Physics 111

Exam #2

October 13, 2023

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}| = (5kg)\times\left(2\,\frac{m}{s}\right) = 10\,\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 6 points.

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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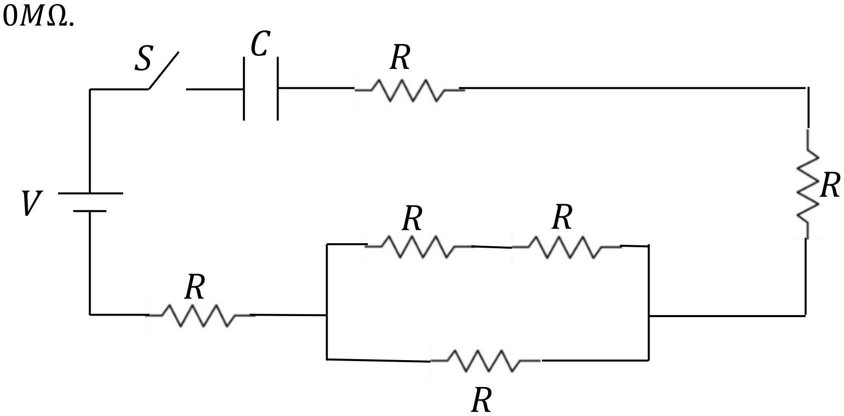
1. Consider the network of resistors shown below. The network is connected to a $V_B = 1000V$ battery and an initially uncharged capacitor C and a switch S. Assume all the resistors are the same in every way, unless otherwise specified, and have a resistance $R = 100M\Omega$.



a. What is the time constant for the circuit? Hint: the capacitor is constructed out of two square plates with sides of length $L = 50cm$ and separated by $0.5mm$ of air.

$$C = \frac{\kappa\epsilon_0 A}{d} = \frac{8.85 \times 10^{-12}\frac{C^2}{Nm^2} \times (0.5m)^2}{0.5 \times 10^{-3}m} = 4.43 \times 10^{-9}F$$

$$R_{eq} = R + R + R + \left(\frac{1}{R} + \frac{1}{2R}\right)^{-1} = \frac{11}{3}R = \frac{11}{3} \times 100M\Omega = 366.7M\Omega$$

$$\tau = R_{eq}C = 366.7 \times 10^6\Omega \times 4.43 \times 10^{-9}F = 1.62s$$

b. When the switch S is closed, the capacitor begins to charge through the network of resistors. What is the maximum current that flows when the battery is connected to the circuit, the maximum charge that can be stored on the capacitor, and the total energy stored in the capacitor when fully charged?

$$I_{max} = \frac{V_{max}}{R_{eq}} = \frac{1000V}{366.7 \times 10^6\Omega} = 2.7 \times 10^{-6}A$$

$$Q_{max} = CV_{max} = 4.43 \times 10^{-9}F \times 1000V = 4.43 \times 10^{-6}C$$

$$U_{max} = \frac{1}{2}CV_{max}^2 = \frac{1}{2} \times 4.43 \times 10^{-9}F \times (1000V)^2 = 0.0022J = 2.2mJ$$

c. After a long time, the capacitor becomes fully charged. At this point, the capacitor is disconnected from the battery and the network of resistors. The capacitor is then connected to an unknown single resistor and this resistor is not any of the resistors from part a. When connected to this unknown single resistor, the fully charged capacitor begins to discharge and it is found that it takes $16\mu s$ for the capacitor to lose 75% of its initial stored energy. What was the resistance of the unknown resistor?

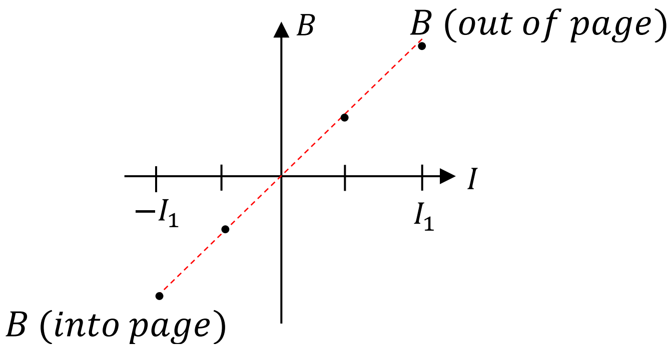
$$U_f = \frac{1}{2}CV(t)^2 = \frac{1}{2}C\left(V_{max}e^{-\frac{t}{RC}}\right)^2 = U_i e^{-\frac{2t}{RC}}$$

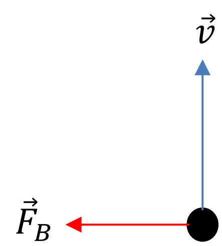
$$\rightarrow R_{unk} = \frac{2t}{\ln\left(\frac{U_f}{U_i}\right)C} = \frac{2 \times 16 \times 10^{-6}s}{\ln(0.25) \times 4.43 \times 10^{-9}F} = 5211\Omega$$

d. What is the length of the conducting wire in the unknown resistor if the resistor has a circular cross-section with radius $r = 5\mu m$ and is made out of platinum with resistivity $9.82 \times 10^{-8}\Omega m$.

$$R_{unk} = \frac{\rho L}{A} \rightarrow L = \frac{R_{unk}A}{\rho} = \frac{5211\Omega \times \pi(5 \times 10^{-6})^2}{9.82 \times 10^{-8}\Omega m} = 4.2m$$

2. Suppose you were in the lab making measurements of the magnetic field in a region of space as a function of current flowing through some circular coils of wire connected to a battery. The data taken are plotted below where the current has a positive value it is flowing in a counterclockwise fashion around the circuit, while a negative value corresponds to a clockwise flow. Suppose that a current $I = +0.5I_1$ was flowing and that an unknown charged particle (either a proton or electron) in the region of space where the magnetic field was being measured was moving toward the top of the page. In this case the magnetic force on the unknown charged particle is F_B in magnitude and to the left as shown below.





a. What is the identity of the unknown charged particle and what change to the current only in the wire would make the magnitude of the magnetic force be twice the original value and the direction of the magnetic force point to the right? Be sure to fully explain your answer using as many physics ideas as possible.

By the right-hand rule, the unknown charge must be an electron.

The magnetic force is $F_B = qvB$ and if you want to double the force then the magnetic field (which is related to the current) would have to be doubled. The new current would have to be $I_{new} = 2I_{old} = 2(0.5I_1) = I_1$. To change the direction of the magnetic force on the electron, the direction of the magnetic field would have to change from pointing out of the page to pointing into the page by the right-had rule.

b. After careful experimentation it is found that the magnitude of the magnetic field was found to obey $B = \frac{\mu_0 NI}{2R}$, where $N = 130$ is the number of turns of the wire, I is the current flowing, and $R = 2.5m$ is the radius of the circular loops of wire. If the current flowing is $I = +1.25A$ and the magnetic force was measured to be $F_B = 9.9 \times 10^{-18}N$, what was the speed of the unknown charge in the magnetic field?

$$B = \frac{\mu_0 NI}{2R} = \frac{4\pi \times 10^{-7}\frac{Tm}{A} \times 130 \times 1.25A}{2 \times 2.5m} = 4.1 \times 10^{-5}T$$

$$F_B = qvB \rightarrow v = \frac{F_B}{qB} = \frac{9.9 \times 10^{-18}N}{1.6 \times 10^{-19}C \times 4.1 \times 10^{-5}T} = 1.5 \times 10^6\frac{m}{s}$$

c. What is the radius and period of the charged particles orbit?

$$F_B = qvB = m\frac{v^2}{R} \rightarrow R = \frac{mv}{qB} = \frac{9.11\times10^{-31}kg\times1.5\times10^6\frac{m}{s}}{1.6\times10^{-19}C\times4.1\times10^{-5}T} = 0.21m$$

$$v = \frac{2\pi R}{T} \rightarrow T = \frac{2\pi R}{v} = \frac{2\pi \times 0.21m}{1.5 \times 10^6\frac{m}{s}} = 8.8 \times 10^{-7}s = 88\mu s$$

d. Suppose that a magnetic field $\vec{B}$ were directed everywhere out of the plane of the page with a constant magnitude B. A small coil of wire of cross-sectional area A is oriented in several ways described below. Of the configurations described below, explain if a current will flow or not in the small coil of wire. For those with currents, determine the direction of the current flow in the coil of wire.

1. The coil of wire is in the plane of the page with its normal perpendicular to the page. That is, the normal to the loop of wire is pointing out of the page at you. The coil of wire is rotated in the plane of the page at a constant rate clockwise.

   As the coil rotates in the plane of the page there is no change in magnetic flux since the normal to the loop and the magnetic field are parallel to each other. Since there is no change in magnetic flux there is nothing to undo, so no current is produced in the coil.

2. The coil of wire is in the plane of the page with its normal pointing perpendicular to the page. That is, the normal to the loop of wire is pointing out of the page at you. The magnetic field is rotated so that it now points into the plane of the page from pointing out of the plane and has the same magnitude.

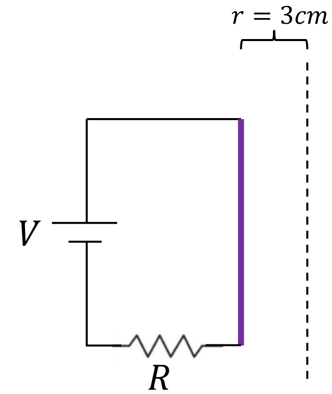
   As the magnetic field $\vec{B}$ pointing out of the page is rotated, the magnetic flux decreases to zero and then increases from zero to $\vec{B}$ pointing into the page. Only the direction of the magnetic field changes, the magnitude remains constant, but the magnetic flux changes. To undo the decrease in magnetic flux out of the loop as $\vec{B}$ decreases, a counterclockwise current will be produced. In addition, to undo the increase in magnetic flux into the loop as $\vec{B}$ points into the loop, a counterclockwise current will also flow. Thus, the current flow will be counterclockwise.

3. The coil of wire is in the plane of the page with its normal pointing perpendicular to the page. That is, the normal to the loop of wire is pointing out of the page at you. The coil of wire is stretched in such a way that the area of the coil increases from A to $2A$.

   Since the area of the coil changes there are more magnetic field lines that can pass through the cross-section of the loop and thus there is an increase in magnetic flux. To undo the increase in magnetic flux we need a clockwise current to flow to produce a magnetic field that points into the page to undo the increase in magnetic flux out of the page.

3. Consider the circuit shown below right where a $100V$ battery is connected to a 50Ω resistor.

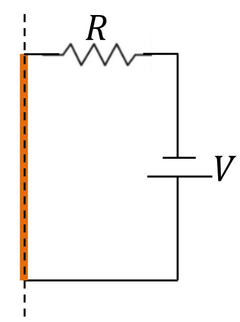
   a. Along the dashed line located $3cm$ to the right of the circuit, what is the magnitude and direction of the magnetic field due to the long straight wire segment colored purple?



By the right-hand rule, since the current is flowing clockwise around the circuit the magnetic field will point out of the page at the dashed line.

$$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7}\frac{Tm}{A} \times \left(\frac{100V}{50\Omega}\right)}{2\pi \times 0.03m} = 1.33 \times 10^{-5}T$$

   b. Suppose a second circuit was constructed by connecting a $220V$ battery to a 250Ω resistor. This circuit was placed $3cm$ to the right of the first circuit in the same plane. A $10cm$ segment of wire colored orange is placed along the dashed line. What is the net magnetic force on this segment of wire?



The direction of the magnetic force is given by the right-hand rule. Since the magnetic field points out of the page and the current flows clockwise, the magnetic force must point to the right.

The magnitude of the magnetic force:

$$F_B = ILB = \left(\frac{220V}{250\Omega}\right) \times 0.1m \times 1.33 \times 10^{-5}T = 1.2 \times 10^{-6}N$$

c. The segment of orange wire in part b was made from platinum and has a length $L = 10cm$ and radius $r = 2mm$. What is the drift velocity of charge carriers in the segment of platinum wire if each platinum atom donates one valence electron to the current? Some relevant data for platinum: $\rho_{Pt} = 21.45\frac{g}{cm^3}$, $\rho = 9.82 \times 10^{-8}\Omega m$, and $M = 191.1\frac{g}{mol}$.

$$I = \frac{V}{R} = \frac{220V}{250\Omega} = 0.88A$$

$$n = \left(\frac{\rho_{Pt} N_A}{M}\right)$$
$$n = \left[\left(\frac{21.45\frac{g}{cm^3} \times 6.022 \times 10^{23}\frac{Pt\ atoms}{mol}}{191.1\frac{g}{mol}}\right) \times 1\frac{valence\ e}{atom}\right] \times \left(\frac{100cm}{1m}\right)^3$$
$$n = 6.8 \times 10^{28} m^{-3}$$

$$I = neAv_d \rightarrow v_d = \frac{I}{neA}$$
$$I = \frac{0.88A}{6.8 \times 10^{28} m^{-3} \times 1.6 \times 10^{-19}C \times \pi(2 \times 10^{-3}m)^2}$$

$$v_d = 6.5 \times 10^{-6}\frac{m}{s} = 6.5\frac{\mu m}{s}$$

d. What Hall voltage would be measured across a diameter of the wire?

$$V_{hall} = wv_d B = 2rv_d B = 2 \times 2 \times 10^{-3}m \times 6.5 \times 10^{-6}\frac{m}{s} \times 1.33 \times 10^{-5}T$$
$$V_{Hall} = 3.5 \times 10^{-13}V$$

Physics 111 Formula Sheet

Electrostatics

$F = k\frac{q_1 q_2}{r^2}$

$\vec{F} = q\vec{E}; \quad E_{pc} = k\frac{q}{r^2}; \quad E_{plate} = \frac{q}{\epsilon_0 A}$

$E = -\frac{\Delta V}{\Delta x}$

$V_{pc} = k\frac{q}{r}$

$U_e = k\frac{q_1 q_2}{r} = qV$

$W = -q\Delta V = -\Delta U_e = \Delta K$

Magnetism

$\vec{F} = q\vec{v} \times \vec{B} \rightarrow F = qvB\sin\theta$

$\vec{F} = I\vec{L} \times \vec{B} \rightarrow F = ILB\sin\theta$

$V_{Hall} = wv_d B$

$B = \frac{\mu_0 I}{2\pi r}$

$\varepsilon = \Delta V = -N\frac{\Delta\phi_B}{\Delta t}$

$\phi_B = BA\cos\theta$

Electric Circuits - Capacitors

$Q = CV; \quad C = \frac{\kappa\epsilon_0 A}{d}$

$C_{parallel} = \sum_{i=1}^{N} C_i$

$\frac{1}{C_{series}} = \sum_{i=1}^{N}\frac{1}{C_i}$

$Q_{charging}(t) = Q_{max}\left(1 - e^{-\frac{t}{\tau}}\right)$

$Q_{discharging}(t) = Q_{max}e^{-\frac{t}{\tau}}$

$I(t) = I_{max}e^{-\frac{t}{\tau}} = \frac{Q_{max}}{\tau}e^{-\frac{t}{\tau}}$

$\tau = RC$

$U_C = \frac{1}{2}qV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$

Electric Circuits - Resistors

$I = \frac{\Delta Q}{\Delta t}$

$I = neAv_d; \quad n = \frac{\rho N_A}{m}$

$V = IR$

$R = \frac{\rho L}{A}$

$R_{series} = \sum_{i=1}^{N} R_i$

$\frac{1}{R_{parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$

$P = \frac{\Delta E}{\Delta t} = IV = I^2 R = \frac{V^2}{R}$

Light as a Wave

$c = f\lambda$

$S(t) = \frac{\text{Energy}}{\text{time}\times\text{Area}} = c\epsilon_0 E^2(t) = c\frac{B^2(t)}{\mu_0}$

$I = S_{avg} = \frac{1}{2}c\epsilon_0 E_{max}^2 = c\frac{B_{max}^2}{2\mu_0}$

$P = \begin{cases} \frac{S}{c}; & \text{absorbed} \\ \frac{2S}{c}; & \text{reflected} \end{cases}$

$S = S_0\cos^2\theta$

$v = \frac{c}{n}$

$\theta_{\text{incident}} = \theta_{\text{reflected}}$

$n_1\sin\theta_1 = n_2\sin\theta_2$

$P = \frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i}$

$M = -\frac{d_i}{d_0}; \quad |M| = \frac{h_i}{h_0}$

Light as a Particle/Relativity

$E = hf = \frac{hc}{\lambda}$

$K_{max} = hf - \phi$

$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$

$\frac{1}{E'} = \frac{1}{E} + \frac{(1-\cos\phi)}{E_{rest}}; \quad E_{rest} = mc^2$

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$p = \gamma mv$

$E_{total} = E_{rest} + K = \gamma mc^2$

$K = (\gamma - 1)mc^2$

$E_{total}^2 = p^2c^2 + m^2c^4$

Nuclear Physics

$N = N_0 e^{-\lambda t}$

$m = m_0 e^{-\lambda t}$

$A = A_0 e^{-\lambda t}$

$A = \lambda N$

$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

Constants

$g = 9.8 \frac{m}{s^2}$

$1e = 1.6 \times 10^{-19} C$

$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$

$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$

$1eV = 1.6 \times 10^{-19} J$

$\mu_0 = 4\pi \times 10^{-7} \frac{Tm}{A}$

$c = 3 \times 10^8 \frac{m}{s}$

$h = 6.63 \times 10^{-34} Js = 4.14 \times 10^{-15} eVs$

$N_A = 6.02 \times 10^{23}$

$1u = 1.66 \times 10^{-27} kg = 931.5 \frac{MeV}{c^2}$

$m_p = 1.67 \times 10^{-27} kg = 937.1 \frac{MeV}{c^2}$

$m_n = 1.69 \times 10^{-27} kg = 948.3 \frac{MeV}{c^2}$

$m_e = 9.11 \times 10^{-31} kg = 0.511 \frac{MeV}{c^2}$

Physics 110 Formulas

$\vec{F} = m\vec{a}; \quad F_G = \frac{GM_1 m_2}{r^2}; \quad F_s = -ky; \quad a_c = \frac{v^2}{r}$

$W = -\Delta U_g - \Delta U_s = \Delta K$

$U_g = mgy$

$U_s = \frac{1}{2} k y^2$

$K = \frac{1}{2} m v^2$

$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2} \vec{a} t^2$

$\vec{v}_f = \vec{v}_i + \vec{a} t$

$v_f^2 = v_i^2 + 2 a_r \Delta r$

Common Metric Units

$nano\ (n) = 10^{-9}$

$micro\ (\mu) = 10^{-6}$

$milli\ (m) = 10^{-3}$

$centi\ (c) = 10^{-2}$

$kilo\ (k) = 10^{3}$

$mega\ (M) = 10^{6}$

Geometry/Algebra

Circles: $A = \pi r^2$ $\quad C = 2\pi r = \pi$

Spheres: $A = 4\pi r^2$ $\quad V = \frac{4}{3}\pi r^3$

Triangles: $A = \frac{1}{2} bh$

Quadratics: $ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$