## Physics 111

## Exam \#1

November 5, 2021

Name $\qquad$

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}|=(5 \mathrm{~kg}) \times\left(2 \frac{\mathrm{~m}}{\mathrm{~s}}\right)=10 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each multiple-choice question is worth 3 points and each free-response part is worth 7 points.

| Problem \#1 | $/ 24$ |
| :---: | :---: |
| Problem \#2 | $/ 24$ |
| Problem \#3 | $/ 24$ |
| Total | $/ 72$ |

1. A mass spectrometer uses magnetic fields to separate and determine atomic masses. Consider the mass-spectrometer shown on the right. Singly ionized charges $(q=+e)$ are accelerated from rest through a potential difference $\Delta V=-V_{a c c}$ (not shown in the diagram). These charges then pass through a set of crossed electric and magnetic fields (Region I) and then finally into a region of magnetic field only (Region II) where they bend in the presence of the magnetic field and strike one of the two detectors.

a. The region of crossed electric and magnetic fields is used to select out ions with a particular speed. To do this we require the charge to pass through region I undeflected, meaning that it neither rises nor falls. If the electric field in region I has a magnitude $E=9.1 \times 10^{8} \frac{\mathrm{~V}}{\mathrm{~m}}$, what is the direction of the electric field needed for the charges to go undeflected and what is the speed of the charges through region I? Assume that the magnetic field everywhere has a magnitude $23 T$.

Assuming up is the positive y-direction we have that $F_{B}$ points up the page, so to have the charge go undeflected through region $\mathrm{I}, F_{E}$ has to point down the page and thus the direction of the electric field must be from the upper plate to the lower plate or down the page. We have:

$$
F_{B}-F_{E}=m a_{y}=0 \rightarrow e v B-e E=0 \rightarrow v=\frac{E}{B}=\frac{9.1 \times 10^{8} \frac{\mathrm{~V}}{\mathrm{~m}}}{23 T}=4 \times 10^{7} \frac{\mathrm{~m}}{\mathrm{~s}}
$$

Since $q$ is positive, the electric field and electric force on $q$ point in the same direction. The electric field points down the plane of the page between the two plates.
b. What is the mass of the ion (in atomic mass units) that passes through region I into region II and which detector does the ion strike (upper or lower)? Assume that the distance from where the ion enters region II to where it strikes a detector is $D=$ $0.84 m$.
$F_{B}=e v B=m a_{c}=\frac{m v^{2}}{R}$
$m=\frac{e R B}{v}=\frac{1.6 \times 10^{-19} C \times \frac{0.84 m}{2} \times 23 T}{4 \times 10^{7 \frac{m}{s}}}$
$m=3.864 \times 10^{-26} \mathrm{~kg} \times \frac{1 \mathrm{amu}}{1.66 \times 10^{-27} \mathrm{~kg}}=23.4 \mathrm{amu}$
By the right-had-rule, the charge strikes the upper detector.
c. Through what magnitude of potential difference $V_{a c c}$ were the charges accelerated?

$$
\begin{aligned}
& W=-q \Delta V=e V_{a c c}=(\gamma-1) m c^{2} \rightarrow V_{a c c}=(\gamma-1) \frac{m c^{2}}{e} \\
& V_{a c c}=\left(\frac{1}{\left.\sqrt{1-\frac{v^{2}}{c^{2}}}-1\right) \frac{m c^{2}}{e}}\right. \\
& V_{a c c}=\left(\left(\frac{1}{\sqrt{1-\frac{\left(4 \times 10^{7} \frac{m}{s}\right)^{2}}{\left(3 \times 10^{8} \frac{m}{s}\right)^{2}}}}\right)-1\right) \frac{\left(23.4 a m u \times 931.5 \frac{\frac{\mathrm{MeV}}{\mathrm{c}^{2}}}{a m u}\right) c^{2}}{e}=196 \mathrm{MV}
\end{aligned}
$$

d. In region II the charges bend in the magnetic field. The time of flight of the charge is given by which of the following?

1. $T=\frac{e B}{\pi m}$.
2. $T=\frac{e R B}{\pi m}$.
3. $T=\frac{\pi m}{e B}$.
4. $T=\frac{\pi m}{e R B}$.
5. None of the above give the time-of-flight of the charge in the magnetic field.
6. Suppose that a beam of green light $(\lambda=545 \mathrm{~nm})$ in air is incident at an angle of $\theta_{1}=$ $41^{\circ}$ on a block of glass $\left(n_{g}=1.6\right)$ formed into an equilateral triangle as shown below.

a. At what angle does the light enter back into the air after passing through the glass, $\theta_{\text {exit }}$ ?

$$
\begin{aligned}
& n_{a} \sin \theta_{1}=n_{g} \sin \theta_{2} \rightarrow \theta_{2}=\sin ^{-1}\left(\frac{n_{a}}{n_{g}} \sin \theta_{1}\right)=\sin ^{-1}\left(\frac{1.0}{1.6} \sin 41\right)=24.2^{0} \\
& 90=\theta_{2}+\alpha \rightarrow \alpha=90-\theta_{2}=90^{0}-24.2^{0}=65.8^{0} \\
& \alpha+\beta+60^{0}=180^{0} \rightarrow \beta=120^{0}-\alpha=120^{0}-65.8^{0}=54.2^{0} \\
& n_{g} \sin \theta_{3}=n_{a} \sin \theta_{\text {exit }} \rightarrow \theta_{\text {exit }}=\sin ^{-1}\left(\frac{n_{g}}{n_{a}} \sin \theta_{3}\right) \\
& \beta+\theta_{3}=90^{0} \rightarrow \theta_{3}=90^{0}-\beta=90^{0}-54.4^{0}=35.8^{0} \\
& \theta_{\text {exit }}=\sin ^{-1}\left(\frac{1.6}{1.0} \sin 35.8\right)=69.4^{0}
\end{aligned}
$$

b. Suppose that the glass above had a smaller index of refraction than 1.6 , but greater than 1.0. Which of the following statements would be true if the index of refraction of the glass were smaller?

1. The speed of light in the glass would be decrease.
2. The wavelength of green light in the glass would decrease.
3. The frequency of the green light in the glass would increase.
4. The frequency of the green light in the glass would decrease.
5. None of the above statements are true.
c. Suppose that two converging lenses are used in combination to form an image of an object. The object is placed 28 mm to the left of converging lens \#1 which has a 24 mm focal length. At a distance of 180 mm to the right of converging lens \#1, converging lens \#2is placed. If converging lens \#2 has a focal length of 15 mm , where will the final image be produced with respect to lens \#2?

Converging lens \#1:
$\frac{1}{f_{1}}=\frac{1}{d_{o 1}}+\frac{1}{d_{i 1}} \rightarrow \frac{1}{d_{i 1}}=\frac{1}{f_{1}}-\frac{1}{d_{o 1}}=\frac{1}{24 m m}-\frac{1}{28 m m} \rightarrow d_{i 1}=168 \mathrm{~mm}$
Object distance for lens \#2:
$D=d_{o 2}+d_{i 1} \rightarrow d_{o 2}=D-d_{i 1}=180 \mathrm{~mm}-168 \mathrm{~mm}=12 \mathrm{~mm}$
Converging lens \#2:
$\frac{1}{f_{2}}=\frac{1}{d_{o 2}}+\frac{1}{d_{i 2}} \rightarrow \frac{1}{d_{i 2}}=\frac{1}{f_{2}}-\frac{1}{d_{o 2}}=\frac{1}{15 \mathrm{~mm}}-\frac{1}{12 m \mathrm{~m}} \rightarrow d_{i 1}=-60 \mathrm{~mm}$
The final image will be to the left of lens \#2, and thus is a virtual image.
d. What is the size of original object if the final image has a height of 4.7 cm ?

$$
\begin{aligned}
& M_{1}=\frac{d_{i 1}}{d_{o 1}}=\frac{168 \mathrm{~mm}}{28 \mathrm{~mm}}=6 \\
& M_{2}=\frac{d_{i 2}}{d_{o 2}}=\frac{60 \mathrm{~mm}}{12 \mathrm{~mm}}=5 \\
& M_{\text {total }}=6 \times 5=30 \rightarrow M_{\text {total }}=\frac{h_{i f}}{h_{o}} \rightarrow h_{o}=\frac{h_{i f}}{M_{\text {total }}}=\frac{4.7 \mathrm{~cm}}{30}=0.16 \mathrm{~cm}=1.6 \mathrm{~mm}
\end{aligned}
$$

3. Consider the following situation in which a 2000 turn coil of tungsten wire is oriented in the plane of the page as shown below. The coil of wire has a radius $R=$ 10.5 cm .

a. At a distance of $r=200 \mathrm{~m}$ from the coil, a lightning bolt suddenly strikes. If the current in the lightning bolt rises from $I_{i}=0 A$ to $I_{f}=3.1 M A$ over a time of $\Delta t=$ $8 \mu s$, what is the potential difference induced across the coil of wire?
$\varepsilon=\left|-N \frac{\Delta \phi_{B}}{\Delta t}\right|=\left|-N A \cos \theta \frac{\Delta B}{\Delta t}\right|=N A \frac{\Delta B}{\Delta t}=\frac{2000 \times \pi(0.105 \mathrm{~m})^{2} \times 0.0031 T}{8 \times 10^{-6} s}$
$\varepsilon=2.7 \times 10^{4} \mathrm{~V}$
where, $\Delta B=B_{f}-B_{i}=B_{f}=\frac{\mu_{0} I}{2 \pi r}=\frac{4 \pi \times 10^{-7 \frac{T m}{A} \times 3.1 \times 10^{6} A}}{2 \pi \times 200 \mathrm{~m}}=0.0031 \mathrm{~T}$
b. What is the magnitude and direction of the current induced in the wire loop? Suppose that the wire was made out of tungsten ( $\rho_{W}=5.6 \times 10^{-8} \Omega m$ ) and had a diameter of 3 mm .
$R=\frac{\rho L}{A}=\frac{\rho_{W} \times N \times 2 \pi R}{\pi r_{\text {wire }}^{2}}=\frac{5.6 \times 10^{-8} \Omega m \times 2000 \times 2 \pi \times 0.105 \mathrm{~m}}{\pi\left(1.5 \times 10^{-3} \mathrm{~m}\right)^{2}}=10.5 \Omega$
$I=\frac{\varepsilon}{R}=\frac{2.6 \times 10^{4} V}{10.5 \Omega}=2487 A$
The magnetic field is pointing out of the page through the wire loops and thus the magnetic flux is increasing out of the page through the loops of wire. We need a magnetic field produced by the loops of wire to point into the page to undo the change in magnetic flux. Thus, the current will flow clockwise in the loops of wire.
c. How much energy is dissipated as heat in the loop of wire as the lightning bolt builds up?

$$
P=\frac{\Delta E}{\Delta t}=I^{2} R \rightarrow \Delta E=I^{2} R \Delta t=(2487 A)^{2} \times 10.5 \Omega \times 8 \times 10^{-6} s=520 J
$$

d. A bar magnet is dropped into a hollow vertical aluminum tube. Suppose that the north pole of the magnet is pointing down into the tube and you look down into the tube as the magnet falls. As the magnet falls, you are looking at the south pole. At any instant, as viewed by you, which of the following statements is true?

1. Current will circulate around the tube counterclockwise below the magnet and the magnet will experience a magnetic force pointing up.
2. Current will circulate around the tube clockwise below the magnet and the magnet will experience a magnetic force pointing up.
3. Current will circulate around the tube counterclockwise below the magnet and the magnet will experience a magnetic force pointing down.
4. Current will circulate around the tube clockwise below the magnet and the magnet will experience a magnetic force pointing down.
5. There is not enough information to answer this question.

Physics 111 Formula Sheet
Electrostatics
$F=k \frac{q_{1} q_{2}}{r^{2}}$
$\vec{F}=q \vec{E} ; \quad E_{p c}=k \frac{q}{r^{2}} ; \quad E_{\text {plate }}=\frac{q}{\epsilon_{0} A}$
Magnetism
$\vec{F}=q \vec{v} \times \vec{B} \rightarrow F=q v B \sin \theta$
$\vec{F}=I \vec{L} \times \vec{B} \rightarrow F=I L B \sin \theta$
$E=-\frac{\Delta V}{\Delta x}$
$B=\frac{\mu_{0} I}{2 \pi r}$
$V=k \frac{q}{r}$
$U_{e}=k \frac{q_{1} q_{2}}{r}=q V$
$W=-q \Delta V=-\Delta U_{e}=\Delta K$
Electric Circuits - Capacitors
$Q=C V ; \quad C=\frac{\kappa \epsilon_{0} A}{d}$
$C_{\text {parallel }}=\sum_{i=1}^{N} C_{i}$
$\frac{1}{C_{\text {series }}}=\sum_{i=1}^{N} \frac{1}{C_{i}}$
$Q_{\text {charging }}(t)=Q_{\max }\left(1-e^{-\frac{t}{\tau}}\right)$
$Q_{\text {discharging }}(t)=Q_{\text {max }} e^{-\frac{t}{\tau}}$
$I(t)=I_{\max } e^{-\frac{t}{\tau}}=\frac{Q_{\max }}{\tau} e^{-\frac{t}{\tau}}$
$\tau=R C$
$U_{C}=\frac{1}{2} q V=\frac{1}{2} C V^{2}=\frac{Q^{2}}{2 C}$
Light as a Wave
$c=f \lambda$
$S(t)=\frac{\text { Energy }}{\text { time } \times \text { Area }}=c \epsilon_{0} E^{2}(t)=c \frac{B^{2}(t)}{\mu_{0}}$
$I=S_{\text {avg }}=\frac{1}{2} c \epsilon_{0} E_{\text {max }}^{2}=c \frac{B_{\text {max }}^{2}}{2 \mu_{0}}$
$P= \begin{cases}\frac{S}{c} ; & \text { absorbed } \\ \frac{2 S}{c} ; & \text { reflected }\end{cases}$
$S=S_{0} \cos ^{2} \theta$
$v=\frac{c}{n}$
$\theta_{\text {incident }}=\theta_{\text {reflected }}$
$n_{1} \sin \theta_{1}=n_{2} \sin \theta_{2}$
$P=\frac{1}{f}=\frac{1}{d_{0}}+\frac{1}{d_{i}}$
$M=-\frac{d_{i}}{d_{0}} ; \quad|M|=\frac{h_{i}}{h_{0}}$
$\phi_{B}=B A \cos \theta$

Electric Circuits - Resistors
$I=\frac{\Delta Q}{\Delta t}$
$I=n e A v_{d} ; \quad n=\frac{\rho N_{A}}{m}$
$V=I R$
$R=\frac{\rho L}{A}$
$R_{\text {series }}=\sum_{i=1}^{N} R_{i}$
$\frac{1}{R_{\text {parallel }}}=\sum_{i=1}^{N} \frac{1}{R_{i}}$
$P=\frac{\Delta E}{\Delta t}=I V=I^{2} R=\frac{V^{2}}{R}$

Light as a Particle/Relativity
$E=h f=\frac{h c}{\lambda}$
$K_{\text {max }}=h f-\phi$
$\Delta \lambda=\lambda^{\prime}-\lambda=\frac{h}{m c}(1-\cos \phi)$
$\frac{1}{E^{\prime}}=\frac{1}{E}+\frac{(1-\cos \phi)}{E_{\text {rest }}} ; \quad E_{\text {rest }}=m c^{2}$
$\gamma=\frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}}$
$p=\gamma m v$
$E_{\text {total }}=E_{\text {rest }}+K=\gamma m c^{2}$
$K=(\gamma-1) m c^{2}$
$E_{\text {total }}^{2}=p^{2} c^{2}+m^{2} c^{4}$

Nuclear Physics

$$
\begin{aligned}
& N=N_{0} e^{-\lambda t} \\
& m=m_{0} e^{-\lambda t} \\
& A=A_{0} e^{-\lambda t} \\
& A=\lambda N \\
& t_{\frac{1}{2}}=\frac{\ln 2}{\lambda}
\end{aligned}
$$

## Constants

$g=9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$
$1 e=1.6 \times 10^{-19} \mathrm{C}$
$k=\frac{1}{4 \pi \epsilon_{0}}=9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{\mathrm{C}^{2}}$
$\epsilon_{0}=8.85 \times 10^{-12} \frac{\mathrm{C}^{2}}{\mathrm{Nm}^{2}}$
$1 \mathrm{eV}=1.6 \times 10^{-19} \mathrm{~J}$
$\mu_{0}=4 \pi \times 10^{-7 \frac{T m}{A}}$
$c=3 \times 10^{8} \frac{\mathrm{~m}}{\mathrm{~s}}$
$h=6.63 \times 10^{-34} \mathrm{~J} \cdot \mathrm{~s}=4.14 \times 10^{-15} \mathrm{eV} \cdot \mathrm{s}$
$N_{A}=6.02 \times 10^{23}$
$1 u=1.66 \times 10^{-27} \mathrm{~kg}=931.5 \frac{\mathrm{MeV}}{\mathrm{c}^{2}}$
$m_{p}=1.67 \times 10^{-27} \mathrm{~kg}=937.1 \frac{\mathrm{MeV}}{\mathrm{c}^{2}}$
$m_{n}=1.69 \times 10^{-27} \mathrm{~kg}=948.3 \frac{\mathrm{MeV}}{\mathrm{c}^{2}}$
$m_{e}=9.11 \times 10^{-31} \mathrm{~kg}=0.511 \frac{\mathrm{MeV}}{\mathrm{c}^{2}}$

Physics 110 Formulas
$\vec{F}=m \vec{a} ; \quad F_{G}=\frac{G M_{1} m_{2}}{r^{2}} ; \quad F_{S}=-k y ; \quad a_{c}=\frac{v^{2}}{r}$
$W=-\Delta U_{g}-\Delta U_{s}=\Delta K$
$U_{g}=m g y$
$U_{s}=\frac{1}{2} k y^{2}$
$K=\frac{1}{2} m v^{2}$
$\vec{r}_{f}=\vec{r}_{i}+\vec{v}_{i} t+\frac{1}{2} \vec{a} t^{2}$
$\vec{v}_{f}=\vec{v}_{i}+\vec{a} t$
$v_{f}^{2}=v_{i}^{2}+2 a_{r} \Delta r$

Geometry/Algebra
Circles: $\quad A=\pi r^{2} \quad C=2 \pi r=\pi$
Spheres: $\quad A=4 \pi r^{2} \quad V=\frac{4}{3} \pi r^{3}$
Triangles: $\quad A=\frac{1}{2} b h$
Quadratics: $\quad a x^{2}+b x+c=0 \rightarrow x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$

Periodic Table of the Elements

https://www.wuwm.com/post/periodic-table-elements-turns-150\#stream/0

