{0.5m} = 1.6\times10^{-6}T.$$

   b. Suppose that a copper penny (diameter $D = 19.1mm$ and thickness $t = 1.52mm$) were placed at the midpoint between the two wires. If the current in the leftmost wire were decreased from its maximum value to zero over a time $\Delta t = 1.2ms$, what are the magnitude and direction of the induced current in the penny? (Assume that the resistance of cupper is $R_{Cu} = 8.9\times10^{-8}\Omega$. Data on the penny are taken from http://www.usmint.gov/about_the_mint/?action=coin_specifications)

   The induced current is given by Ohm's law $I = \frac{\varepsilon}{R}$, where the induced potential difference is calculated from Faraday's law.

   $$|\varepsilon| = \left|N\frac{\Delta(BA\cos\theta)}{\Delta t}\right| = A\frac{\Delta B}{\Delta t} = \left(\pi r_p^2\right)\frac{\Delta B}{\Delta t}$$

   $$\varepsilon = \left|\pi\left(\frac{19.1\times10^{-3}m}{2}\right)^2\left(\frac{\frac{1.6\times10^{-6}T}{2} - 1.6\times10^{-6}T}{1.2\times10^{-3}s}\right)\right| = 1.91\times10^{-7}V.$$

   The current is therefore, $I = \frac{\varepsilon}{R} = \frac{1.91\times10^{-7}V}{8.9\times10^{-8}\Omega} = 2.15A$ clockwise to oppose the decrease in magnetic flux.

c. The energy per unit volume is defined by the quantity $\frac{IE}{A}$, where I is the current in Amperes, E is the electric field in volts per meter, and A is the cross sectional area of the penny in square meters. Using this definition, what is the total energy E_{total} that is dissipated by the penny due to the induced current flow in the penny?

$\frac{IE}{A}$ is an energy density. To get the total energy we'll have to at least multiply this by the volume of the penny. Working out the units we see that $\frac{IE}{A} \times volume$ is a power, so to calculate the total energy, we multiply by the time that the field changes. Therefore,

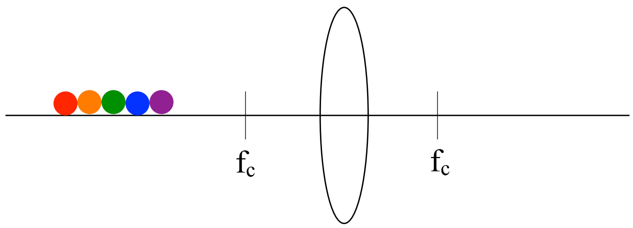
$$E_{total} = \left(\frac{IE}{A}\right) \times Volume \times time = \left(\frac{I\varepsilon}{Ad}\right) \times Ad \times \Delta t = I\varepsilon\Delta t .$$

$$E_{total} = I\varepsilon\Delta t = 2.15A \times 1.91 \times 10^{-7} V \times 1.2 \times 10^{-3} s = 4.9 \times 10^{-10} J$$

d. Suppose now that the current in both wires were to go to zero over the same amount of time Δt. In this case,

1. $I_{induced}$ would $\uparrow$ and E_{total} would $\uparrow$.
2. $I_{induced}$ would $\uparrow$ and E_{total} would $\downarrow$.
3. $I_{induced}$ would $\downarrow$ and E_{total} would $\uparrow$.
4. $I_{induced}$ would $\downarrow$ and E_{total} would $\downarrow$.

3. Suppose that you have the following situation in which you have a series of colored lights placed to the left of a converging lens, where the violet light is closer to the lens than the red light as shown below.



a. Given the situation above, what type of image is formed and what is the orientation of the colors of lights as seen in the image?
   1. The image is real and the order of the colors is  .
   2. The image is real and the order of the colors is  .
   3. The image is virtual and the order of the colors is  .
   4. The image is virtual and the order of the colors is  .

b. If the focal length of the lens is $f_c = 20cm$ and the violet light is located a distance $d_0 = 36cm$ to the left of the lens (as shown in the figure), what is the lateral magnification defined by $M = -\frac{L_i}{L_o}$? Assume that the object's length is $L_0 = 5cm$.

For the violet light,

$$\frac{1}{d_{oV}} + \frac{1}{d_{iV}} = \frac{1}{f_c} \rightarrow d_{iV} = \left(\frac{1}{f_c} - \frac{1}{d_{oV}}\right)^{-1} = \left(\frac{1}{20cm} - \frac{1}{36cm}\right)^{-1} = 45cm$$

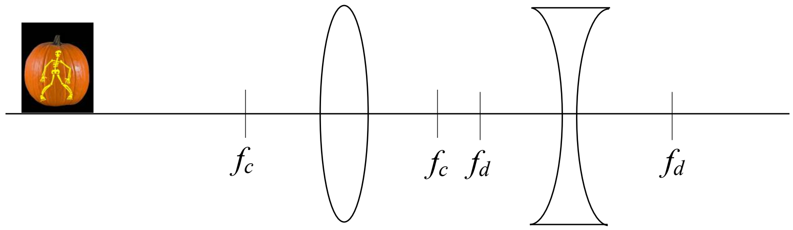
For the red light,

$$\frac{1}{d_{oR}} + \frac{1}{d_{iR}} = \frac{1}{f_c} \rightarrow d_{i1} = \left(\frac{1}{f_c} - \frac{1}{d_{oR}}\right)^{-1} = \left(\frac{1}{20cm} - \frac{1}{36cm + 5cm}\right)^{-1} = 39cm$$

The magnification here using the definition is

$$M = -\frac{L_i}{L_o} = -\left(\frac{45cm - 39cm}{5cm}\right) = -1.2$$

c. Suppose you have the arrangement of lenses shown below. The focal length of the converging lens is $f_c = 24cm$, the focal length of the diverging lens is $f_d = 12cm$, and a distance of $d = 40cm$ separates the lenses. Where will the final image be located with respect to the diverging lens and what will be the image properties if the object is located $100cm$ to the left of the converging lens?



The first image of the object is given by the thin lens equation. We have

$$\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{f_c} \rightarrow d_{i1} = \left(\frac{1}{f_c} - \frac{1}{d_{o1}}\right)^{-1} = \left(\frac{1}{24cm} - \frac{1}{100cm}\right)^{-1} = 31.6cm$$ to the right of the converging lens. This real image is de-magnified by an amount,

$$M = -\frac{d_{i1}}{d_{o1}} = -\frac{31.5cm}{100cm} = -0.315$$. This image becomes the object for the second lens. The final image location is given again by the thin lens equation.

$$\frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f_d} \rightarrow d_{i2} = \left(\frac{1}{f_d} - \frac{1}{d_{o2}}\right)^{-1} = \left(-\frac{1}{12cm} - \frac{1}{40cm - 31.5cm}\right)^{-1} = -5cm.$$

Thus the virtual image is located 5 cm to the left of the diverging lens. This image is further de-magnified by an amount $M = -\frac{d_{i1}}{d_{o1}} = -\frac{(-5cm)}{8.5cm} = 0.59$ and the final virtual image will be smaller than the object by an amount $M_T = M_1 M_2 = -0.315 \times 0.59 = -0.19$.

d. Which of the following would change in part c if the just the lenses were switched, everything else remaining the same?

1. Nothing associated with the final image would change.
2. The final image would become real, inverted with respect to the object, and have a magnification $M > 1$.
3. The final image would become real, inverted with respect to the object, and have a magnification $M < 1$.
4. The final image would become virtual, upright with respect to the object, and have a magnification $M > 1$.
5. The final image would become virtual, upright with respect to the object and have a magnification $M < 1$.
6. The final image would change, but in a way that it's properties cannot be predicted.

Physics 111 Equation Sheet

Electric Forces, Fields and Potentials

$$\vec{F} = k\frac{Q_1Q_2}{r^2}\hat{r}$$
$$\vec{E} = \frac{\vec{F}}{q}$$
$$\vec{E}_Q = k\frac{Q}{r^2}\hat{r}$$
$$PE = k\frac{Q_1Q_2}{r}$$
$$V(r) = k\frac{Q}{r}$$
$$E_x = -\frac{\Delta V}{\Delta x}$$
$$W = -q\Delta V$$

Magnetic Forces and Fields

$$F = qvB\sin\theta$$
$$F = IlB\sin\theta$$
$$\tau = NIAB\sin\theta = \mu B\sin\theta$$
$$PE = -\mu B\cos\theta$$
$$B = \frac{\mu_0 I}{2\pi r}$$
$$\varepsilon_{induced} = -N\frac{\Delta\phi_B}{\Delta t} = -N\frac{\Delta(BA\cos\theta)}{\Delta t}$$

Constants

$g = 9.8\,\frac{m}{s^2}$

$1e = 1.6\times10^{-19}C$

$k = \frac{1}{4\pi\varepsilon_o} = 9\times10^9\,\frac{Nm^2}{C^2}$

$\varepsilon_o = 8.85\times10^{-12}\,\frac{C^2}{Nm^2}$

$1eV = 1.6\times10^{-19}J$

$\mu_o = 4\pi\times10^{-7}\,\frac{Tm}{A}$

$c = 3\times10^8\,\frac{m}{s}$

$h = 6.63\times10^{-34}Js$

$m_e = 9.11\times10^{-31}kg = \frac{0.511MeV}{c^2}$

$m_p = 1.67\times10^{-27}kg = \frac{937.1MeV}{c^2}$

$m_n = 1.69\times10^{-27}kg = \frac{948.3MeV}{c^2}$

$1amu = 1.66\times10^{-27}kg = \frac{931.5MeV}{c^2}$

$N_A = 6.02\times10^{23}$

$Ax^2 + Bx + C = 0 \rightarrow x = \frac{-B\pm\sqrt{B^2-4AC}}{}$

Electric Circuits

$$I = \frac{\Delta Q}{\Delta t}$$
$$V = IR = I\left(\frac{\rho L}{A}\right)$$
$$R_{series} = \sum_{i=1}^{N} R_i$$
$$\frac{1}{R_{parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$$
$$P = IV = I^2R = \frac{V^2}{R}$$
$$Q = CV = \left(\frac{\kappa\varepsilon_0 A}{d}\right)V = (\kappa C_0)V$$
$$PE = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}$$
$$Q_{charge}(t) = Q_{max}\left(1 - e^{-\frac{t}{RC}}\right)$$
$$Q_{discharge}(t) = Q_{max}e^{-\frac{t}{RC}}$$
$$C_{parallel} = \sum_{i=1}^{N} C_i$$
$$\frac{1}{C_{series}} = \sum_{i=1}^{N}\frac{1}{C_i}$$

Light as a Wave

$$c = f\lambda = \frac{1}{\sqrt{\varepsilon_o\mu_o}}$$
$$S(t) = \frac{energy}{time\times area} = c\varepsilon_o E^2(t) = c\frac{B^2(t)}{\mu_0}$$
$$I = S_{avg} = \tfrac{1}{2}c\varepsilon_o E_{max}^2 = c\frac{B_{max}^2}{2\mu_0}$$
$$P = \frac{S}{c} = \frac{Force}{Area}$$
$$S = S_o\cos^2\theta$$
$$v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{n}$$
$$\theta_{inc} = \theta_{refl}$$
$$n_1\sin\theta_1 = n_2\sin\theta_2$$
$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$
$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$
$$M_{total} = \prod_{i=1}^{N} M_i$$
$$d\sin\theta = m\lambda \text{ or } \left(m+\tfrac{1}{2}\right)\lambda$$
$$a\sin\phi = m'\lambda$$

Light as a Particle & Relativity

$$E = hf = \frac{hc}{\lambda} = pc$$
$$KE_{max} = hf - \phi = eV_{stop}$$
$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\phi)$$
$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$p = \gamma mv$$
$$E_{total} = KE + E_{rest} = \gamma mc^2$$
$$E_{total}^2 = p^2c^2 + m^2c^4$$
$$E_{rest} = mc^2$$
$$KE = (\gamma - 1)mc^2$$

Nuclear Physics

$$E_{binding} = (Zm_p + Nm_n - m_{rest})c^2$$
$$\frac{\Delta N}{\Delta t} = -\lambda N_o \rightarrow N(t) = N_o e^{-\lambda t}$$
$$A(t) = A_o e^{-\lambda t}$$
$$m(t) = m_o e^{-\lambda t}$$
$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

Geometry

$Circles: \quad C = 2\pi r = \pi D \qquad A = \pi r^2$

$Triangles: \quad A = \tfrac{1}{2}bh$

$Spheres: \quad A = 4\pi r^2 \qquad V = \tfrac{4}{3}\pi r^3$

Misc. Physics 110 Formulae

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\Delta(m\vec{v})}{\Delta t} = m\vec{a}$$
$$\vec{F} = -k\vec{y}$$
$$\vec{F}_C = m\frac{v^2}{R}\hat{r}$$
$$W = \Delta KE = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right) = -\Delta PE$$
$$PE_{gravity} = mgy$$
$$PE_{spring} = \tfrac{1}{2}ky^2$$
$$x_f = x_i + v_{ix}t + \tfrac{1}{2}a_x t^2$$
$$v_{fx} = v_{ix} + a_x t$$
$$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x$$