Physics 111

Exam #2

October 13, 2025

Name____________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So, erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}| = (5kg)\times\left(2\frac{m}{s}\right) = 10\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 6 points.

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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1. Bradycardia is a cardiac condition in which the heart rate (or the number of heart beats per minute) is too slow, generally at a rate lower than $60\,bpm$. Bradycardia may be caused by a wide array of factors including age, heart disease, low thyroid function (hypothyroidism), and possibly problems with the sinoatrial node (the heart's natural pacemaker). To correct for bradycardia, one may be prescribed medication or one may have an artificial pacemaker installed. The artificial cardiac pacemaker is a tunable resistor-capacitor circuit.

   a. To construct the pacemaker, the capacitor is built out of two square metal plates with sides of length $l = 30mm$ separated by a gap of $d = 1mm$. The space between the gap between the plates is filled with an insulating material with a dielectric constant $\kappa = 22{,}600$. The resistor in the circuit is variable and can be set externally to control the heartbeat and the capacitor discharges through the resistor every time the capacitor reaches 90% of its maximum charge. Suppose that the physician requires the heart rate to be $68\,bpm$, what resistance would need to be set to accomplish this?

$$t = \frac{1min}{68beats} \times \frac{60s}{1min} = 0.88s$$

$$C = \frac{\kappa \epsilon_0 A}{d} = \frac{22600 \times 8.85 \times 10^{-12} \frac{C^2}{Nm^2} \times (30 \times 10^{-3} m)^2}{1 \times 10^{-3} m} = 1.8 \times 10^{-7} F$$

$$Q(t) = 0.9Q_{max} = Q_{max}\left(1 - e^{-\frac{t}{\tau}}\right) \to \tau = RC = -\frac{t}{\ln\left(1 - \frac{0.9Q_{max}}{Q_{max}}\right)}$$

$$R = -\frac{t}{C \ln\left(1 - \frac{0.9Q_{max}}{Q_{max}}\right)} = -\frac{0.88s}{1.8 \times 10^{-7} F \ln(0.1)} = 2.1 \times 10^{6} \Omega = 2.1 M\Omega$$

   b. If the physician instead wanted to make the heart rate something greater than $68\,bpm$, explain what thing(s) could be done to the RC circuit in the pacemaker to make the heart rate increase?

   If the HR were to increase then this would lower the charging time, meaning we would need to reach of the full charge faster. This is controlled by the time constant of the circuit. To charge faster we need a shorter time constant and since the capacitor is a fixed value we can alter the value of the variable resistor in the circuit. To decrease the time constant, we'd have to decrease the value of the resistor in the circuit.

c. Unfortunately, many people suffer a heart attack as a result of an undiagnosed heart condition. Suppose that you are a paramedic and you are called to an accident scene where a person is in full cardiac arrest. To attempt to revive the victim you use a defibrillator. A defibrillator is constructed out of two square metal paddles with sides of length $l = 15cm$. The system is charged using a voltage source rated at $5000V$. When placed on the chest the capacitor system discharges through the heart. Suppose that the separation between the paddles when placed on the chest is $d = 5cm$ and that the resistance of the body is approximately 5000Ω. How much energy is stored in the fully charged capacitor before it is discharged through the body and how long does it take to discharge the defibrillator to 1% of its initial stored charge?

$$C = \frac{\kappa\epsilon_0 A}{d} = \frac{1 \times 8.85 \times 10^{-12}\frac{C^2}{Nm^2} \times (15 \times 10^{-2}m)^2}{5 \times 10^{-2}m} = 4 \times 10^{-12}F$$

$$U_{max} = \frac{1}{2}CV^2 = \frac{1}{2} \times 4 \times 10^{-12}F \times (5000V)^2 = 5 \times 10^{-5}J = 50mJ$$

$$Q(t) = Q_{max}e^{-\frac{t}{\tau}} \rightarrow t = -\tau \ln\left(\frac{Q(t)}{Q_{max}}\right)$$

$$t = -5000\Omega \times 4 \times 10^{-12}F \times \ln\left(\frac{0.01Q_{max}}{Q_{max}}\right) = 1.1 \times 10^{-7}s$$

d. Suppose that one of the cables that connects the defibrillator paddles to the defibrillator controller is shown below. The cable is parallel to and $3cm$ above the ground. When current flows though the cable, it does so from right to left (taken as east to west). In this region of space, the Earth's magnetic field is $50\mu T$ and points into the page (taken to be north). Suppose that as paramedic is charging the defibrillator, the maximum current in the cable is $24A$ and the time constant for the charging process is $2s$. At a time $t = 4s$, what is the net magnetic field on the ground directly below the cable?

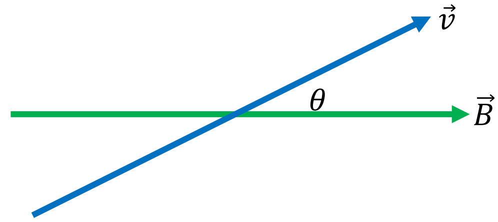
Taking out of the page as the positive direction we have:

$$B_{net} = B_{cable} - B_{earth} = \frac{\mu_0 I}{2\pi r} - B_{earth}$$

$$B_{net} = \frac{4\pi \times 10^{-7}\frac{Tm}{A} \times 24A \times e^{-\frac{4s}{2s}}}{2\pi \times 0.03m} - 50 \times 10^{-6}T = -2.8 \times 10^{-5}T$$

$$B_{net} = 2.8 \times 10^{-5}T = 28\mu T \text{ into the page.}$$

2. A charged particle of unknown sign and mass $m = 5 \times 10^{-25} kg$ is launched with a velocity $\vec{v}$ at an angle θ with respect to the magnetic field $\vec{B}$ shown below. The radius of the circular orbit about the magnetic field is $R = 10cm$ and the pitch parallel to the magnetic field $L = 40cm$.



a. At what angle θ does the velocity vector $\vec{v}$ make with respect to the magnetic field $\vec{B}$?

$$v_{\parallel} = v\cos\theta = \frac{L}{T}$$
$$v_{\perp} = v\sin\theta = \frac{2\pi R}{T}$$

$$\frac{v_{\perp}}{v_{\parallel}} = \tan\theta = \frac{2\pi R}{L} \rightarrow \theta = \tan^{-1}\left(\frac{2\pi R}{L}\right) = \tan^{-1}\left(\frac{2\pi \times 0.1m}{0.4m}\right) = 57.5^0$$

b. Suppose the charge has magnitude $q = 12e$ and that the charge was accelerated from rest through a potential difference $|\Delta V| = 500V$. What is the orbital period of the charge's motion about the magnetic field? To earn full credit, you cannot simply quote a formula and evaluate it. You need to derive an expression first, then evaluate that expression.

$$W = -q\Delta V = \Delta K = \tfrac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2 \times [12 \times 1.6 \times 10^{-19}C] \times 500V}{5 \times 10^{-25}kg}}$$

$$v = 6.2 \times 10^4 \frac{m}{s}$$

$$v_{\perp} = v\sin\theta = \frac{2\pi R}{T} \rightarrow T = \frac{2\pi R}{v\sin\theta} = \frac{2\pi \times 0.1m}{6.2 \times 10^4 \frac{m}{s} \sin 57.5} = 1.2 \times 10^{-5}s$$

Or,

$$v_{\parallel} = v\cos\theta = \frac{L}{T} \rightarrow T = \frac{L}{v\cos\theta} = \frac{0.4m}{6.2 \times 10^4 \frac{m}{s} \cos 57.5} = 1.2 \times 10^{-5}s$$

c. What was the magnitude of the magnetic field that the charge interacted with?

$$F = qv_{\perp}B = m\frac{v_{\perp}^2}{R} \rightarrow B = \frac{mv_{\perp}}{qR} = \frac{5 \times 10^{-25}kg \times 6.2 \times 10^4\frac{m}{s}\sin 57.5}{12 \times 1.6 \times 10^{-19}C \times 0.1m} = 0.14T$$

Or,

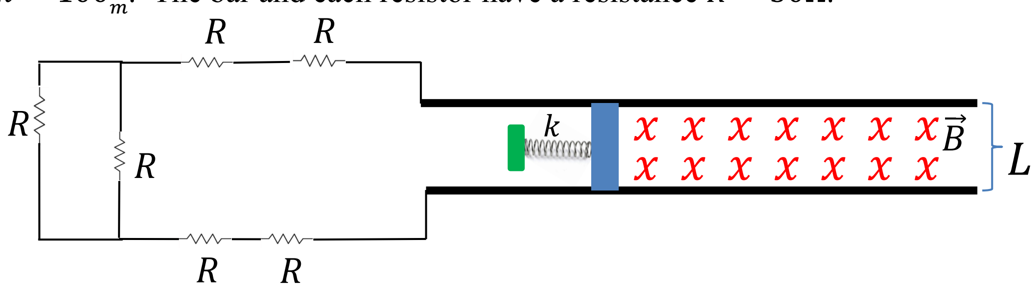
$$F = qv_{\perp}B = \frac{mv_{\perp}^2}{R} \rightarrow B = \frac{mv_{\perp}}{qR} = \frac{m}{qR}\left(\frac{2\pi R}{T}\right) = \frac{2\pi m}{qT}$$

$$B = \frac{2\pi \times 5 \times 10^{-25}kg}{12 \times 1.6 \times 10^{-19}C \times 1.2 \times 10^{-5}s} = 0.14T$$

d. Suppose that we look down the magnetic field lines (so that $\vec{B}$ points into the page). The interaction of the charge with the magnetic field produces a clockwise circular orbit about the magnetic field. Using this information, determine and explain fully, the sign of the charge. Simply stating positive or negative will earn no credit. You must explain your reasoning for picking positive or negative.

Using the right-hand rule, fingers point into the page and the velocity points into the page. From this the force must be up if the particle undergoes a clockwise circular orbit about the magnetic field. From the RHR, the back of the hand is pointing up the page and thus the charge must be negative.

3. Consider the network of resistors connected to a set of parallel rails. A bar of mass $m = 250g$ and length $L = 40cm$ (colored blue) is connected to a spring with stiffness $k = 100\frac{N}{m}$. The bar and each resistor have a resistance $R = 30\Omega$.



a. What is the equivalent resistance of the circuit?

The left-most resistors are in parallel and the equivalent resistance of this combination is:

$$\frac{1}{R'} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R} \rightarrow R' = \frac{R}{2}$$

This resistor is in series with the remaining four resistors and the bar. The equivalent resistance of the circuit is therefore:

$$R_{eq} = R_{bar} + R + R + R + R + \frac{R}{2} = 5.5R = 5.5 \times 30\Omega = 165\Omega$$

b. The bar is pushed against the spring compressing the spring from its equilibrium position by $10cm$. If the bar is released from rest and leaves the spring when the spring reaches its equilibrium position, what is the speed of the bar when it loses contact with the spring? Hint: The spring force and potential energy are given by $F = -kx$ and $U = \frac{1}{2}kx^2$ respectively.

$$W = -\Delta U_e = \Delta K \rightarrow \tfrac{1}{2}kx_i^2 = \tfrac{1}{2}mv_f^2 \rightarrow v_f = \sqrt{\frac{k}{m}}x_i = \sqrt{\frac{100\frac{N}{m}}{0.25kg}} \times 0.1m = 2\frac{m}{s}$$

c. What externally applied force would be needed to keep the bar moving at the constant speed determined in part b, through the external magnetic field $B = 0.5T$?

Taking to the right as the positive x-direction:

$$F_{ext} - F_B = ma_x = 0 \rightarrow F_{ext} = F_B = IlB = \left(\frac{Blv}{R_{eq}}\right) lB = \frac{B^2 l^2 v}{R_{eq}}$$

$F_{ext} = \frac{(0.5T)^2 (0.4m)^2 \times 2\frac{m}{s}}{165\Omega} = 4.9 \times 10^{-4} N$ to the right.

d. What is the drift speed of charge carriers in the bar if the bar is made out of tungsten with dimensions $W \times H \times L = 2mm \times 2mm \times 400mm$. Tungsten donates 2 charge carriers per atom and here is some data on tungsten: ${}^{183}_{74}W$: $\rho_W = 19250\frac{kg}{m^3}$, $\rho = 5.6 \times 10^{-8}\Omega m$, and $m_W = 0.183\frac{kg}{mole}$.

$$I = neAv_d \rightarrow v_d = \frac{I}{neA} = \frac{1}{neA}\left(\frac{Blv}{R_{eq}}\right)$$

$$v_d = \frac{0.5T \times 0.4m \times 2\frac{m}{s}}{1.3 \times 10^{29} m^{-3} \times 1.6 \times 10^{-19} C \times (2 \times 10^{-3} m)^2 \times 165\Omega} = 2.9 \times 10^{-8}\frac{m}{s}$$

Where, the number density of charge carriers is determined from:

$$n = \left(\frac{\rho_W N_A}{m_W}\right) \times 2 = \left(\frac{19250\frac{kg}{m^3}}{0.183\frac{kg}{mole}} \times 6.02 \times 10^{23}\frac{Watoms}{mol}\right) \times 2\frac{charge\ carriers}{W\ atom}$$

$$n = 1.3 \times 10^{29} m^{-3}$$

Physics 111 Formula Sheet

Electrostatics

$F = k\frac{q_1 q_2}{r^2}$

$\vec{F} = q\vec{E}; \quad E_{pc} = k\frac{q}{r^2}; \quad E_{plate} = \frac{q}{\epsilon_0 A}$

$E = -\frac{\Delta V}{\Delta x}$

$V_{pc} = k\frac{q}{r}$

$U_e = k\frac{q_1 q_2}{r} = qV$

$W = -q\Delta V = -\Delta U_e = \Delta K$

Magnetism

$\vec{F} = q\vec{v} \times \vec{B} \rightarrow F = qvB\sin\theta$

$\vec{F} = I\vec{L} \times \vec{B} \rightarrow F = ILB\sin\theta$

$V_{Hall} = wv_d B$

$B = \frac{\mu_0 I}{2\pi r}$

$\varepsilon = \Delta V = -N\frac{\Delta\phi_B}{\Delta t}$

$\phi_B = BA\cos\theta$

Electric Circuits - Capacitors

$Q = CV; \quad C = \frac{\kappa\epsilon_0 A}{d}$

$C_{parallel} = \sum_{i=1}^{N} C_i$

$\frac{1}{C_{series}} = \sum_{i=1}^{N} \frac{1}{C_i}$

$Q_{charging}(t) = Q_{max}\left(1 - e^{-\frac{t}{\tau}}\right)$

$Q_{discharging}(t) = Q_{max}e^{-\frac{t}{\tau}}$

$I(t) = I_{max}e^{-\frac{t}{\tau}} = \frac{Q_{max}}{\tau}e^{-\frac{t}{\tau}}$

$\tau = RC$

$U_C = \frac{1}{2}qV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$

Electric Circuits - Resistors

$I = \frac{\Delta Q}{\Delta t}$

$I = neAv_d; \quad n = \frac{\rho N_A}{m}$

$V = IR$

$R = \frac{\rho L}{A}$

$R_{series} = \sum_{i=1}^{N} R_i$

$\frac{1}{R_{parallel}} = \sum_{i=1}^{N} \frac{1}{R_i}$

$P = \frac{\Delta E}{\Delta t} = IV = I^2 R = \frac{V^2}{R}$

Light as a Wave

$c = f\lambda$

$S(t) = \frac{\text{Energy}}{\text{time}\times\text{Area}} = c\epsilon_0 E^2(t) = c\frac{B^2(t)}{\mu_0}$

$I = S_{avg} = \frac{1}{2}c\epsilon_0 E_{max}^2 = c\frac{B_{max}^2}{2\mu_0}$

$P = \begin{cases} \frac{S}{c}; & \text{absorbed} \\ \frac{2S}{c}; & \text{reflected} \end{cases}$

$S = S_0\cos^2\ \theta$

$v = \frac{c}{n}$

$\theta_{\text{incident}} = \theta_{\text{reflected}}$

$n_1\sin\theta_1 = n_2\sin\theta_2$

$P = \frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i}$

$M = \frac{d_i}{d_0}; \quad |M| = \frac{h_i}{h_0}$

Light as a Particle/Relativity

$E = hf = \frac{hc}{\lambda}$

$K_{max} = hf - \phi$

$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$

$\frac{1}{E'} = \frac{1}{E} + \frac{(1-\cos\phi)}{E_{rest}}; \quad E_{rest} = mc^2$

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$p = \gamma mv$

$E_{total} = E_{rest} + K = \gamma mc^2$

$K = (\gamma - 1)mc^2$

$E_{total}^2 = p^2c^2 + m^2c^4$

Nuclear Physics

$N = N_0 e^{-\lambda t}$

$m = m_0 e^{-\lambda t}$

$A = A_0 e^{-\lambda t}$

$A = \lambda N$

$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

Constants

$g = 9.8\frac{m}{s^2}$

$1e = 1.6 \times 10^{-19} C$

$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$

$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$

$1eV = 1.6 \times 10^{-19} J$

$\mu_0 = 4\pi \times 10^{-7} \frac{Tm}{A}$

$c = 3 \times 10^8 \frac{m}{s}$

$h = 6.63 \times 10^{-34} Js = 4.14 \times 10^{-15} eVs$

$N_A = 6.02 \times 10^{23}$

$1u = 1.66 \times 10^{-27} kg = 931.5 \frac{MeV}{c^2}$

$m_p = 1.67 \times 10^{-27} kg = 937.1 \frac{MeV}{c^2}$

$m_n = 1.69 \times 10^{-27} kg = 948.3 \frac{MeV}{c^2}$

$m_e = 9.11 \times 10^{-31} kg = 0.511 \frac{MeV}{c^2}$

Physics 110 Formulas

$\vec{F} = m\vec{a}; \quad F_G = \frac{GM_1 m_2}{r^2}; \quad F_s = -ky; \quad a_c = \frac{v^2}{r}$

$W = -\Delta U_g - \Delta U_s = \Delta K$

$U_g = mgy$

$U_s = \frac{1}{2}ky^2$

$K = \frac{1}{2}mv^2$

$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$

$\vec{v}_f = \vec{v}_i + \vec{a}t$

$v_f^2 = v_i^2 + 2a_r \Delta r$

Common Metric Units

$nano\ (n) = 10^{-9}$

$micro\ (\mu) = 10^{-6}$

$milli\ (m) = 10^{-3}$

$centi\ (c) = 10^{-2}$

$kilo\ (k) = 10^3$

$mega\ (M) = 10^6$

Geometry/Algebra

Circles: $A = \pi r^2$ $C = 2\pi r = \pi$

Spheres: $A = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$

Triangles: $A = \frac{1}{2}bh$

Quadratics: $ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$