Physics 111

Exam #1

November 2, 2018

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg)\times(2\,\tfrac{m}{s}) = 10\,\tfrac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 3 points and each free-response part is worth 9 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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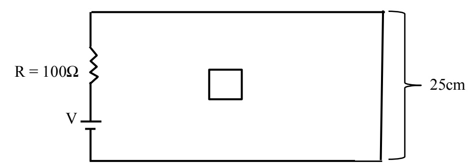
1. Magnetism

   a. Consider the circuit shown below where a variable voltage battery is connected to a 100Ω resistor. The upper and lower wires are very long and a small rectangular loop of wire is placed at the midpoint between the upper and lower wires. The small wire loop has dimensions $10mm \times 5mm$ and a resistance $0.7m\Omega$. If the battery potential varies from $0V$ to $75V$ in $0.5s$, what is the magnitude and direction of the current flow induced in the small wire loop?



As the potential of the battery in the circuit increases, the current in the circuit increases. This increases the magnetic flux through the loop of wire and to oppose the increase in magnetic flux, the loop of wire will produce a CCW current flow by the right-hand rule.

The current:

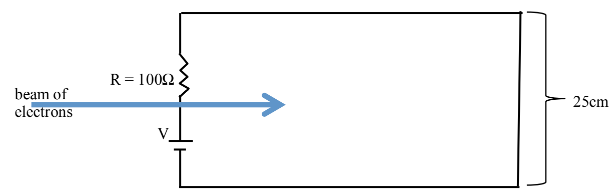
$$I = \frac{\varepsilon}{R_{loop}} = \frac{1}{R_{loop}}\left[\left|-N\frac{\Delta BA\cos\theta}{\Delta t}\right|\right] = \frac{A}{R_{loop}}\left|\frac{\Delta B}{\Delta t}\right|;$$

$$\Delta B = B_f - B_i = \left(\frac{\mu_0 I_f}{2\pi r_{upper}} + \frac{\mu_0 I_f}{2\pi r_{lower}}\right) - \left(\frac{\mu_0 I_i}{2\pi r_{upper}} + \frac{\mu_0 I_i}{2\pi r_{lower}}\right) = 2\left(\frac{\mu_0 I}{2\pi r_{upper}}\right)$$

$$\rightarrow \Delta B = 2\left(\frac{\mu_0 \frac{V}{R_{circuit}}}{2\pi r_{upper}}\right) = \frac{4\times10^{-7}\,\frac{Tm}{A}\times\left(75V/100\Omega\right)}{0.125m} = 2.4\times10^{-6}T$$

$$\therefore I = \frac{A}{R_{loop}}\left|\frac{\Delta B}{\Delta t}\right| = \frac{\left(10\times10^{-3}m\times5\times10^{-3}m\right)\times2.4\times10^{-6}T}{0.75\times10^{-3}\Omega\times0.5s} = 3.43\times10^{-7}A = 343nA$$

b. Suppose the small wire loop were removed and a beam of electrons were instead incident at a constant rate of $3\times10^{15}\,\frac{electrons}{s}$ from the left toward the right across the top of the circuit? If the beam passes across the circuit at the midpoint between the upper and lower wires, what magnetic force would the beam of electrons feel? Assume that the electrons are accelerated from rest through a potential difference $4MV$ by an accelerator located somewhere on the far left not shown. Assume that the battery has reached its maximum potential of $75V$ and remains constant.



$$W=-q\Delta V=-(-e)\left[4MV-0MV\right]=4MeV$$

$$W=(\gamma-1)mc^2\rightarrow\gamma=\frac{W}{mc^2}+1=\frac{4MeV}{0.511\frac{meV}{c^2}c^2}+1=8.828$$

$$\gamma=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\rightarrow v=\sqrt{1-\frac{1}{\gamma^2}}c=\sqrt{1-\frac{1}{(8.828)^2}}c=0.994c$$

$$\left|\vec{F}\right|=qvB=1.6\times10^{-19}C\times0.994c\times2.4\times10^{-6}T=1.15\times10^{-16}N$$

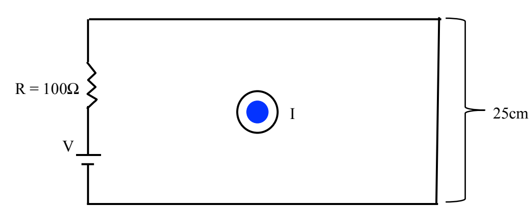
and the direction of the force is toward the lower wire by the right-hand rule.

c. A charge q moves in through a magnetic field $\vec{B}$ with its velocity vector $\vec{v}$ oriented at an angle θ with respect to the magnetic field. As the angle θ between the velocity vector and the magnetic field increases which of the following is true?

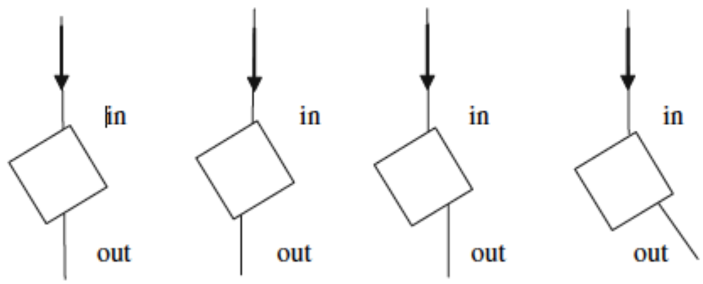
1. The orbital radius and orbital period (the time it takes to make one loop about the magnetic field line) both increase.
2. The orbital radius and orbital period (the time it takes to make one loop about the magnetic field line) would both decrease.
3. The orbital radius would increase while the orbital period (the time it takes to make one loop about the magnetic field line) would decrease.
4. The orbital radius would decrease while the orbital period (the time it takes to make one loop about the magnetic field line) would increase.
5. The orbital radius would increase while the orbital period (the time it takes to make one loop about the magnetic field line) would remain constant because the period is independent of the angle between the velocity vector and the magnetic field.
6. The orbital radius would decrease while the orbital period (the time it takes to make one loop about the magnetic field line) would remain constant because the period is independent of the angle between the velocity vector and the magnetic field.

d. Consider the circuit from part a above, redrawn below, in which the wire loop is removed and a long straight wire is placed at the midpoint between the upper and lower wires in such a way that the current flowing in the wire is up out of the plane of the page at you while the circuit lies in the plane of the page. In this case, which of the following is true?

1. The magnetic force would cause the wire to move to the right.
2. The magnetic force would cause the wire to move to the left.
3. The magnetic force would cause the wire to move up the page.
4. The magnetic force would cause the wire to move down the page.
5. The magnetic force on the wire is zero and the wire will not move.

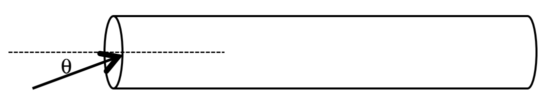


2. Light as a wave
   a. A $0.5mW$ laser green laser pointer $(\lambda = 550nm)$ is aimed at a glass cube shown below. Which of the following best illustrates the direction of the laser beam when it emerges from the glass cube?



1. 2. 3. 4.

   b. Suppose that you wanted to guide red laser light $(\lambda = 630nm)$ down a piece of plastic $n_p = 1.2$. In the diagram below, at what angle of incidence θ with respect to the normal on the front surface of the plastic should the red laser light be incident so that the light will propagate entirely in the plastic? Assume that the plastic is surrounded on all sides by air.



$$n_p \sin\theta_C = n_a \sin 90 \rightarrow \theta_C = \sin^{-1}\frac{n_a}{n_p} = \sin^{-1}\frac{1.0}{1.2} = 56.4^0$$

$$\theta_2 + \theta_C = 90 \rightarrow \theta_2 = 90 - \theta_C = 33.6^0$$

$$n_a \sin\theta = n_p \sin\theta_2 \rightarrow \theta = \sin^{-1}\left(\frac{n_p}{n_a}\sin\theta_2\right) = \sin^{-1}\left(\frac{1.2}{1.0}\sin 33.6^0\right) = 41.6^0$$

c. Suppose that an optical device were constructed out of two lenses. The first lens has a focal length of $f_1 = +1cm$ and a second lens with focal length $f_2 = +25cm$ is placed a distance $D = 30cm$ to the right of the first lens. A $1mm$ tall object is located $d_{01} = 1.1cm$ to the left of lens 1. With respect to lens 2, where will the final image be located and what will be the size of the final image?

Lens 1:

$$\frac{1}{f_1} = \frac{1}{d_{o1}} + \frac{1}{d_{i1}} \rightarrow d_{i1} = \left(\frac{1}{f_1} - \frac{1}{d_{o1}}\right)^{-1} = \left(\frac{1}{1cm_1} - \frac{1}{1.1cm}\right)^{-1} = 11cm$$

$$M_1 = -\frac{d_{i1}}{d_{o1}} = -\frac{11cm}{1.1cm} = -10 \rightarrow h_{i1} = M_1 h_o = -10 \times 1mm = -10mm$$

Lens 2:

$$D = d_{o2} + d_{i1} \rightarrow d_{o2} = D - d_{i1} = 30cm - 11cm = 19cm$$

$$\frac{1}{f_2} = \frac{1}{d_{o2}} + \frac{1}{d_{i2}} \rightarrow d_{i2} = \left(\frac{1}{f_2} - \frac{1}{d_{o2}}\right)^{-1} = \left(\frac{1}{25cm} - \frac{1}{19cm}\right)^{-1} = -79.2cm$$

$$M_2 = -\frac{d_{i2}}{d_{o2}} = -\frac{-79.2cm}{19cm} = 4.2 \rightarrow h_{i2} = M_2 h_{o2} = M_2 h_{i1} = 4.2 \times -10mm = -42mm$$

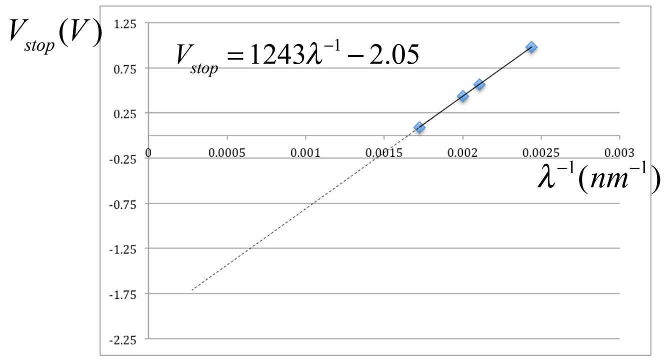
The final image is a virtual image located $79.2cm$ to the left of the second lens. The final image is inverted with respect to the original object and it is magnified by a factor of 42.

d. Which of the following methods could be used to determine the focal length of a diverging lens?

1. Place an object at a distance d_0 from the diverging lens and measure the real image distance d_i. Then use the thin lens equation to determine the focal length of the diverging lens.
2. Set several object distances d_0 and measure the corresponding real image distances d_i. Then from a plot of d_0 versus d_i, the slope will tell you the focal length of the lens.
3. Use a second diverging lens with a known focal length (placed a distance D from the first) to produce a real image of an object. From the data acquired use the thin lens equation twice to determine the unknown focal length for the diverging lens.
4. Use a converging lens with a known focal length (placed a distance D from the first) to produce a real image of the object. From the data acquired use the thin lens equation twice to determine the unknown focal length for the diverging lens.
5. There is no experimental method that can be used to measure the focal lengths of diverging lenses.

3. Light as a particle

Consider the plot shown below for data taken for a photoelectric effect experiment with an unknown metal, where the stopping potential difference is plotted as a function of the inverse of the wavelength of light incident on the metal.



a. From the plot and the table below, what is the identity of the metal used for the emitter in the experiment?
   1. Lithium.
   2. Rubidium.
   3. Cesium.
   4. Calcium.
   5. Barium.

Metal	$\phi(eV)$
Cesium	1.95
Rubidium	2.05
Lithium	2.36
Barium	2.52
Calcium	2.87

b. From the graph, the slope of the line relates which fundamental constants?

   1. Planck's constant and the speed of light.
   2. Planck's constant, the speed of light, and the charge of the electron.
   3. Planck's constant and the charge of the electron
   4. The speed of light and the charge of the electron.
   5. Stopping potential difference and the wavelength of light.

c. Suppose that you want to perform a photoelectric effect experiment with $100nm$ light incident on a platinum surface $\phi = 5.63eV$. What are the speed of the ejected electrons (if any are ejected) and what potential difference would be needed to stop the ejected electrons from hitting the collector? If no electrons are ejected with this wavelength of light, what is the maximum wavelength that could be used to eject electrons and what potential difference would be needed to stop them from hitting the collector in this case? If there are ejected electrons, express their speeds as a fraction of the speed of light.

$$K = \frac{hc}{\lambda} - \phi = \left[\left(\frac{6.63\times10^{-34}\,Js\times3\times10^{8}\,\tfrac{m}{s}}{100\times10^{-9}\,m}\right)\times\frac{1eV}{1.6\times10^{-19}\,J}\right] - 5.63eV = 6.8eV$$

$$K = eV_{stop} = 6.8eV \rightarrow V_{stop} = 6.8V$$

$$K = (\gamma - 1)mc^2 \rightarrow \gamma = \frac{K}{mc^2} + 1 = \frac{6.8eV}{511000\,\tfrac{eV}{c^2}\,c^2} + 1 = 1.0000133 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\rightarrow v = \sqrt{1-\frac{1}{\gamma^2}}\,c = \sqrt{1-\frac{1}{(1.0000133)^2}}\,c = 0.0052c$$

d. Suppose that instead of using the $100nm$ light in part c for the photoelectric effect experiment you decide instead to shine the light onto a wall in the room. The light makes a circular spot with a diameter of $4mm$ and has an intensity of $100\,\frac{mW}{m^2}$. How many photons per second strike the wall?

$$S = \frac{Energy}{time \bullet Area} = \frac{NE_{photon}}{time \bullet Area}$$

$$\rightarrow \frac{N}{t} = \frac{SA}{E} = \frac{100\times10^{-3}\,\frac{W}{m^2}\times\pi\left(2\times10^{-3}\,m\right)^2}{6.63\times10^{-34}\,Js\times3\times10^{8}\,\frac{m}{s}\Big/100\times10^{-9}\,m} = 6.3\times10^{11}\,\frac{photons}{s}$$

Physics 111 Equation Sheet

Electric Forces, Fields and Potentials

$$\vec{F} = k\frac{Q_1Q_2}{r^2}\hat{r}$$

$$\vec{E} = \frac{\vec{F}}{q}$$

$$\vec{E}_Q = k\frac{Q}{r^2}\hat{r}$$

$$PE = k\frac{Q_1Q_2}{r}$$

$$V(r) = k\frac{Q}{r}$$

$$E_x = -\frac{\Delta V}{\Delta x}$$

$$W = -q\Delta V = -q\left[V_f - V_i\right]$$

Magnetic Forces and Fields

$$F = qvB\sin\theta$$

$$F = IlB\sin\theta$$

$$\tau = NIAB\sin\theta = \mu B\sin\theta$$

$$PE = -\mu B\cos\theta$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\varepsilon_{induced} = -N\frac{\Delta\phi_B}{\Delta t} = -N\frac{\Delta(BA\cos\theta)}{\Delta t}$$

Constants

$$g = 9.8\,\tfrac{m}{s^2}$$

$$1e = 1.6\times10^{-19}C$$

$$k = \frac{1}{4\pi\varepsilon_o} = 9\times10^9\,\tfrac{Nm^2}{C^2}$$

$$\varepsilon_o = 8.85\times10^{-12}\,\tfrac{C^2}{Nm^2}$$

$$1eV = 1.6\times10^{-19}J$$

$$\mu_o = 4\pi\times10^{-7}\,\tfrac{Tm}{A}$$

$$c = 3\times10^8\,\tfrac{m}{s}$$

$$h = 6.63\times10^{-34}Js$$

$$m_e = 9.11\times10^{-31}kg = \frac{0.511MeV}{c^2}$$

$$m_p = 1.67\times10^{-27}kg = \frac{937.1MeV}{c^2}$$

$$m_n = 1.69\times10^{-27}kg = \frac{948.3MeV}{c^2}$$

$$1amu = 1.66\times10^{-27}kg = \frac{931.5MeV}{c^2}$$

$$N_A = 6.02\times10^{23}$$

$$Ax^2 + Bx + C = 0 \rightarrow x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

Electric Circuits

$$I = \frac{\Delta Q}{\Delta t}$$

$$V = IR = I\left(\frac{\rho L}{A}\right)$$

$$R_{series} = \sum_{i=1}^{N} R_i$$

$$\frac{1}{R_{parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$$

$$P = IV = I^2R = \frac{V^2}{R}$$

$$Q = CV = \left(\frac{\kappa\varepsilon_0 A}{d}\right)V = (\kappa C_0)V$$

$$W = U = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}$$

$$Q_{\text{charge}}(t) = Q_{\max}\left(1 - e^{-\frac{t}{RC}}\right)$$

$$Q_{\text{discharge}}(t) = Q_{\max}e^{-\frac{t}{RC}}$$

$$C_{parallel} = \sum_{i=1}^{N} C_i$$

$$\frac{1}{C_{series}} = \sum_{i=1}^{N}\frac{1}{C_i}$$

Light as a Particle & Relativity

$$E = hf = \frac{hc}{\lambda} = pc$$

$$KE_{\max} = hf - \phi = eV_{stop}$$

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\phi)$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$p = \gamma mv$$

$$E_{total} = KE + E_{rest} = \gamma mc^2$$

$$E_{total}^2 = p^2c^2 + m^2c^4$$

$$E_{rest} = mc^2$$

$$KE = (\gamma-1)mc^2$$

Geometry

Circles: $C = 2\pi r = \pi D \qquad A = \pi r^2$

Triangles: $A = \tfrac{1}{2}bh$

Spheres: $A = 4\pi r^2 \qquad V = \tfrac{4}{3}\pi r^3$

Light as a Wave

$$c = f\lambda = \frac{1}{\sqrt{\varepsilon_o\mu_o}} = nv$$

$$S(t) = \frac{energy}{time\times area} = c\varepsilon_o E^2(t) = c\frac{B^2(t)}{\mu_0}$$

$$I = S_{avg} = \tfrac{1}{2}c\varepsilon_o E_{\max}^2 = c\frac{B_{\max}^2}{2\mu_0}$$

$$P = \begin{cases} \dfrac{S}{c} \\ \dfrac{2S}{c} \end{cases} = \frac{Force}{Area}$$

$$S = S_o\cos^2\theta$$

$$v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{n}$$

$$\theta_{inc} = \theta_{refl}$$

$$n_1\sin\theta_1 = n_2\sin\theta_2$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$M_{total} = \prod_{i=1}^{N} M_i$$

$$S = S_o e^{-\mu x}$$

Nuclear Physics

$$E_{binding} = (Zm_p + Nm_n - m_{rest})c^2$$

$$\frac{\Delta N}{\Delta t} = -\lambda N_o \rightarrow N(t) = N_o e^{-\lambda t}$$

$$A(t) = A_o e^{-\lambda t}$$

$$m(t) = m_o e^{-\lambda t}$$

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

Misc. Physics 110 Formulae

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\Delta(m\vec{v})}{\Delta t} = m\vec{a}$$

$$\vec{F} = -k\vec{y}$$

$$\vec{F}_C = m\frac{v^2}{R}\hat{r}$$

$$W = \Delta KE = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right) = -\Delta PE$$

$$PE_{gravity} = mgy$$

$$PE_{spring} = \tfrac{1}{2}ky^2$$

$$x_f = x_i + v_{ix}t + \tfrac{1}{2}a_x t^2$$

$$v_{fx} = v_{ix} + a_x t$$

$$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x$$