Physics 111

Exam #2

October 14, 2024

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $\left|\vec{p}\right| \approx m\left|\vec{v}\right| = (5kg)\times\left(2\,\frac{m}{s}\right) = 10\,\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 6 points.

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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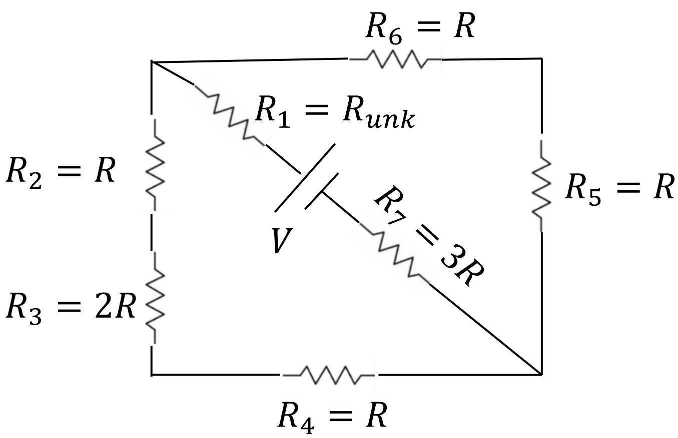
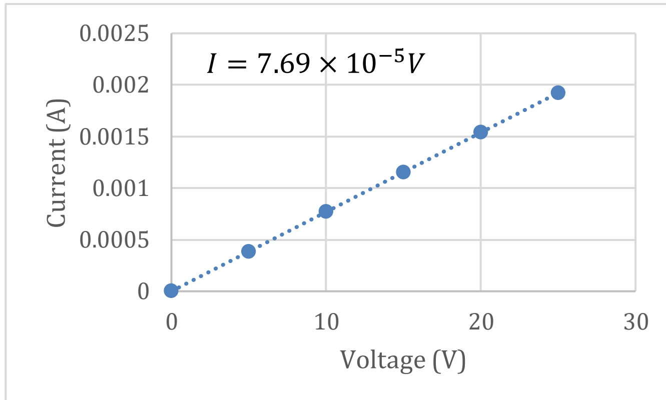
1. Consider the circuit below with some resistors wired to a battery. The resistors have various values, but each of the known resistors is either $R = 1k\Omega$ or a multiple of this value. There is one resistor $R_1 = R_{unk}$, whose resistance is unknown.

   a. Consider the plot below where data were taken on different batteries placed in the circuit and the total current produced by that battery. Using this circuit and the plot, what is the value of the unknown resistor R_1?





Resistors R_5 and R_6 are in series:
$R_{56} = R_5 + R_6 = 1000\Omega + 1000\Omega = 2000\Omega$

Resistors R_2, R_3, and R_4 are in series:
$R_{234} = R_2 + R_3 + R_4 = 1000\Omega + 2(1000\Omega) + 1000\Omega = 4000\Omega$

Resistors R_{234} and R_{56} are in parallel:
$$\frac{1}{R_{23456}} = \frac{1}{R_{234}} + \frac{1}{R_{56}} = \frac{1}{4000\Omega} + \frac{1}{2000\Omega} = \frac{3}{4000\Omega} \rightarrow R_{23456} = \frac{4000\Omega}{3} = 1333.3\Omega$$

Resistors R_1, R_{23456} and R_7 are in series:
$R_{eq} = R_1 + R_{23456} + R_7 \rightarrow R_1 = R_{unk} = R_{eq} - R_{234567} - R_7$

The equivalent resistance is the inverse of the slope of the graph:
$$R_{eq} = \frac{1}{7.69 \times 10^{-5}\Omega^{-1}} = 13003.9\Omega$$

$\rightarrow R_1 = R_{unk} = 13003.9\Omega - 1333.3\Omega - 3(1000\Omega) = 8670.6\Omega$

b. What is the current through resistor R_5 if the circuit were connected to a $100V$ battery?

$$V - V_{R_1} - V_{R_7} - V_{R_{23456}} = 0 \rightarrow V_{R_{23456}} = V - V_{R_1} - V_{R_7} = V - I_{total}(R_1 + R_7)$$

$$V_{R_{23456}} = 100V - 0.0077A\big(8670.6\Omega + 3(1000\Omega)\big) = 10.1V$$

$$V_{R_{23456}} = V_{R_{234}} = V_{R_{56}} = 10.1V$$

$$I_{R_{56}} = I_{R_5} = I_{R_6} = \frac{I_{R_{56}}}{R_{56}} = \frac{10.1V}{2(1000\Omega)} = 0.0051A = 5.1mA$$

We calculated $I_{total} = \frac{V}{R_{eq}} = \frac{100V}{13003.9} = 0.0077A = 7.7mA$ or we could have used the curve fit: $I = (7.69 \times 10^{-5}\Omega^{-1})V = 7.69 \times 10^{-5}\Omega^{-1} \times 100V = 0.0077A = 7.7mA$

c. Suppose that resistors R_2, R_3, and R_4 in the original circuit in part 1a were light bulbs. Rank the bulbs in order of least to most bright and explain how you arrived at the conclusions you did for each bulb. Hint: The brightness is proportional to the energy dissipated across the bulb.

The energy dissipated (per unit time) is the electric power.

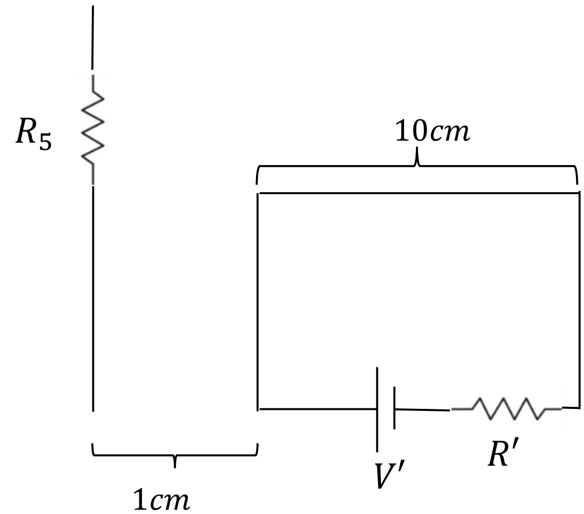
$$P_{R_2} = I_{R_{234}}^2 R_2 = I_{R_{234}}^2 R$$

$$P_{R_3} = I_{R_{234}}^2 R_3 = I_{R_{234}}^2 (2R) = 2P_{R_2} = 2P_{R_4}$$

$$P_{R_4} = I_{R_{234}}^2 R_4 = I_{R_{234}}^2 R$$

Since there is more energy dissipated across resistor R_3, resistor R_3 is brighter than resistors R_2 and R_4. Resistors R_2 and R_4 have the same energy dissipated and thus have the same brightness. So, in terms of least to most bright: $R_3 > R_2 = R_4$.

d. Suppose a square loop of wire with sides of length $L = 10cm$ and total resistance $R' = 3k\Omega$ is connected to a $V' = 50V$ battery. This circuit is placed a distance of $1cm$ to the right of resistor R_5, from the original circuit in part 1a, as shown below. What net force does the square loop of wire feel? The remainder of the original circuit is not needed and not shown for clarity.



The current in the loop flows clockwise and has a magnitude $I' = \frac{V'}{R'} = \frac{50V}{3000\Omega} = 0.0167A$.

The force on the top (pointing down) and bottom (pointing up) cancels everywhere across the top and bottom of the wire loop.

The force on the left edge:

$$F_{left} = I'LB = I'L\left(\frac{\mu_0 I}{2\pi r_{left}}\right)$$

$$F_{left} = \frac{4\pi \times 10^{-7}\frac{Tm}{A} \times 0.0167A \times 0.0051A \times 0.1m}{2\pi \times 0.01m} = 1.70 \times 10^{-10}N$$

And by the RHR, the direction of the force on the left side (for the current clockwise and up on the left and the magnetic field out of the page) is to the ***RIGHT***.

The force on the right edge:

$$F_{right} = I'LB = I'L\left(\frac{\mu_0 I}{2\pi r_{right}}\right)$$

$$F_{right} = \frac{4\pi \times 10^{-7}\frac{Tm}{A} \times 0.0167A \times 0.0051A \times 0.1m}{2\pi \times 0.11m} = 1.54 \times 10^{-11}N$$

And by the RHR, the direction of the force on the right side (for the current clockwise and down on the right and the magnetic field out of the page) is to the ***LEFT***.

The net force:

$F_{net} = F_{left} - F_{right} = 1.70 \times 10^{-10}N - 1.54 \times 10^{-11}N = 1.55 \times 10^{-10}N$ to the right.

2. A proton with speed v enters a uniform $B = 1T$ magnetic field at an angle θ measured with respect to the direction of the magnetic field

   a. What is the orbital period of the proton about the external magnetic field?

      The perpendicular component of the velocity dictates the motion about the magnetic field.

      $v_\perp = v\sin\theta = \frac{2\pi R}{T}$ and $F = qv_\perp B = m\frac{v_\perp^2}{R} \rightarrow v_\perp = \frac{qRB}{m}$. Equating these two expressions we get:

$$\frac{2\pi R}{T} = \frac{qRB}{m} \rightarrow T = \frac{2\pi m}{qB} = \frac{2\pi \times 1.67 \times 10^{-27} kg}{1.6 \times 10^{-19} C \times 1T} = 6.56 \times 10^{-8} s$$

   b. Suppose that the orbital radius and pitch of the proton was $R = 6cm$ and $L = 47cm$ respectively. At what velocity (magnitude v and direction θ) did the proton enter the magnetic field? Hint: calculate the components of the velocity perpendicular ($v_\perp$) and parallel ($v_\parallel$) to the magnetic field.

$$v_\perp = \frac{2\pi R}{T} = \frac{2\pi \times 0.06m}{6.56 \times 10^{-8} s} = 5.75 \times 10^6 \frac{m}{s}$$

$$v_\parallel = \frac{L}{T} = \frac{0.47m}{6.56 \times 10^{-8} s} = 7.16 \times 10^6 \frac{m}{s}$$

$$v = \sqrt{v_\parallel^2 + v_\perp^2} = \sqrt{(7.16)^2 + (5.75)^2} \times 10^6 \frac{m}{s} = 9.18 \times 10^6 \frac{m}{s} \text{ in magnitude.}$$

      The direction:

$$v_\perp = v\sin\theta \rightarrow \theta = \sin^{-1}\left(\frac{v_\perp}{v}\right) = \sin^{-1}\left(\frac{5.75 \times 10^6 \frac{m}{s}}{9.18 \times 10^6 \frac{m}{s}}\right) = 38.8^0$$

      Or from

$$v_\parallel = v\cos\theta \rightarrow \theta = \cos^{-1}\left(\frac{v_\parallel}{v}\right) = \cos^{-1}\left(\frac{7.16 \times 10^6 \frac{m}{s}}{9.18 \times 10^6 \frac{m}{s}}\right) = 38.7^0$$

c. The motion of the proton about the external magnetic field is termed uniform circular. That means that the magnitude of the velocity, the speed v, does not change as the proton orbits. What does change is the direction of the proton's velocity due to the proton interacting with the external magnetic field. This interaction exerts a force (and thus an acceleration) on the proton. Explain why the motion is uniform, meaning that the speed does not change and from this, how much work is done on the proton by the external magnetic field.

The motion is uniform meaning that the speed of the proton does not change as the proton orbits about the external magnetic field. For the speed of the proton to change, there would need to be a force (or a component of a force) either parallel to or antiparallel to the velocity. Since the force is a centripetal force and is perpendicular to the velocity, there is never a component of the magnetic force parallel to or antiparallel to the velocity. Thus, the speed never changes.

From this, the work done on the proton by the external magnetic field is the force times the displacement times the cosine of the angle between the force and the displacement (which is parallel to the velocity.). Since none of the force points in the direction of the velocity, the angle between $\vec{F}$ and $\Delta\vec{r}(\propto \vec{v})$ is $\theta = 90^0$. Thus, the work done ($W = F\Delta r\cos\theta = F\Delta r\cos 90 = 0$) is zero. Since the work done is zero, the change in kinetic energy of the proton is zero and the final speed of the proton is the same as the initial speed of the proton.

d. What is the maximum orbital radius the proton could undergo about the external magnetic field and at what angle, measured with respect to the magnetic field, does this maximum radius occur?

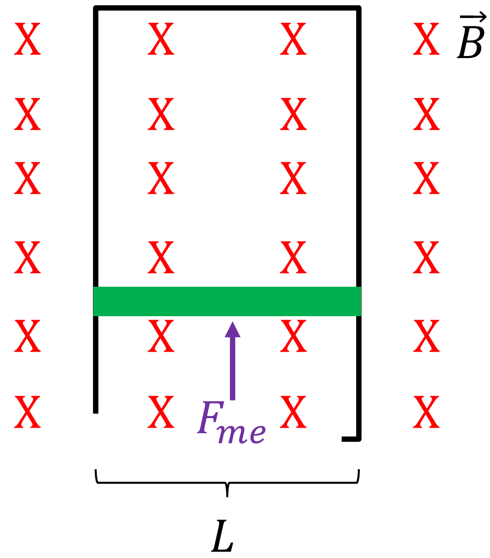
$$F = qv_\perp B = m\frac{v_\perp^2}{R} \rightarrow R = \frac{mv_\perp}{qB} = \frac{mv\sin\theta}{qB}$$

The maximum radius occurs when the angle θ between $\vec{v}$ and $\vec{B}$is 90^0 and the radius:

$$R = \frac{mv_\perp}{qB} = \frac{mv\sin\theta}{qB} = \frac{1.67\times10^{-27}kg\times9.18\times10^6\frac{m}{s}\sin 90}{1.6\times10^{-19}C\times1T}$$

$$R = 0.0958m = 9.58cm$$

3. Consider a set of parallel vertical low-friction metal rails is attached to a wall as shown below. A uniform $B = 1.5T$ magnetic field points into the wall and is parallel to the normal to the loop, which also points into the wall. A bar of mass $m = 250g$ and length $L = 1m$ is pushed up the rails at a constant speed v.



a. If the bar has resistance $R = 0.75\Omega$, what upward force would I have to apply (F_{me}) so that the bar moves up the rails at a constant speed $v = 2.4\frac{m}{s}$? Assume my force acts vertically upward parallel to the wall.

$$F_{net} = F_{me} - F_W - F_B = ma_y = 0$$

$$F_{me} = F_W + F_B = mg + ILB$$

$$F_{me} = mg + \left(\frac{\varepsilon}{R}\right) LB = mg + \left(\frac{BL\cos\theta}{R}\frac{\Delta y}{\Delta t}\right) LB$$

$$F_{me} = mg + \left(\frac{\varepsilon}{R}\right) LB = mg + \left(\frac{BLv\cos 0}{R}\right) LB$$

$$F_{me} = mg + \frac{B^2L^2v}{R}$$

$$F_{me} = 0.25kg \times 9.8\frac{m}{s^2} + \frac{(1.5T)^2(1m)^2 \times 2.4\frac{m}{s}}{0.75\Omega} = 9.65N$$

b. What is the magnitude and direction of the current induced in the bar as it is pushed up the rails at a constant speed $v = 2.4\frac{m}{s}$? Be sure to fully explain your reasoning behind your choice for the direction of the current.

$$I = \frac{\varepsilon}{R} = \frac{BLv}{R} = \frac{1.5T \times 1m \times 2.4\frac{m}{s}}{0.75\Omega} = 4.8A$$

As the bar moves up the page, the magnetic flux is decreasing. To undo the decrease in magnetic flux, I need a magnetic field that also points into the page. This will induce a current to flow through the bar from right-to-left, or clockwise.

c. Suppose at the very top of the rails you hold the bar at rest. Then at some moment of your choosing, you let go of the bar. If the bar falls from rest vertically down along the rails, what is the terminal speed of the bar? Assume that the bar always maintains contact with the rails as it falls.

$$F_{net,y} = -F_W + F_B = ma_y = 0$$

$$F_W = F_B \rightarrow mg = ILB = \left(\frac{\varepsilon}{R}\right)LB \rightarrow mg = \left(\frac{BLv}{R}\right)LB = \frac{B^2L^2v_{term}}{R}$$

$$v_{term} = \frac{mgR}{B^2L^2} = \frac{0.25kg \times 9.8\frac{m}{s^2} \times 0.75\Omega}{(1.5T)^2(1m)^2} = 0.82\frac{m}{s}$$

d. When the bar reaches terminal speed, what is the magnitude and direction of the electric field induced in the bar? Be sure to fully explain your choice for the direction of the induced electric field in the bar.

As the bar falls, the magnetic flux through the loop of wire bounded by the bar is increasing. To undo the increase in magnetic flux, I need a magnetic field generated by the loop to point out of the page. To get the magnetic field to point out of the page, the current in the bar must flow from left-to-right or counterclockwise. This means the electric field must point from left-to-right across the bar with magnitude

$$E = -\frac{\Delta V}{\Delta x} = -\frac{\left(V_{right} - V_{left}\right)}{L} = \frac{V}{L} = \frac{\varepsilon}{L} = \frac{BLv}{L} = Bv = 1.5T \times 0.82\frac{m}{s} = 1.23\frac{N}{C}$$

Physics 111 Formula Sheet

Electrostatics

$F = k\frac{q_1 q_2}{r^2}$

$\vec{F} = q\vec{E}; \quad E_{pc} = k\frac{q}{r^2}; \quad E_{plate} = \frac{q}{\epsilon_0 A}$

$E = -\frac{\Delta V}{\Delta x}$

$V_{pc} = k\frac{q}{r}$

$U_e = k\frac{q_1 q_2}{r} = qV$

$W = -q\Delta V = -\Delta U_e = \Delta K$

Magnetism

$\vec{F} = q\vec{v} \times \vec{B} \rightarrow F = qvB\sin\theta$

$\vec{F} = I\vec{L} \times \vec{B} \rightarrow F = ILB\sin\theta$

$V_{Hall} = wv_d B$

$B = \frac{\mu_0 I}{2\pi r}$

$\varepsilon = \Delta V = -N\frac{\Delta\phi_B}{\Delta t}$

$\phi_B = BA\cos\theta$

Electric Circuits - Capacitors

$Q = CV; \quad C = \frac{\kappa\epsilon_0 A}{d}$

$C_{parallel} = \sum_{i=1}^{N} C_i$

$\frac{1}{C_{series}} = \sum_{i=1}^{N}\frac{1}{C_i}$

$Q_{charging}(t) = Q_{max}\left(1 - e^{-\frac{t}{\tau}}\right)$

$Q_{discharging}(t) = Q_{max}e^{-\frac{t}{\tau}}$

$I(t) = I_{max}e^{-\frac{t}{\tau}} = \frac{Q_{max}}{\tau}e^{-\frac{t}{\tau}}$

$\tau = RC$

$U_C = \frac{1}{2}qV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$

Electric Circuits - Resistors

$I = \frac{\Delta Q}{\Delta t}$

$I = neAv_d$

$n = \left(\frac{\rho_m N_A}{m}\right) \times \frac{\text{charge carriers donated}}{\text{atom}}$

$V = IR$

$R = \frac{\rho L}{A}$

$R_{series} = \sum_{i=1}^{N} R_i$

$\frac{1}{R_{parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$

$P = \frac{\Delta E}{\Delta t} = IV = I^2 R = \frac{V^2}{R}$

Light as a Wave

$c = f\lambda$

$S(t) = \frac{\text{Energy}}{\text{time}\times\text{Area}} = c\epsilon_0 E^2(t) = c\frac{B^2(t)}{\mu_0}$

$I = S_{avg} = \frac{1}{2}c\epsilon_0 E_{max}^2 = c\frac{B_{max}^2}{2\mu_0}$

$P = \begin{cases} \frac{S}{c}; & \text{absorbed} \\ \frac{2S}{c}; & \text{reflected} \end{cases}$

$S = S_0\cos^2\theta$

$v = \frac{c}{n}$

$\theta_{\text{incident}} = \theta_{\text{reflected}}$

$n_1\sin\theta_1 = n_2\sin\theta_2$

$P = \frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i}$

$M = \frac{d_i}{d_0}; \quad |M| = \frac{h_i}{h_0}$

Light as a Particle/Relativity

$E = hf = \frac{hc}{\lambda}$

$K_{max} = hf - \phi$

$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$

$\frac{1}{E\prime} = \frac{1}{E} + \frac{(1-\cos\phi)}{E_{rest}}; \quad E_{rest} = mc^2$

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$p = \gamma mv$

$E_{total} = E_{rest} + K = \gamma mc^2$

$K = (\gamma - 1)mc^2$

$E_{total}^2 = p^2c^2 + m^2c^4$

Nuclear Physics

$N = N_0 e^{-\lambda t}$

$m = m_0 e^{-\lambda t}$

$A = A_0 e^{-\lambda t}$

$A = \lambda N$

$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

Constants

$g = 9.8\frac{m}{s^2}$

$1e = 1.6 \times 10^{-19} C$

$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$

$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$

$1eV = 1.6 \times 10^{-19} J$

$\mu_0 = 4\pi \times 10^{-7} \frac{Tm}{A}$

$c = 3 \times 10^8 \frac{m}{s}$

$h = 6.63 \times 10^{-34} Js = 4.14 \times 10^{-15} eVs$

$N_A = 6.02 \times 10^{23}$

$1u = 1.66 \times 10^{-27} kg = 931.5 \frac{MeV}{c^2}$

$m_p = 1.67 \times 10^{-27} kg = 937.1 \frac{MeV}{c^2}$

$m_n = 1.69 \times 10^{-27} kg = 948.3 \frac{MeV}{c^2}$

$m_e = 9.11 \times 10^{-31} kg = 0.511 \frac{MeV}{c^2}$

Physics 110 Formulas

$\vec{F} = m\vec{a}; \quad F_G = \frac{GM_1 m_2}{r^2}; \quad F_s = -ky; \quad a_c = \frac{v^2}{r}$

$W = -\Delta U_g - \Delta U_s = \Delta K$

$U_g = mgy$

$U_s = \frac{1}{2}ky^2$

$K = \frac{1}{2}mv^2$

$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$

$\vec{v}_f = \vec{v}_i + \vec{a}t$

$v_f^2 = v_i^2 + 2a_r \Delta r$

Common Metric Units

$nano\ (n) = 10^{-9}$

$micro\ (\mu) = 10^{-6}$

$milli\ (m) = 10^{-3}$

$centi\ (c) = 10^{-2}$

$kilo\ (k) = 10^{3}$

$mega\ (M) = 10^{6}$

Geometry/Algebra

Circles:	$A = \pi r^2$	$C = 2\pi r = \pi$
Spheres:	$A = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$
Triangles:	$A = \frac{1}{2}bh$	
Quadratics:	$ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$