Physics 111

Exam #2

October 30, 2020

Name_____

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}| = (5kg) \times \left(2\frac{m}{s}\right) = 10\frac{kg \cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple-choice questions are worth 3 points and each free-response part is worth 7 points.

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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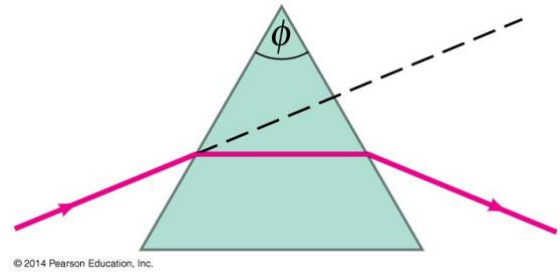
1. Reflection, Refraction and Lenses

a. The upper (or apex) angle of a prism is $\phi = 75^0$. What is the minimum indecent angle that light can make on the left face of the prism for the ray to emerge on the opposite side? That is, what is the minimum angle on the left surface such that the light will not be totally internally reflected if the index of refraction of the prism is $n = 1.58$?



On the left face:
$n_{air} \sin\theta_{air} = n \sin\theta_L$

On the right face:
$n \sin\theta_R = n_{air} \sin\theta'_{air} = 1 \times \sin 90 = 1$
$\rightarrow 1.58 \sin\theta_R = 1 \rightarrow \theta_R = 39.3$

The triangle:
$90 - \theta_L + 90 - \theta_R + 75 = 180 \rightarrow \theta_L + \theta_R = 75$
$\rightarrow \theta_L = 75 - \theta_R = 75 - 39.3 = 35.7$

$\rightarrow n_{air} \sin\theta_{air} = n \sin\theta_L = 1.58 \sin 35.7 = 0.922 \rightarrow \theta_{air} = 67.3$

b. Consider making a camera by using a lens with an unknown focal length. To determine the focal length of the lens you are using, you hold it over a piece of paper with some writing on it. The paper is $18.5cm$ away from the lens and the writing is seen to be magnified by a factor of 5.5. Then the lens is put into the camera and is used to take a picture of an object $0.62m$ tall. The image on the film is found to be $3.5cm$ tall. How far away from the object was the lens in the camera?

Focal length of the lens:
$\frac{1}{f_c} = \frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{d_0} - \frac{1}{Md_o} = \frac{1}{18.5cm} - \frac{1}{5.5 \times 18.5cm} \rightarrow f_c = 22.6cm$

The camera:
$M = \frac{h_i}{h_0} = \frac{0.035m}{0.62m} = 0.057$
$M = \frac{d_i}{d_0} \rightarrow d_i = Md_0$
$$\frac{1}{f_c} = \frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{d_0} + \frac{1}{Md_o} = \frac{1}{d_0}\left(1 + \frac{1}{0.057}\right)$$
$\rightarrow d_0 = 18.5 f_c = 18.5 \times 22.6cm = 419cm = 4.2m$

c. Two lenses are used to form an image of an object. Lens 1 ($f_1 = 124mm$) is placed to the left of lens 2 ($f_2 = 66mm$) by an amount D. A $1cm$ tall object is placed a distance of $72mm$ to the left of lens 1. A real image is produced on a screen $87.2mm$ to the right of lens 2. What was the separation, D, between the lenses and what is the final image height of the object?

For lens 1:

$$\frac{1}{f_{c1}} = \frac{1}{d_{01}} + \frac{1}{d_{i1}} \rightarrow \frac{1}{124mm} = \frac{1}{72mm} + \frac{1}{d_{1i}} \rightarrow d_{i1} = -171.7mm$$

For lens 2:

$$\frac{1}{f_c} = \frac{1}{d_0} + \frac{1}{d_i} \rightarrow \frac{1}{66mm} = \frac{1}{d_{02}} + \frac{1}{87.2mm} \rightarrow d_{02} = 271.5mm$$

$$d_{02} = D + d_{i1} \rightarrow D = d_{02} - d_{i1} = 271.5mm - 171.7mm = 99.8mm$$

$$\frac{h_{if}}{h_0} = M_T = M_1 M_2 = \left|\left(-\frac{d_{i2}}{d_{o2}}\right)\left(-\frac{d_{i1}}{d_{o1}}\right)\right| = \left(\frac{171.7mm}{72mm}\right)\left(\frac{87.2mm}{271.5mm}\right) = 0.77$$

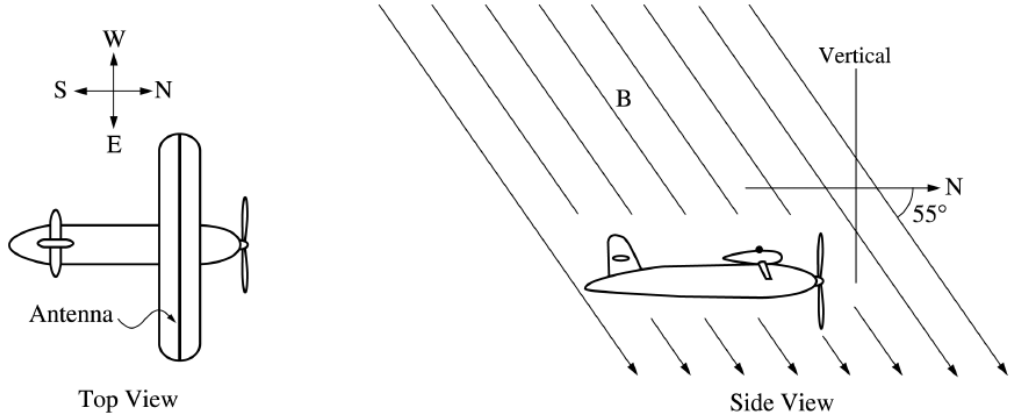
$$h_{if} = M_T h_0 = 0.77 \times 1cm = 0.77cm$$

d. Suppose that the lens from part b were placed under water, where $n_{water} > n_{air}$ and $n_{lens} < n_{water}$. A parallel beam of light is incident on the lens under water. Which of the following gives the focal length of the lens under water (f_{water}) compared to the focal length of the lens in air (f_{air})?

1. $f_{water} > f_{air}$.
2. $f_{water} = f_{air}$.
3. $f_{water} < f_{air}$.
4. The focal length cannot be determined from the information given.

2. Airplanes in a magnetic field

An airplane has an aluminum antenna attached to its wing that extends $15m$ from wingtip to wingtip. The plane is traveling north at $300\frac{m}{s}$ in a region where the Earth's magnetic field has both a vertical component and a northward component, as shown below. The net magnetic field is at an angle of 55^0 measured with respect to the horizontal and the magnitude of the magnetic field is $6 \times 10^{-5}T$.



a. What is the magnetic force on electrons in the antennae?

$F = qvB\sin\theta = 1.6 \times 10^{-19}C \times 300\frac{m}{s} \times 6 \times 10^{-5}T \sin 55 = 2.4 \times 10^{-21}N$

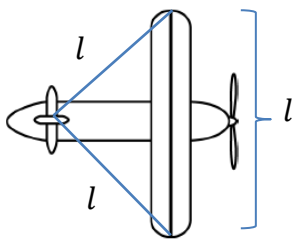
Direction: By the right-hand-rule the electrons move down the page toward the right wingtip.

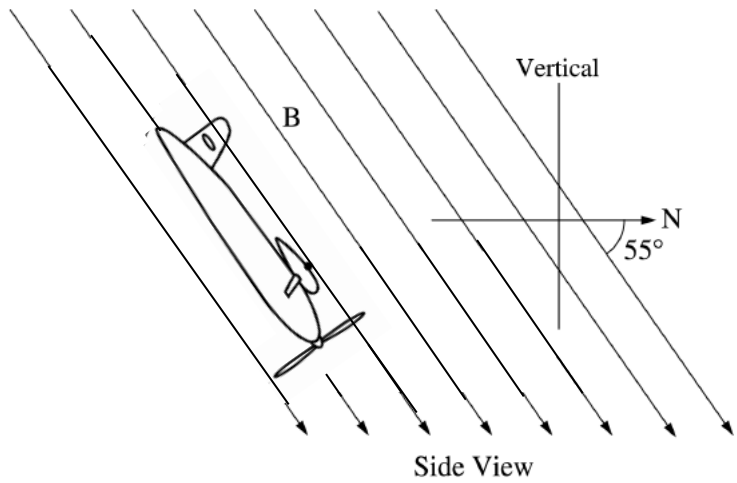
b. What is the net electric field generated in the antenna and what is the potential difference across the antenna?

$F = qE \rightarrow E = \frac{F}{q} = \frac{2.4\times10^{-21}N}{1.6\times10^{-19}C} = 0.015\frac{N}{C}$ up the page toward the left wingtip.

$E = \left|-\frac{\Delta V}{\Delta y}\right| = \frac{\Delta V}{\Delta y} \rightarrow \Delta V = E\Delta y = 0.015\frac{N}{C} \times 15m = 0.23V$

c. Suppose that the ends of the antenna are now connected by conducting wires (in blue) so that a closed circuit is formed and further, suppose the plane decided to tip its nose downward so that it flies along and in the direction of the magnetic field as shown below. What is the magnitude and direction of the current induced in the loop of wire if the loop has a resistance of $R = 100\Omega$? Assume that the plane takes $10s$ to transition from flying horizontal to flying along the magnetic field and that the wires each have length $l = 15m$.





$$\varepsilon = \left|-N\frac{\Delta(BA\cos\theta)}{\Delta t}\right|$$

$$\varepsilon = \left|BA\frac{\Delta\cos\theta}{\Delta t}\right|$$

$$\varepsilon = \left|6\times10^{-5}T\times2\left(\frac{1}{2}\times\frac{15m}{2}\times13m\right)\times\frac{\cos 90-\cos 35}{10s}\right| = 4.8\times10^{-4}V$$

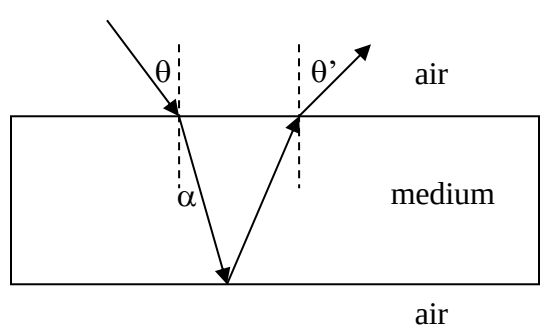
Where the height of the triangle is: $\sin 60 = \frac{height}{l} \rightarrow height = l\sin 60 = 15m\sin 60 = 13m$

$$I = \frac{\varepsilon}{R} = \frac{4.8\times10^{-4}V}{100\Omega} = 4.8\times10^{-6}A = 4.8\mu A$$

The direction of the induced current is CW to undo the decrease in magnetic flux.

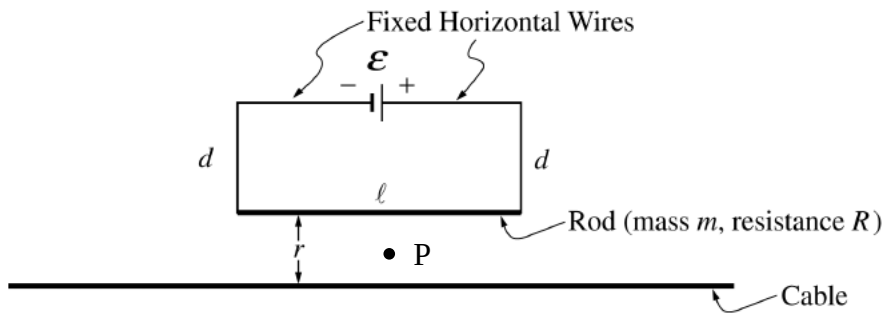
d. A beam of light shines from air into a transparent medium having two parallel surfaces. Part of the beam is reflected from the second surface as shown below. The index of refraction of the medium is *1.5*. Which of the following describes the relationship between the angles of incidence and reflection and the relationship between the angles of incidence and refraction?

a. $\theta < \theta'$ and $\theta < \alpha$
b. $\theta = \theta'$ and $\theta > \alpha$
c. $\theta = \theta'$ and $\theta < \alpha$
d. $\theta > \theta'$ and $\theta > \alpha$



3. Magnetic forces and fields

The circuit shown below consists of a battery with potential difference $\varepsilon = 1000V$ in series with a rod of length $l = 5m$, mass $m = 140g$, and resistance $R = \frac{1}{2}\Omega$. The rod is suspended by vertical connecting wires of length $d = 100cm$, and the horiztonal wire that is connnected to the battery is fixed. All of the connecting wires have negligible resistance and mass. The rod is a distance $r = 1cm$ above a conducting cable. The cable is very long and is located directly below and parallel to the rod.



a. What magnitude and direction of current flowing in the cable (I_C) will produce no tension in the vertical connecting wires?

$$F_B - F_W = ma_y = 0 \rightarrow F_B = F_W \rightarrow I_R l B_C = mg \rightarrow \frac{\mu_o I_C}{2\pi r} = \frac{mg}{I_R l}$$

$$I_C = \frac{2\pi r m g}{\mu_0 I_R l} = \frac{2\pi \times 0.01m \times 0.140kg \times 9.8\frac{m}{s^2}}{4\pi \times 10^{-7}\frac{Tm}{A} \times \left(\frac{1000V}{0.5\Omega}\right) \times 5m} = 6.86A$$

The direction of the current in the cable would be from left to right across the page to produce a magnetic field out of the page at the rod and an upward magnetic force on the rod.

b. What is the net magnetic field at point P in the diagram, midway between the rod and the cable? Hint: Assume that $d \gg r$.

The magnetic field of the cable at point P points out of the page and the magnetic field of the rod also points out of the page. Thus, the net magnetic field will point out of the page with magnitude:

$$B_{net,P} = B_C + B_R = \frac{\mu_o I_C}{2\pi r'} + \frac{\mu_o I_R}{2\pi r'} = \frac{\mu_o}{2\pi\left(\frac{r}{2}\right)}(I_C + I_R) = \frac{\mu_0}{\pi r}(I_C + I_R)$$

$$B_{net,P} = \frac{4\pi \times 10^{-7}\frac{Tm}{A}}{\pi(0.01m)}(6.86A + 2000A) = 0.081T$$

c. Suppose that a beam of protons are accelerated from rest by a potential difference of $16MV$. At the end of the accelerating region, assume that the proton beam enters a uniform magnetic field with magnitude and direction given by your result to part b. If the velocity of the protons is perpendicular to the magnetic field, what are the orbital radius and period of the proton beam?

$$W = -q\Delta V = -(e)(0V - 16MV) = 16MeV$$
$$W = (\gamma - 1)m_p c^2 = (\gamma - 1)\left(937.1\frac{MeV}{c^2}\right)c^2 = 937.1MeV(\gamma - 1) = 16MeV$$
$$\gamma = \frac{16MeV}{937.1MeV} + 1 = 1.0172 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \rightarrow v = \sqrt{1 - \frac{1}{\gamma^2}}c = \sqrt{1 - \frac{1}{(1.0172)^2}}c = 0.18c$$

$$F = qvB = \frac{m_p v^2}{R} \rightarrow R = \frac{mv}{qB} = \frac{1.67 \times 10^{-27}kg \times 0.18 \times 3 \times 10^8\frac{m}{s}}{1.6 \times 10^{-19}C \times 0.081T} = 7.1m$$

$$v = \frac{2\pi R}{T} \rightarrow T = \frac{2\pi R}{v} = \frac{2\pi \times 7.1m}{0.18 \times 3 \times 10^8\frac{m}{s}} = 8.23 \times 10^{-7}s$$

d. Which of the following is true if the intensity of an electromagnetic wave doubles?

1. The electric field must increase by a factor of 2.
2. The magnetic field must increase by a factor of 2.
3. The electric and magnetic fields must each increase by a factor of $\sqrt{2}$.
4. The electric and magnetic fields must decrease by a factor of $\sqrt{2}$.
5. None of the above statements are correct.

Physics 111 Equation Sheet

Electric Forces, Fields and Potentials

$$\vec{F} = k\frac{Q_1Q_2}{r^2}\hat{r}$$

$$\vec{E} = \frac{\vec{F}}{q}$$

$$\vec{E}_Q = k\frac{Q}{r^2}\hat{r}$$

$$PE = k\frac{Q_1Q_2}{r}$$

$$V(r) = k\frac{Q}{r}$$

$$E_x = -\frac{\Delta V}{\Delta x}$$

$$W = -q\Delta V = -q\left[V_f - V_i\right]$$

Magnetic Forces and Fields

$$F = qvB\sin\theta$$

$$F = IlB\sin\theta$$

$$\tau = NIAB\sin\theta = \mu B\sin\theta$$

$$PE = -\mu B\cos\theta$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\varepsilon_{induced} = -N\frac{\Delta\phi_B}{\Delta t} = -N\frac{\Delta(BA\cos\theta)}{\Delta t}$$

Constants

$$g = 9.8\,\tfrac{m}{s^2}$$

$$1e = 1.6\times10^{-19}C$$

$$k = \frac{1}{4pe_o} = 9\times10^{9}\,\tfrac{Nm^2}{C^2}$$

$$e_o = 8.85\times10^{-12}\,\tfrac{C^2}{Nm^2}$$

$$1eV = 1.6\times10^{-19}J$$

$$m_0 = 4p\times10^{-7}\,\tfrac{Tm}{A}$$

$$c = 3\times10^{8}\,\tfrac{m}{s}$$

$$h = 6.63\times10^{-34}Js$$

$$m_e = 9.11\times10^{-31}kg = \frac{0.511MeV}{c^2}$$

$$m_p = 1.67\times10^{-27}kg = \frac{937.1MeV}{c^2}$$

$$m_n = 1.69\times10^{-27}kg = \frac{948.3MeV}{c^2}$$

$$1amu = 1.66\times10^{-27}kg = \frac{931.5MeV}{c^2}$$

$$N_A = 6.02\times10^{23}$$

$$Ax^2 + Bx + C = 0 \rightarrow x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

Electric Circuits

$$I = \frac{DQ}{Dt} = neav_d$$

$$V = IR = I\left(\frac{rL}{A}\right)$$

$$R_{series} = \sum_{i=1}^{N} R_i$$

$$\frac{1}{R_{parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$$

$$P = IV = I^2R = \frac{V^2}{R}$$

$$Q = CV = \left(\frac{ke_0A}{d}\right)V = \left(kC_0\right)V$$

$$W = U = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}$$

$$Q_{charge}(t) = Q_{max}\left(1 - e^{-\frac{t}{RC}}\right)$$

$$Q_{discharge}(t) = Q_{max}e^{-\frac{t}{RC}}$$

$$C_{parallel} = \sum_{i=1}^{N} C_i$$

$$\frac{1}{C_{series}} = \sum_{i=1}^{N}\frac{1}{C_i}$$

Light as a Particle & Relativity

$$E = hf = \frac{hc}{\lambda} = pc$$

$$KE_{max} = hf - \phi = eV_{stop}$$

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\phi)$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$p = \gamma mv$$

$$E_{total} = KE + E_{rest} = \gamma mc^2$$

$$E_{total}^2 = p^2c^2 + m^2c^4$$

$$E_{rest} = mc^2$$

$$KE = (\gamma - 1)mc^2$$

Geometry

Circles: $C = 2pr = pD \qquad A = pr^2$

Triangles: $A = \tfrac{1}{2}bh$

Spheres: $A = 4pr^2 \qquad V = \tfrac{4}{3}pr^3$

Light as a Wave

$$c = f\lambda = \frac{1}{\sqrt{\varepsilon_o\mu_o}}$$

$$S(t) = \frac{energy}{time\times area} = c\varepsilon_o E^2(t) = c\frac{B^2(t)}{\mu_0}$$

$$I = S_{avg} = \tfrac{1}{2}c\varepsilon_o E_{max}^2 = c\frac{B_{max}^2}{2\mu_0}$$

$$P = \frac{S}{c} = \frac{Force}{Area} \;;\; P = \frac{2S}{c}$$

$$S = S_o\cos^2\theta$$

$$v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{n}$$

$$\theta_{inc} = \theta_{refl}$$

$$n_1\sin\theta_1 = n_2\sin\theta_2$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$M_{total} = \prod_{i=1}^{N} M_i$$

$$d\sin\theta = m\lambda \text{ or } \left(m+\tfrac{1}{2}\right)\lambda$$

$$a\sin\phi = m'\lambda$$

Nuclear Physics

$$E_{binding} = \left(Zm_p + Nm_n - m_{rest}\right)c^2$$

$$\frac{\Delta N}{\Delta t} = -\lambda N_o \rightarrow N(t) = N_o e^{-\lambda t}$$

$$A(t) = A_o e^{-\lambda t}$$

$$m(t) = m_o e^{-\lambda t}$$

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

Misc. Physics 110 Formulae

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\Delta(mv)}{\Delta t} = m\vec{a}$$

$$\vec{F} = -k\vec{y}$$

$$\vec{F}_C = m\frac{v^2}{R}\hat{r}$$

$$W = \Delta KE = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right) = -\Delta PE$$

$$PE_{gravity} = mgy$$

$$PE_{spring} = \tfrac{1}{2}ky^2$$

$$x_f = x_i + v_{ix}t + \tfrac{1}{2}a_x t^2$$

$$v_{fx} = v_{ix} + a_x t$$

$$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x$$