Physics 111

Exam #2

February 27, 2015

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg)\times\left(2\,\frac{m}{s}\right) = 10\,\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 3 points and each free-response part is worth 7 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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1. Two $1m$ long wires are oriented horizontally across the page separated by a distance $33cm$. The upper and lower wires have currents of $3A$ running to the right.

   a. What is the net magnetic field at a point $1m$ above the upper wire? (Hint: Recall magnetic fields are vectors quantities.)

      By the right hand rule, both magnetic fields are pointing out of the page, so let the positive direction be out of the page. We have

      $$B_{net} = B_u + B_l = \frac{\mu_0 I_u}{2\pi r_{u,P}} + \frac{\mu_0 I_l}{2\pi r_{l,P}} = \frac{\mu_0 I}{2\pi}\left(\frac{1}{r_{u,P}} + \frac{1}{r_{l,P}}\right)$$

      $$B_{net} = \frac{4\pi \times 10^{-7}\,\frac{Tm}{A} \times 3A}{2\pi}\left(\frac{1}{1m} + \frac{1}{1.33m}\right) = 1.05 \times 10^{-6}T$$

      directed out of the page.

   b. What is the force on the lower wire due to the current flowing in the upper wire? (Hint: Recall magnetic forces are vectors quantities.)

      The magnetic force in magnitude is given by

      $$F = I_l L_l B_{l,u} = I_l L_l\left(\frac{\mu_o I_u}{2\pi r}\right) = \frac{\mu_o L I^2}{2\pi r} = \frac{2\times 10^{-7}\,\frac{Tm}{A} \times 1m \times (3A)^2}{0.33m} = 5.46 \times 10^{-6} N$$ while

      the direction is towards the upper wire by the RHR.

c. Suppose that a 100-turn coil of wire were placed at a point $0.5m$ above the upper wire with its normal pointing out of the page at you. Assuming that the magnetic field is uniform over the coil of wire and that the current in the wires decreases in time according to $I(t) = 3 - 0.3t$ (where I is in Ampere's and the current varies over the interval $0 \le t \le 10s$.) What current (magnitude and direction) is induced in the coil of wire? (Suppose that the coil is constructed out of tungsten wire ($\rho_W = 5.6\times10^{-8}\,\Omega m$) and that is $0.5mm$ in diameter and is wound 100 times into a circle with diameter of $25cm$.)

The resistance of the wire is:

$$R = \frac{\rho L}{A} = \frac{5.6\times10^{-8}\,\Omega m \times \left(100\times 2\pi(0.125m)\right)}{\pi\left(0.25\times10^{-3}m\right)^2} = 22.4\Omega$$

The induced potential difference is:

$$\varepsilon = \left|N\frac{\Delta(BA\cos\theta)}{\Delta t}\right| = 100\times\pi(0.125m)^2\left|\frac{\Delta B}{\Delta t}\right|$$

$$\Delta B = B_f - B_i = \left|\frac{\mu_0}{2\pi}\left(\frac{1}{0.5m}+\frac{1}{0.83m}\right)[0-3]\right| = 1.92\times10^{-6}T$$

$$\therefore \varepsilon = 100\times\pi(0.125m)^2\left|\frac{1.92\times10^{-6}T}{10s}\right| = 4.2\times10^{-8}V$$

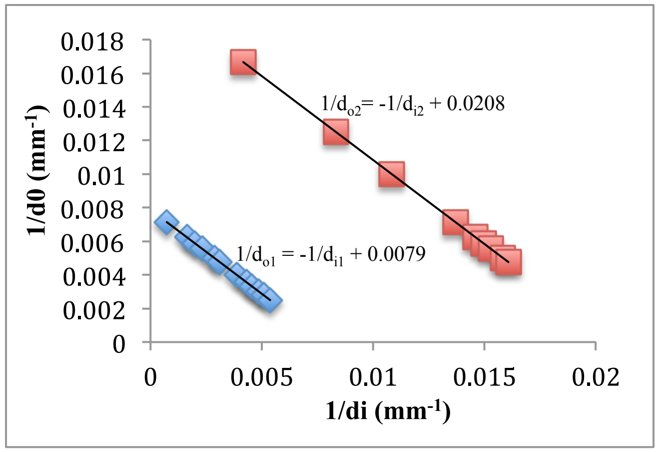
The current, by Ohm's law is: $I = \dfrac{\varepsilon}{R} = \dfrac{5.2\times10^{-7}V}{22.4\Omega} = 2.3\times10^{-8}A\ CCW$ to oppose the change in magnetic flux.

d. Suppose that the coil of wire in part c did not remain stationary as the current decreases with time but that as the current is decreasing in time across the coil of wire, the coil of wire were moved down the plane of the page towards the upper wire. In this case the direction of the current flow in the coil of wire would

1. be clockwise to oppose the decrease in magnetic flux.
2. be clockwise to oppose the increase in magnetic flux.
3. be counterclockwise to oppose the decrease in magnetic flux.
4. be counterclockwise to oppose the increase in magnetic flux.
5. not be able to be determined since the velocity of the coil is not known.

2. Data collected on object and image distances for two lenses are shown below. Suppose that lens #1 is placed to the left of lens #2 and that the two lenses are separated by $D = 32cm$.

   a. If a $0.5cm$ tall object is placed $12cm$ to *the right* of lens #2, where will the final image be located and what will be the size of the final image?



$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \to \frac{1}{d_o} = \frac{1}{d_i} - \frac{1}{f}$$

Lens #1:

$$\frac{1}{f_1} = 0.0079mm^{-1} \to f_1 = 126.6mm$$

Lens #2

$$\frac{1}{f_2} = 0.0208mm^{-1} \to f_2 = 48.1mm$$

$$\frac{1}{f_2} = \frac{1}{d_{o1}} + \frac{1}{d_{i1}} \to \frac{1}{d_{i1}} = \frac{1}{4.8cm} - \frac{1}{12cm} \to d_{i1} = 8cm.$$

$$D = d_{02} + d_{i1} \to d_{02} = D - d_{i1} = 32cm - 8cm = 24cm.$$

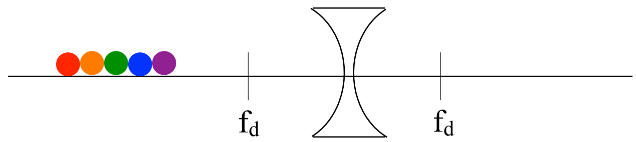
$$\frac{1}{f_1} = \frac{1}{d_{o2}} + \frac{1}{d_{i2}} \to \frac{1}{d_{i2}} = \frac{1}{12.7cm} - \frac{1}{24cm} \to d_{i2} = 27cm$$

$$M_{total} = M_1 M_2 = \frac{d_{i1}}{d_{o1}} \times \frac{d_{i2}}{d_{02}} = \left(\frac{8cm}{12cm}\right) \times \left(\frac{27cm}{24cm}\right) = 0.75 = \frac{h_{if}}{h_o}$$

$$h_{if} = M_{total} h_o = 0.75 \times 0.5cm = 0.375cm$$

   b. Keeping the two lenses in part a in the same locations and separated by the same distance, and that the object were moved and placed to the left of lens #1 at a distance of $12cm$. With respect to the object, the final image produced using the two lenses above would be
      1. real and upright.
      2. real and inverted.
      3. virtual and upright.
      4. virtual and inverted.

c. Suppose that you have the following situation in which you have a series of colored lights are placed to the left of a diverging lens, where the violet light is closer to the lens than the red light shown below. If the focal length of the lens is $f_d = -20cm$ and the violet light is located a distance $d_0 = 36cm$ to the left of the lens (as shown in the figure), what are the order of the colors in the image and what is the lateral magnification defined by $M = \frac{L_i}{L_o}$? Assume that the object's length is $L_0 = 5cm$.



The image location of the violet light:

$$\frac{1}{f_d} = \frac{1}{d_{ov}} + \frac{1}{d_{iv}} \rightarrow d_{iv} = \left(\frac{1}{f_d} - \frac{1}{d_{ov}}\right)^{-1} = \left(-\frac{1}{20cm} - \frac{1}{36cm}\right)^{-1} = -12.9cm$$

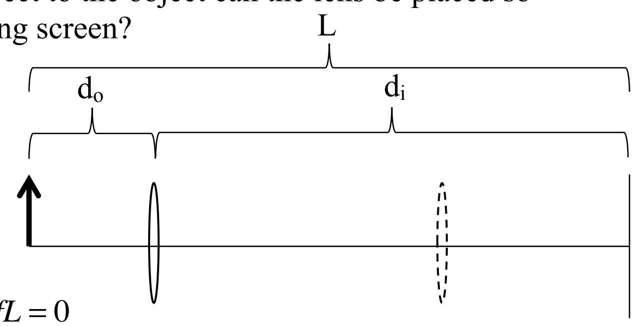
The image location of the red light:

$$\frac{1}{f_d} = \frac{1}{d_{or}} + \frac{1}{d_{ir}} \rightarrow d_{ir} = \left(\frac{1}{f_d} - \frac{1}{d_{or}}\right)^{-1} = \left(-\frac{1}{20cm} - \frac{1}{41cm}\right)^{-1} = -13.4cm.$$

Since the violet light is closer to the lens than the red, the order of the colors is preserved.

The magnification is: $M = \frac{L_i}{L_o} = \frac{13.4cm - 12.9cm}{5cm} = 0.1$

d. Suppose instead of the lens system in part a, you have a single object and a single viewing screen that are at a fixed separation $L = 66cm$. A single converging lens ($f = 4.8cm$) can be placed somewhere between the object and the viewing screen. At what location(s) with respect to the object can the lens be placed so that a sharp image forms on the viewing screen?



$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} + \frac{1}{L - d_o} \rightarrow d_o^2 - d_o L + fL = 0$$

$$d_0 = \frac{L \pm \sqrt{L^2 - 4fL}}{2} = \frac{66cm \pm \sqrt{(66cm)^2 - 4(4.8cm \times 66cm)}}{2} = \begin{cases} 60.8cm \\ 5.2cm \end{cases}$$

3. A beam of horizontally polarized light is incident on a polarizer with an intensity of $S_0 = 25\,\frac{mW}{cm^2}$. The transmission axis of the polarizer is oriented at 30^0 with respect to the vertical.

   a. The light that passes through the polarizer is incident on a viewing screen. What is the magnitude of the force that is exerted on the screen by the beam of light if the beam of light makes a circular spot with diameter $1cm$?

   The initial intensity is given as: $S_0 = 25\,\frac{mW}{cm^2} = \frac{25\times10^{-3}W}{cm^2}\times\left(\frac{100cm}{1m}\right)^2 = 250\,\frac{W}{m^2}$

   When this light passes through the polarizer the intensity of the light that emerges is: $S = S_0\cos^2\theta = 250\,\frac{W}{m^2}\cos^2 60 = 62.5\,\frac{W}{m^2}$

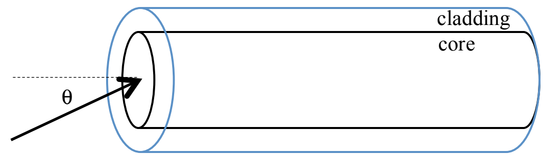
   From the radiation pressure we can calculate the force that the light exerts on the viewing screen:

   $$P = \frac{2S}{c} = \frac{F}{A} \rightarrow F = \frac{2SA}{c} = \frac{2\times62.5\,\frac{W}{m^2}\times\left[\pi\left(0.5\times10^{-2}m\right)^2\right]}{3\times10^8\,\frac{m}{s}} = 3.3\times10^{-11}N.$$

   b. Suppose that you wish to rotate the direction of polarization of the incident horizontally polarized beam by 90° (so that the electric field is pointing vertically after the light passes through the polarizer). To get the maximum transmitted intensity, you should use how many Polaroid sheets?

   1. One
   2. Two
   3. Three
   4. As many as possible
   5. There is no way to rotate the polarization from horizontal to vertical.

c. Suppose that the light that emerges from the polarizer is allowed to be incident on an optical fiber, as shown below. The optical fiber is constructed of two pieces of glass with different refractive indices. The core of the fiber has an index of $n_{core} = 1.62$ refraction while the cladding (the material surrounding the core) has an index of refraction $n_{cladding} = 1.52$. The entire fiber is surrounded by air and is shown below. At what angle θ would the beam of light from the polarizer have to be incident so that the light stays contained in the core of the fiber?



On the upper surface:

$$n_{core} \sin\theta_{core} = n_{cladding} \sin\theta_{cladding}$$

$$1.62 \sin\theta_c = 1.52 \sin 90 \to \theta_c = \sin^{-1}\left(\frac{1.52}{1.62}\right) = 69.8^0$$

The angle of refraction on the front surface is obtained from the geometry:

$$\theta_2 + \theta_c = 90^0 \to \theta_2 = 90^0 - \theta_c = 20.2^0 .$$

On the front surface:

$$n_{air} \sin\theta = n_{core} \sin\theta_{core}$$

$$1 \sin\theta = 1.62 \sin\theta_2 \to \theta = \sin^{-1}\left(1.62 \sin\left(20.2^0\right)\right) = 34^0$$

c. Suppose that light from the polarizer were incident at an angle θ on a slab of material with index of refraction n_{mat} surrounded on all sides by air as shown below. What is the minimum index of refraction that the material can have so that the light is totally internally reflected?

At the upper surface:

$$n_{mat} \sin\theta_c = n_{air} \sin\theta_{air} \to n_{mat} \sin\theta_c = 1.00 \sin 90 \to \sin\theta_c = \frac{1}{n_{mat}}$$

At the front surface:

$$n_{air} \sin\theta = n_{mat} \sin\left(90 - \theta_c\right) = n_{mat} \cos\theta_c$$

$$1 = n_{mat} \cos\theta_c \to \cos\theta_c = \frac{1}{n_{mat}}$$

Dividing the two results: $\tan\theta_c = 1 \to \theta_c = 45^0$ and therefore,

$$\sin\theta_c = \sin 45 = \frac{1}{n_{mat}} \Rightarrow n_{mat} = 1.41 .$$

Physics 111 Equation Sheet

Electric Forces, Fields and Potentials

$$\vec{F} = k\frac{Q_1Q_2}{r^2}\hat{r}$$

$$\vec{E} = \frac{\vec{F}}{q}$$

$$\vec{E}_Q = k\frac{Q}{r^2}\hat{r}$$

$$PE = k\frac{Q_1Q_2}{r}$$

$$V(r) = k\frac{Q}{r}$$

$$E_x = -\frac{\Delta V}{\Delta x}$$

$$W = -q\Delta V$$

Magnetic Forces and Fields

$$F = qvB\sin\theta$$

$$F = ILB\sin\theta$$

$$\tau = NIAB\sin\theta = \mu B\sin\theta$$

$$PE = -\mu B\cos\theta$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\varepsilon_{induced} = -N\frac{\Delta\phi_B}{\Delta t} = -N\frac{\Delta(BA\cos\theta)}{\Delta t}$$

Constants

$g = 9.8\,\frac{m}{s^2}$

$1e = 1.6\times10^{-19}C$

$k = \frac{1}{4\pi\varepsilon_o} = 9\times10^9\,\frac{Nm^2}{C^2}$

$\varepsilon_o = 8.85\times10^{-12}\,\frac{C^2}{Nm^2}$

$1eV = 1.6\times10^{-19}J$

$\mu_o = 4\pi\times10^{-7}\,\frac{Tm}{A}$

$c = 3\times10^8\,\frac{m}{s}$

$h = 6.63\times10^{-34}Js$

$m_e = 9.11\times10^{-31}kg = \frac{0.511MeV}{c^2}$

$m_p = 1.67\times10^{-27}kg = \frac{937.1MeV}{c^2}$

$m_n = 1.69\times10^{-27}kg = \frac{948.3MeV}{c^2}$

$1amu = 1.66\times10^{-27}kg = \frac{931.5MeV}{c^2}$

$N_A = 6.02\times10^{23}$

$Ax^2 + Bx + C = 0 \rightarrow x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$

Electric Circuits

$$I = \frac{\Delta Q}{\Delta t}$$

$$V = IR = I\left(\frac{\rho L}{A}\right)$$

$$R_{series} = \sum_{i=1}^{N} R_i$$

$$\frac{1}{R_{parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$$

$$P = IV = I^2R = \frac{V^2}{R}$$

$$Q = CV = \left(\frac{\kappa\varepsilon_0 A}{d}\right)V = (\kappa C_0)V$$

$$PE = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}$$

$$Q_{\text{charge}}(t) = Q_{\max}\left(1-e^{-\frac{t}{RC}}\right)$$

$$Q_{\text{discharge}}(t) = Q_{\max}e^{-\frac{t}{RC}}$$

$$C_{parallel} = \sum_{i=1}^{N} C_i$$

$$\frac{1}{C_{series}} = \sum_{i=1}^{N}\frac{1}{C_i}$$

Light as a Wave

$$c = f\lambda = \frac{1}{\sqrt{\varepsilon_o\mu_o}} = nv$$

$$S(t) = \frac{energy}{time\times area} = c\varepsilon_o E^2(t) = c\frac{B^2(t)}{\mu_0}$$

$$I = S_{avg} = \tfrac{1}{2}c\varepsilon_o E_{\max}^2 = c\frac{B_{\max}^2}{2\mu_0}$$

$$P_{abs} = \frac{S}{c} = \frac{Force}{Area}$$

$$P_{refl} = \frac{2S}{c} = \frac{Force}{Area}$$

$$S = S_o\cos^2\theta$$

$$\theta_{inc} = \theta_{refl}$$

$$n_1\sin\theta_1 = n_2\sin\theta_2$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$M_{total} = \prod_{i=1}^{N} M_i$$

$$S = S_0 e^{-\mu x}$$

$$HU = \frac{\mu_t - \mu_w}{\mu_w}$$

Light as a Particle & Relativity

$$E = hf = \frac{hc}{\lambda} = pc$$

$$KE_{\max} = hf - \phi = eV_{stop}$$

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\phi)$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$p = \gamma mv$$

$$E_{total} = KE + E_{rest} = \gamma mc^2$$

$$E_{total}^2 = p^2c^2 + m^2c^4$$

$$E_{rest} = mc^2$$

$$KE = (\gamma - 1)mc^2$$

Geometry

$Circles:\ C = 2\pi r = \pi D \qquad A = \pi r^2$

$Triangles:\ A = \tfrac{1}{2}bh$

$Spheres:\ A = 4\pi r^2 \qquad V = \tfrac{4}{3}\pi r^3$

Nuclear Physics

$$E_{binding} = (Zm_p + Nm_n - m_{rest})c^2$$

$$\frac{\Delta N}{\Delta t} = -\lambda N_o \rightarrow N(t) = N_o e^{-\lambda t}$$

$$A(t) = A_o e^{-\lambda t}$$

$$m(t) = m_o e^{-\lambda t}$$

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

Misc. Physics 110 Formulae

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\Delta(m\vec{v})}{\Delta t} = m\vec{a}$$

$$\vec{F} = -k\vec{y}$$

$$\vec{F}_C = m\frac{v^2}{R}\hat{r}$$

$$W = \Delta KE = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right) = -\Delta PE$$

$$PE_{gravity} = mgy$$

$$PE_{spring} = \tfrac{1}{2}ky^2$$

$$x_f = x_i + v_{ix}t + \tfrac{1}{2}a_x t^2$$

$$v_{fx} = v_{ix} + a_x t$$

$$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x$$