## Physics 111

Exam \#1

## February 14, 2014

Name $\qquad$

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}|=(5 \mathrm{~kg}) \times\left(2 \frac{\mathrm{~m}}{\mathrm{~s}}\right)=10 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 3 points and each free-response part is worth 9 points

| Problem \#1 | $/ 24$ |
| :---: | :---: |
| Problem \#2 | $/ 24$ |
| Problem \#3 | $/ 24$ |
| Total | $/ 72$ |

I affirm that I have carried out my academic endeavors with full academic honesty.

1. Consider the circuit below. A collection of resistors is connected to a battery rated at $V=13.8 \mathrm{~V}$.

a. What are the equivalent resistance of the circuit and the total current produce by the battery?

Resistors $4 \Omega, 16 \Omega$ and $48 \Omega$ are in parallel. Their effective resistance is given by

$$
\frac{1}{R_{41648}}=\frac{1}{R_{4}}+\frac{1}{R_{16}}+\frac{1}{R_{48}}=\frac{1}{4 \Omega}+\frac{1}{16 \Omega}+\frac{1}{48 \Omega}=\frac{12+3+1}{48 \Omega}=\frac{16}{48 \Omega} \rightarrow R_{41648}=3 \Omega .
$$

This equivalent resistor is in series with the $3 \Omega$ resistor in the original circuit.
Thus the equivalent resistance of the circuit is $R_{e q}=R_{3}+R_{41648}=3 \Omega+3 \Omega=6 \Omega$.
The total current is given by Ohm's law: $I=\frac{V}{R_{e q}}=\frac{13.8 \mathrm{~V}}{6 \Omega}=2.3 \mathrm{~A}$.
b. What are the currents through all of the resistors and the potential difference across all resistors?

The potential drop across the $3 \Omega$ resistor is given by $V_{R_{3}}=I_{\text {total }} R_{3}=2.3 \mathrm{~A} \times 3 \Omega=6.9 \mathrm{~V}$. The potential drop across the resistor combination $R_{41648}$ is also 6.9 V . Thus $V_{R_{4}}=V_{R_{16}}=V_{R_{48}}=6.9 \mathrm{~V}$. The currents through each resistor are given by Ohm's law. Thus $I_{4}=\frac{V}{R_{4}}=\frac{6.9 \mathrm{~V}}{4 \Omega}=1.725 \mathrm{~A}$, $I_{16}=\frac{V}{R_{16}}=\frac{6.9 \mathrm{~V}}{16 \Omega}=0.431 \mathrm{~A}$, and $I_{48}=\frac{V}{R_{48}}=\frac{6.9 \mathrm{~V}}{48 \Omega}=0.144 \mathrm{~A}$. As a check the sum of these currents adds to the total current as required.
c. When we worked out the theory for capacitors wired in series and parallel we developed two rules based on applying conservation of energy and charge to the circuit as well as the definition of capacitance to determine if the rules we developed made sense. As an example, for two capacitors in series, the effective capacitance adds according to $\frac{1}{C_{e q}}=\sum_{i=1}^{N} \frac{1}{C_{i}}$. This is because the effective distance between the outside plates of both capacitors has increased and therefore the effective capacitance has decreased. We can do the same thing for resistors. Suppose that you have two resistors wired in parallel. The effective resistance decreases according to $\frac{1}{R_{e q}}=\sum_{i=1}^{N} \frac{1}{R_{i}}$. This is primarily due to

1. the length of the overall resistor increasing.
2. the length of the overall resistor decreasing.
the area of the overall resistor increasing.
3. the area of the overall resistor decreasing.
4. the resistivity of the composite system decreasing.
d. Suppose that in your circuit above the wires were made out of copper and further suppose that you want to perform an experiment to investigate the thermal and electrical properties those copper wires. To perform these experiments you take the wire and connect it in series with an ammeter, a power supply $V$ and a switch $S$. You select a voltage and close the switch so that current flows through the circuit and this raises the wire's temperature. The values in the table below show the results of your experimentation where the trials were conducted at different wire temperatures $T$. The initial value of the resistance $R$ was measured at a temperature of $T=293 \mathrm{~K}$ with an ohmmeter. The other values were calculated using Ohm's law. Using the data, which graph below best illustrates the relation between the temperature $T$ and the resistance $R$ ?

| Trial | $\mathrm{T}(\mathrm{K})$ | $\mathrm{V}(\mathrm{V})$ | $\mathrm{I}(\mathrm{A})$ | $\mathrm{R}(\Omega)$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 293 | 0 | 0 | 4.0 |
| 2 | 373 | 4.6 | 0.75 | 6.1 |
| 3 | 473 | 10 | 1.18 | 8.7 |
| 4 | 573 | 18 | 1.60 | 11.3 |
| 5 | 673 | 28 | 2.00 | 13.9 |

a.)

b.

c.

d.

2. Suppose that a proton is moving through a magnetic field and its motion is shown below. If you are at the motion on the left side of the page, the proton orbits in a counter-clockwise circle and also moves (across the page from left to right) away from you. The pitch of the proton's orbit is given as $L=0.1 \mathrm{~m}$, the orbital period of the proton $T=0.037 \mu s$, and the perpendicular component of the proton's velocity is $v_{\perp}=2.1 \times 10^{6} \frac{\mathrm{~m}}{\mathrm{~s}}$.

a. To produce this motion of the proton the magnetic field that the proton is interacting with is

1. directed toward the left side of the page
2. directed toward the right side of the page.
3. directed out of the page.
4. directed into the page.
b. What are the magnitude of the velocity of the proton, the orbital radius of the proton, and the magnitude of the external magnetic field?

The component of the velocity parallel to the field is given by $v_{/ /}=\frac{L}{T}=\frac{0.1 \mathrm{~m}}{0.037 \times 10^{-6} \mathrm{~s}}=2.7 \times 10^{6} \frac{\mathrm{~m}}{\mathrm{~s}}$. Thus the magnitude of the velocity of the proton is $v=\sqrt{v_{\|}^{2}+v_{\perp}^{2}}=\sqrt{(2.7)^{2}+(2.1)^{2}} \times 10^{6} \frac{\mathrm{~m}}{\mathrm{~s}}=3.4 \times 10^{6} \frac{\mathrm{~m}}{\mathrm{~s}}$.

From the perpendicular component of the velocity, the orbital radius is given as

$$
v_{\perp}=\frac{2 \pi r}{T} \rightarrow r=\frac{v_{\perp} T}{2 \pi}=\frac{2.1 \times 10^{6} \frac{\mathrm{~m}}{s} \times 0.037 \times 10^{-6} \mathrm{~s}}{2 \pi}=0.0124 \mathrm{~m}=12.4 \mathrm{~mm} .
$$

The magnitude of the magnetic field is given from the magnetic force law.

$$
F_{B}=q v_{\perp} B=F_{C}=m \frac{v_{\perp}^{2}}{r} \rightarrow B=\frac{m v_{\perp}}{q r}=\frac{1.67 \times 10^{-27} \mathrm{~kg} \times 2.1 \times 10^{6} \frac{\mathrm{~m}}{\mathrm{~s}}}{1.6 \times 10^{-19} \mathrm{C} \times 0.0124 \mathrm{~m}}=1.77 \mathrm{~T} .
$$

c. Suppose that the magnetic field is turned off at the instant that the proton is at point $A$ and that the proton is coming out of the page at you. At the very instant that the first magnetic field is turned off, a wire oriented vertically oriented and located somewhere to the right of point A suddenly has a current $I$ flowing and this current is running up the plane of the page. Due to this current flowing in the wire, the proton feels a force

1. $F_{B}=\frac{\mu_{o} I}{2 \pi r}$ directed to the left.
2. $F_{B}=\frac{\mu_{o} I}{2 \pi r}$ directed to the right.
3. $F_{B}=\frac{\mu_{0} I}{2 \pi r}$ directed up the plane of the page.
4. $F_{B}=\frac{\mu_{0} I}{2 \pi r}$ directed down the plane of the page.
5. $F_{B}=0$ and the proton feels no magnetic force.
d. Now suppose that you have the setup shown below. The accelerator is used to accelerate an electron and then this electron heads toward the midpoint between the two circuits. What force does the electron feel as it passes between the two circuits along a line through the midpoint between the two circuits? Assume that a long straight wire $\# 1$ is connected to a battery rated at 15 V and a $271 \Omega$ resistor, while another long straight wire $\# 2$ is connected to a $8 V$ battery and a $313 \Omega$ resistor and the two wires ( $\# 1 \& \# 2$ ) are separated by 0.5 m and that each blue segment of wire has a length of 0.25 m .


Current $I_{1}=\frac{V_{1}}{R_{1}}=\frac{15 \mathrm{~V}}{271 \Omega}=0.055 \mathrm{~A}$ produces a magnetic field at the midpoint between the two circuits of $B_{1}=\frac{\mu_{o} I_{i}}{2 \pi r_{1}}=\frac{4 \pi \times 10^{-7} \frac{T m}{A} \times 0.055 \mathrm{~A}}{2 \pi \times 0.25 \mathrm{~m}}=4.4 \times 10^{-8} \mathrm{~T}$ directed out of the page. Current $I_{2}=\frac{V_{2}}{R_{2}}=\frac{8 \mathrm{~V}}{313 \Omega}=0.026 \mathrm{~A}$ produces a magnetic field at the midpoint between the two circuits of $B_{2}=\frac{\mu_{o} I_{2}}{2 \pi r_{2}}=\frac{4 \pi \times 10^{-7} \frac{\mathrm{Tm}}{A} \times 0.026 \mathrm{~A}}{2 \pi \times 0.25 \mathrm{~m}}=2.1 \times 10^{-8} \mathrm{~T}$ directed
out of the page. Thus the magnitude of the force felt by the electron is given by $F_{B}=q v B_{\text {net }}=1.6 \times 10^{-19} \mathrm{C} \times 6.63 \times 10^{6} \frac{\mathrm{~m}}{\mathrm{~s}} \times\left(4.4 \times 10^{-8} \mathrm{~T}+2.1 \times 10^{-8} \mathrm{~T}\right)=6.9 \times 10^{-20} \mathrm{~N}$ and the direction of the magnetic force is toward wire \#2 by the RHR. The speed of the electron used in the calculation is given by

$$
W=-q \Delta V=\Delta K E=\frac{1}{2} m v^{2} \rightarrow v=\sqrt{\frac{2 e V}{m}}=\sqrt{\frac{2 \times 1.6 \times 10^{-19} \mathrm{C} \times 125 \mathrm{~V}}{9.11 \times 10^{-31} \mathrm{~kg}}}=6.63 \times 10^{6} \frac{\mathrm{~m}}{\mathrm{~s}}
$$

3. Experiments to study vision often need to track the movements of a subject's eye. A subject is placed in a magnetic field while wearing a special contact lens that has a coil of fine wire built into it, encircling the edge of the lens and as the eye rotates a current is induced in the coil. Suppose that the wire is made out of copper ( $\rho=$ $1.68 \times 10^{-8} \Omega \mathrm{~m}$ ) with a cross sectional area of $7.85 \times 10^{-7} \mathrm{~m}^{2}$.

a. Suppose that your head is placed between two coils of wire, each of which has radius $r=0.15 \mathrm{~m}$ and 130 -turns of wire, through which a counter-clockwise current $I=1.6 \mathrm{~A}$ is passed. Suppose that the magnetic field is uniform across your head. If you rotate one of your eyes by $10^{\circ}$ in the direction of the magnetic field (in $0.2 s$ ), what are the magnitude and direction of the induced current in the wire loop around the contact lens in your eye? (Hints: For the direction of the induced current, assume you are looking at the patient's eye. The magnetic field at the center of these two coils (and across your eyes) is given by $B=\frac{\mu_{0} N I}{\sqrt{125} r}$ and the two coils are placed such that the normal to the plane of the each coil points toward your ears.)

The external magnetic field is
$B=\frac{\mu_{0} N I}{\sqrt{125} r}=\frac{4 \pi \times 10^{-7} \frac{T m}{A} \times 130 \times 1.6 A}{\sqrt{125} \times 0.15 \mathrm{~m}}=0.00016 T=0.16 \mathrm{mT}$ and the resistance of the coil of wire in the contact lens is

$$
R=\frac{\rho L}{A}=\frac{1.68 \times 10^{-8} \Omega m \times(2 \pi(0.003 \mathrm{~m}))}{7.85 \times 10^{-7} \mathrm{~m}^{2}}=0.004 \Omega=0.4 \mathrm{~m} \Omega .
$$

The induced current is given from Ohm's law using Faraday's law to calculate the induced potential difference across the wire. We have

$$
\begin{aligned}
& I=\frac{\varepsilon}{R}=\frac{1}{R}\left(\frac{\Delta(B A \cos \theta)}{\Delta t}\right)=\frac{B A}{R}\left(\frac{\Delta(\cos \theta)}{\Delta t}\right) \\
& I=\left(\frac{0.00016 T \times \pi(0.003 \mathrm{~m})^{2}}{R}\right)\left(\frac{\cos 80-\cos 90}{0.2 s}\right) \\
& I=\frac{3.93 \times 10^{-9} V}{R}=\frac{3.93 \times 10^{-9} V}{0.0004 \Omega}=9.82 \times 10^{-6} \mathrm{~A}=9.82 \mu \mathrm{~A}
\end{aligned}
$$

and the current flows CW to oppose the change (increase) in magnetic flux.
b. What magnitude of the electric field is produced in the wire of the contact lens and what energy will be dissipated across the loop of wire as heat in the contact lens as the eye rotates through the $10^{\circ}$ angle?

The electric field is given by $|E|=\left|\frac{\Delta V}{\Delta x}\right|=\frac{3.93 \times 10^{-9} \mathrm{~V}}{2 \pi(0.003 \mathrm{~m})}=2.1 \times 10^{-7} \frac{\mathrm{~V}}{\mathrm{~m}}$.
The energy dissipated across the loop of wire as heat is given from
Power $=\frac{\text { energy }}{\text { time }} \rightarrow$ energy $=P t$
energy $=\left(I^{2} R\right) t=\left(9.8 \times 10^{-6} A\right)^{2} \times 0.0004 \Omega \times 0.2 s=7.7 \times 10^{-14} J$
c. Suppose that the wire in the contact lens were made twice as thick as the original wire in the lens. In this situation, the current induced and the energy dissipated as heat in the contact lens would vary according to

1. the induced current and energy dissipated will both increase.
2. the induced current and energy dissipated will both decrease.
3. the induced current will increase and the energy dissipated will decrease.
4. the induced current will decrease and the energy dissipated will increase.
5. the induced current and energy dissipated will remain unchanged.
d. Suppose that you rotate your eyes each through an angle of $-40^{\circ} \leq \theta \leq+40^{\circ}$. Which of the following graphs would best show the induced current produced in the coil of wire in the contact lens in one of your eyes as a function of time?


## Physics 111 Equation Sheet

Electric Forces, Fields and Potentials

$$
\begin{aligned}
& \vec{F}=k \frac{Q_{1} Q_{2}}{r^{2}} \hat{r} \\
& \vec{E}=\frac{\vec{F}}{q} \\
& \vec{E}_{Q}=k \frac{Q}{r^{2}} \hat{r} \\
& P E=k \frac{Q_{1} Q_{2}}{r} \\
& V(r)=k \frac{Q}{r} \\
& E_{x}=-\frac{\Delta V}{\Delta x} \\
& W=-q \Delta V
\end{aligned}
$$

Magnetic Forces and Fields
$F=q \nu B \sin \theta$
$F=I l B \sin \theta$
$\tau=N I A B \sin \theta=\mu B \sin \theta$
$P E=-\mu B \cos \theta$
$B=\frac{\mu_{0} I}{2 \pi r}$
$\varepsilon_{\text {induced }}=-N \frac{\Delta \phi_{B}}{\Delta t}=-N \frac{\Delta(B A \cos \theta)}{\Delta t}$

## Constants

$g=9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$
$1 e=1.6 \times 10^{-19} \mathrm{C}$
$k=\frac{1}{4 \pi \varepsilon_{0}}=9 \times 10^{9} \frac{\frac{\mathrm{Nm}}{}{ }^{2}}{c^{2}}$
$\varepsilon_{o}=8.85 \times 10^{-12} \frac{\mathrm{C}^{2}}{\frac{N_{m}^{2}}{2}}$
$1 \mathrm{eV}=1.6 \times 10^{-19} \mathrm{~J}$
$\mu_{o}=4 \pi \times 10^{-7} \frac{7 m}{A}$
$c=3 \times 10^{8} \frac{\mathrm{~m}}{\mathrm{~s}}$
$h=6.63 \times 10^{-34} \mathrm{Js}$
$m_{e}=9.11 \times 10^{-31} \mathrm{~kg}=\frac{0.511 \mathrm{MeV}}{c^{2}}$
$m_{p}=1.67 \times 10^{-27} \mathrm{~kg}=\frac{937.1 \mathrm{MeV}}{c^{2}}$
$m_{n}=1.69 \times 10^{-27} \mathrm{~kg}=\frac{948.3 \mathrm{MeV}}{c^{2}}$
$1 \mathrm{amu}=1.66 \times 10^{-27} \mathrm{~kg}=\frac{931.5 \mathrm{MeV}}{\mathrm{c}^{2}}$
$N_{A}=6.02 \times 10^{23}$
$A x^{2}+B x+C=0 \rightarrow x=\frac{-B \pm \sqrt{B^{2}-4 A C}}{2 A}$

## Electric Circuits

$$
\begin{aligned}
& I=\frac{\Delta Q}{\Delta t} \\
& V=I R=I\left(\frac{\rho L}{A}\right)
\end{aligned}
$$

$$
R_{\text {series }}=\sum_{i=1}^{N} R_{i}
$$

$$
\frac{1}{R_{\text {parallel }}}=\sum_{i=1}^{N} \frac{1}{R_{i}}
$$

$$
P=I V=I^{2} R=\frac{V^{2}}{R}
$$

$$
Q=C V=\left(\frac{\kappa \varepsilon_{0} A}{d}\right) V=\left(\kappa C_{0}\right) V
$$

$$
P E=\frac{1}{2} Q V=\frac{1}{2} C V^{2}=\frac{Q^{2}}{2 C}
$$

$$
Q_{\text {charge }}(t)=Q_{\max }\left(1-e^{-\frac{t}{R C}}\right)
$$

$$
Q_{\text {discharge }}(t)=Q_{\max } e^{-\frac{t}{R C}}
$$

$$
C_{\text {parallel }}=\sum_{i=1}^{N} C_{i}
$$

$$
\frac{1}{C_{\text {series }}}=\sum_{i=1}^{N} \frac{1}{C_{i}}
$$

Light as a Particle \& Relativity

$$
\begin{aligned}
& E=h f=\frac{h c}{\lambda}=p c \\
& K E_{\max }=h f-\phi=e V_{\text {stop }} \\
& \Delta \lambda=\frac{h}{m_{e} c}(1-\cos \phi) \\
& \gamma=\frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}} \\
& p=\gamma m v \\
& E_{\text {total }}=K E+E_{\text {rest }}=\gamma m c^{2} \\
& E_{\text {total }}^{2}=p^{2} c^{2}+m^{2} c^{4} \\
& E_{\text {rest }}=m c^{2} \\
& K E=(\gamma-1) m c^{2}
\end{aligned}
$$

Geometry
Circles: $C=2 \pi r=\pi D \quad A=\pi r^{2}$
Triangles: $A=\frac{1}{2} b h$
Spheres: $A=4 \pi r^{2} \quad V=\frac{4}{3} \pi r^{3}$

Light as a Wave

$$
\begin{aligned}
& c=f \lambda=\frac{1}{\sqrt{\varepsilon_{o} \mu_{o}}} \\
& S(t)=\frac{\text { energy }}{\text { time } \times \text { area }}=c \varepsilon_{o} E^{2}(t)=c \frac{B^{2}(t)}{\mu_{0}} \\
& I=S_{\text {avg }}=\frac{1}{2} c \varepsilon_{o} E_{\max }^{2}=c \frac{B_{\max }^{2}}{2 \mu_{0}} \\
& P=\frac{S}{c}=\frac{\text { Force }}{\text { Area }} \\
& S=S_{o} \cos ^{2} \theta \\
& v=\frac{1}{\sqrt{\varepsilon \mu}}=\frac{c}{n} \\
& \theta_{\text {inc }}=\theta_{\text {refl }} \\
& n_{1} \sin \theta_{1}=n_{2} \sin \theta_{2} \\
& \frac{1}{f}=\frac{1}{d_{o}}+\frac{1}{d_{i}} \\
& M=\frac{h_{i}}{h_{o}}=-\frac{d_{i}}{d_{o}} \\
& M_{\text {total }}=\prod_{i=1}^{N} M_{i} \\
& d \sin \theta=m \lambda \text { or }\left(m+\frac{1}{2}\right) \lambda \\
& a \sin \phi=m^{\prime} \lambda \\
& \text { Nuclear Physics } \\
& E_{\text {binding }}=\left(Z m_{p}+N m_{n}-m_{\text {rest }}\right) g^{2} \\
& \frac{\Delta N}{\Delta t}=-\lambda N_{o} \rightarrow N(t)=N_{o} e^{-\lambda t} \\
& A(t)=A_{o} e^{-\lambda t} \\
& m(t)=m_{o} e^{-\lambda t} \\
& t_{\frac{1}{2}}=\frac{\ln 2}{\lambda}
\end{aligned}
$$

Misc. Physics 110
Formulae
$\vec{F}=\frac{\Delta \vec{p}}{\Delta t}=\frac{\Delta(m v)}{\Delta t}=m \vec{a}$
$\vec{F}=-k \vec{y}$
$\vec{F}_{C}=m \frac{v^{2}}{R} \hat{r}$
$W=\Delta K E=\frac{1}{2} m\left(v_{f}^{2}-v_{i}^{2}\right)=-\Delta P E$
$P E_{g r a v i t y}=m g y$
$P E_{\text {spring }}=\frac{1}{2} k y^{2}$
$x_{f}=x_{i}+v_{i x} t+\frac{1}{2} a_{x} t^{2}$
$v_{f x}=v_{i x}+a_{x} t$
$v_{f x}^{2}=v_{i x}^{2}+2 a_{x} \Delta x$

