## Physics 111

## Exam \#2

February 24, 2017

Name $\qquad$

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}|=(5 \mathrm{~kg}) \times\left(2 \frac{\mathrm{~m}}{\mathrm{~s}}\right)=10 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 3 points and each free-response part is worth 7 points

| Problem \#1 | $/ 24$ |
| :---: | :---: |
| Problem \#2 | $/ 24$ |
| Problem \#3 | $/ 24$ |
| Total | $/ 72$ |

I affirm that I have carried out my academic endeavors with full academic honesty.

1. Suppose that you had two wire made out of zinc $\left(\rho_{Z n}=7140 \frac{\mathrm{~kg}}{\mathrm{~m}^{3}}\right)$ with length $l$ and radius $r=0.5 \mathrm{~mm}$. The wires are each suspended from two strings of length $L=0.5 \mathrm{~m}$ and negligible mass as shown in the figure below. When a current $I$ flows in each wire as shown in the diagram below the wires repel and the support wires make an angle of $\theta=9^{\circ}$ with respect to the vertical and the center-to-center separation between the wires is $S$. Ignore the proton beam in part a.
a. What is the magnitude of the current $I$ that flows in each wire?

$$
\begin{aligned}
& \sum F_{y}: F_{T} \cos \theta-F_{W}=0 \rightarrow F_{T}=\frac{F_{w}}{\cos \theta}=\frac{m g}{\cos \theta} \\
& \sum F_{x}: F_{T} \sin \theta-F_{B}=0 \rightarrow F_{B}=\frac{F_{w}}{\cos \theta} \sin \theta=m g \tan \theta \\
& I l B=I l\left(\frac{\mu_{o} I}{2 \pi S}\right)=m g \tan \theta \rightarrow I^{2}=\frac{2 \pi m g S \tan \theta}{\mu_{0} l}
\end{aligned}
$$


$\sin \theta=\frac{S / 2}{L} \rightarrow S=2 L \sin \theta=2 \times 0.5 \mathrm{~m} \times \sin 9=0.16 \mathrm{~m}$

$$
\rho=\frac{m}{V}=\frac{m}{\pi r_{\text {wire }}^{2} l} \rightarrow \frac{m}{l}=\rho_{Z n} \pi r_{\text {wire }}^{2}=7140 \frac{\mathrm{~kg}}{\mathrm{~m}^{3}} \times \pi\left(0.5 \times 10^{-3} \mathrm{~m}\right)^{2}=5.6 \times 10^{-3} \frac{\mathrm{~kg}}{\mathrm{~m}}
$$

$$
I=\sqrt{\frac{2 \pi m g S \tan \theta}{\mu_{0} l}}=\sqrt{\frac{2 \pi \times 9.8 \frac{\mathrm{~m}}{s^{2}} \times 5.6 \times 10^{-3} \frac{\mathrm{~kg}}{\mathrm{~m}} \times 0.16 \mathrm{~m} \times \tan 9}{4 \pi \times 10^{-7} \frac{\mathrm{Tm}}{\mathrm{~A}}}}=83.4 \mathrm{~A}
$$

b. Suppose that a beam of protons with a speed of $v=1.4 \times 10^{7} \frac{\mathrm{~m}}{\mathrm{~s}}$ were directed down the midpoint between the wires as shown in the figure. What force would the beam of protons feel?

The direction of the net magnetic field is directed in the negative y-direction and has magnitude

$$
\left|\vec{B}_{\text {net }}\right|=B_{L}+B_{R}=2 B_{L}=2\left(\frac{\mu_{0} I}{2 \pi r}\right)=\frac{2 \mu_{0} I}{\pi S}=\frac{2 \times 4 \pi \times 10^{-7} \frac{T_{m}}{A} \times 83.4 \mathrm{~A}}{\pi \times 0.16 \mathrm{~m}}=4.2 \times 10^{-4} \mathrm{~T}
$$

$|\vec{F}|=q v\left|\vec{B}_{\text {net }}\right|=1.6 \times 10^{-19} \mathrm{C} \times 1.4 \times 10^{7} \frac{\mathrm{~m}}{\mathrm{~s}} \times 4.2 \times 10^{-4} \mathrm{~T}=9.4 \times 10^{-16} \mathrm{~N}$
and the direction is toward the left wire by the right hand rule.
c. Suppose that the current in the wires could somehow be made to increase. In this case, which of the following quantities would change?

1. The angle $\theta$ would decrease.
2. The angle $\theta$ would increase.
(3.) The magnitude of the magnetic force on the protons would increase.
3. The net magnitude of the magnetic field would decrease.
(5.) The separation $S$ between the wires would increase.
d. Suppose that you take one of your wires from above and bend it into a square of length $l=0.3 m$ and $N=400$ turns. You take this coil of wire out into a field and orient it as shown below and wait for lightning to strike. Lightning eventually does strike at a distance of $R=200 \mathrm{~m}$ from the coil of wire and measurement indicate that the current flow in the lightning bolt is $I_{\text {bolt }}=6 \times 10^{6} \mathrm{~A}$ and falls to zero from this maximum value over a time interval of $\Delta t=10.5 \mu \mathrm{~s}$. What are the magnitude and direction of the induced current in the coil of zinc wire? Hint: Model the lightning bolt as a long straight wire and assume that the magnetic field is uniform across the loop of wire. (Hint: The resistivity of zinc is $5.92 \times 10^{-8} \Omega m$.)

$$
\begin{aligned}
& B_{\text {wire }}=\frac{\mu_{0} I}{2 \pi r}=\frac{4 \pi \times 10^{-7} \frac{T_{m}}{A} \times 6 \times 10^{6} A}{2 \pi \times 200 \mathrm{~m}}=6.0 \times 10^{-3} \mathrm{~T} \\
& I_{\text {induced }}=\left|\frac{\varepsilon}{R}\right|=\frac{N A}{R} \frac{\Delta B}{\Delta t}=\frac{400 \times(0.3 \mathrm{~m})^{2}}{36.2 \Omega}\left(\frac{6.0 \times 10^{-3} \mathrm{~T}}{10.5 \times 10^{-6} \mathrm{~s}}\right)=568.3 \mathrm{~A} \\
& R=\frac{\rho l}{A}=\frac{5.92 \times 10^{-8} \Omega \mathrm{~m} \times(400 \times 4 \times 0.3 \mathrm{~m})}{\pi\left(0.5 \times 10^{-3} \mathrm{~m}\right)^{2}}=36.2 \Omega
\end{aligned}
$$

The magnetic field is pointing out of the page and is decreasing with time. To oppose the decrease in magnetic flux a counter-clockwise current will flow.
2. A 10 mW laser pointer is incident on a block of glass 2.0 cm thick. The beam makes a spot that is a circle of diameter $D=2.6 \mathrm{~mm}$.
a. What are the intensity of the spot on the glass block and the maximum electric field amplitude of the incident light?

$$
\begin{aligned}
& S=\frac{\text { Energy }}{\text { time } \cdot \text { area }}=\frac{\text { power }}{\text { area }}=\frac{10 \times 10^{-3} \mathrm{~W}}{\pi\left(1.3 \times 10^{-3} \mathrm{~m}\right)^{2}}=1883.5 \frac{\mathrm{~W}}{\mathrm{~m}^{2}} \\
& S=\frac{1}{2} c \varepsilon_{0} E_{\max }^{2} \rightarrow E_{\max }=\sqrt{\frac{2 S}{c \varepsilon_{0}}}=\sqrt{\frac{2 \times 1883.5 \frac{\mathrm{~W}}{\mathrm{~m}^{2}}}{3 \times 10^{8} \frac{\mathrm{~m}}{\mathrm{~s}} \times 8.85 \times 10^{-12} \frac{\mathrm{C}^{2}}{\mathrm{Nm}^{2}}}}=1191.2 \frac{\mathrm{~N}}{\mathrm{C}}
\end{aligned}
$$

b. Suppose that the light from the laser pointer makes an angle of $\theta=22^{\circ}$ with respect to the normal to the glass' upper surface. What is the perpendicular distance ( $d^{\prime}$ as shown in the diagram below) the beam has been displaced from the direction it was originally aimed when it exits the glass block? The indices of air and glass are $n_{\text {air }}=1.00$ and $n_{\text {glass }}=1.50$ respectively.


$$
\begin{aligned}
& n_{\text {air }} \sin \theta=n_{g} \sin \theta_{2} \rightarrow \theta_{2}=\sin ^{-1}\left(\frac{n_{\text {air }}}{n_{g}} \sin \theta\right)=\sin ^{-1}\left(\frac{1.0}{1.5} \sin 22\right)=14.5^{0} \\
& \theta=\theta_{2}+\theta^{\prime} \rightarrow \theta^{\prime}=\theta-\theta_{2}=22^{0}-14.5^{0}=7.5^{0} \\
& \cos \theta_{2}=\frac{\text { thickness }}{L} \rightarrow L=\frac{\text { thickness }}{\cos \theta_{2}}=\frac{0.02 \mathrm{~m}}{\cos 14.5}=0.021 \mathrm{~m} \\
& \sin \theta^{\prime}=\frac{d^{\prime}}{L} \rightarrow d^{\prime}=L \sin \theta^{\prime}=0.021 \mathrm{~m} \times \sin 7.5=0.0027 \mathrm{~m}=2.7 \mathrm{~mm}
\end{aligned}
$$

c. Suppose instead that the beam from the laser pointer is incident on a polarizer. In one orientation of the polarizer one can see light coming through the polarizer, while in another orientation, no light is seen to come through the polarizer. From this information it can be concluded that
(1.) the laser light is polarized.
2. the laser light is unpolarized.
3. the laser light has a variable intensity.
4. the polarizer is thicker in one spot, so the light does not pass, and thinner in another spot so the light passes.
d. Suppose at the bottom of a 4 foot deep $(1.22 m)$ swimming pool there was a light. When viewing the pool from a deck located on the edge of the pool, how big of a circle would the light make on top of the pools surface? You may calculate either the radius or the diameter of the circle of light, but make sure that you clearly indicate which you are calculating. The indices of air and water are $n_{\text {air }}=1.00$ and $n_{\text {water }}=1.33$ respectively.

On the upper surface, the radius of the light circle will be defined by total internal reflection. At the critical angle the light will not propagate into the air from the water and this defines the radius of the circle.

$$
n_{w} \sin \theta_{c}=n_{\text {air }} \sin 90 \rightarrow \theta_{c}=\sin ^{-1}\left(\frac{n_{\text {air }}}{n_{w}}\right)=\sin ^{-1}\left(\frac{1.00}{1.33}\right)=48.8^{0}
$$

From the geometry we have

$$
\tan \theta_{c}=\frac{r}{\text { depth }} \rightarrow r=\text { depth } \times \tan \theta_{c}=1.22 m \times \tan 48.8=1.4 m
$$

3. Two lenses are used in combination to form an image of an object. A $h_{o}=2 \mathrm{~cm}$ tall object is placed 50 mm to the left of a diverging lens of unknown focal length $f_{D}$. A second converging lens with focal length $f_{c}$ is placed a distance $D=80 \mathrm{~mm}$ to the right of the diverging lens. In this combination, a real image is produced on a screen $d_{i c}=307.4 \mathrm{~mm}$ to the right of the converging lens.
a. Using information from the graph of $\frac{1}{d_{i}}$ versus $\frac{1}{d_{o}}$ below and the information given in the problem, determine the focal length of the diverging lens $f_{D}$.


The $y$-intercept is related to the focal length of the lens.

$$
b=\frac{1}{f_{c}} \rightarrow f_{c}=\frac{1}{b}=\frac{1}{0.0133 \mathrm{~mm}^{-1}}=75.2 \mathrm{~mm}
$$

From the focal length of the converting lens and the real image distance we can calculate distance the object was away from the converging lens. We have:
$\frac{1}{f_{c}}=\frac{1}{d_{02}}+\frac{1}{d_{i 2}} \rightarrow d_{02}=\left(\frac{1}{f_{c}}-\frac{1}{d_{i 2}}\right)^{-1}=\left(\frac{1}{75.2 m m}-\frac{1}{307.4 m m}\right)^{-1}=99.6 \mathrm{~mm}$. The
image distance from the diverging lens can now be calculated from:
$d_{02}=D+d_{i 1} \rightarrow d_{i 1}=d_{02}-D=99.6 \mathrm{~mm}-80 \mathrm{~mm}=19.6 \mathrm{~mm}$. The focal length of the diverging lens is thus

$$
\frac{1}{f_{D}}=\frac{1}{d_{o 1}}+\frac{1}{d_{i 1}} \rightarrow f_{D}=\left(\frac{1}{d_{o 1}}-\frac{1}{d_{i 2}}\right)^{-1}=\left(\frac{1}{50 \mathrm{~mm}}-\frac{1}{19.6 \mathrm{~mm}}\right)^{-1}=-32.2 \mathrm{~mm} .
$$

b. What is the size of the real image produced on the screen?

$$
h_{i f}=M_{T} h_{o}=\left(M_{1} M_{2}\right) h_{0}=\left(\frac{-d_{i 1}}{d_{o 1}}\right)\left(\frac{-d_{i 2}}{d_{o 2}}\right) h_{0}=\left(\frac{-(-19.6 \mathrm{~mm})}{50 \mathrm{~mm}}\right)\left(\frac{-307.4}{99.6 \mathrm{~mm}}\right) \times 2 \mathrm{~cm}=-2.4 \mathrm{~cm}
$$

The real image is 2.4 cm tall and is inverted with respect to the original object.
c. The version of the thin lens equation that we have been derived and have been using in class, in lab, and on the homework is called the Galilean form. Not to be outdone, Newton devised his own version to calculate image distances. Consider the diagram shown below, where the distance between the object and the focal point of the lens is $x$, while the distance between the image and the focal point of the lens is $x^{\prime}$. Show that the Newtonian form of the thin lens equation is given as $x x^{\prime}=f^{2}$ and determine what the magnification would look like in terms of $x$ and $x^{\prime}$.


$$
\begin{aligned}
& \tan \alpha=\frac{h_{0}}{x}=\frac{h_{i}}{f} \rightarrow \frac{h_{i}}{h_{0}}=\frac{f}{x} ; \quad \tan \beta=\frac{h_{0}}{f}=\frac{h_{i}}{x^{\prime}} \rightarrow \frac{h_{i}}{h_{0}}=\frac{x^{\prime}}{f} \\
& \therefore \frac{h_{i}}{h_{0}}=\frac{f}{x}=\frac{x^{\prime}}{f} \rightarrow x x^{\prime}=f^{2}
\end{aligned}
$$

Using the Galilean thin lens equation:

$$
\frac{1}{f}=\frac{1}{d_{0}}+\frac{1}{d_{i}}=\frac{1}{x+f}+\frac{1}{x^{\prime}+f}=\frac{x^{\prime}+f+x+f}{(x+f)\left(x^{\prime}+f\right)}=\frac{2 f+x+x^{\prime}}{x x^{\prime}+x f+x^{\prime} f+f^{2}}
$$

$$
2 f^{2}+x f+x^{\prime} f=x x^{\prime}+x f+x^{\prime} f+f^{2}
$$

$$
\therefore f^{2}=x x^{\prime}
$$

d. Suppose that you have a lens made out of glass with an index of refraction $n_{g}$. A second lens is constructed identical in shape to the first lens, except that this lens has an index of refraction $0.75 n_{g}$. For the second lens the focal length of this lens compared to the focal length of the first lens would
1.) be greater.
2. be less.
3. be the same.
4. be unable to be determined from the information given.

## Physics 111 Equation Sheet

Electric Forces, Fields and Potentials

$$
\begin{aligned}
& \vec{F}=k \frac{Q_{1} Q_{2}}{r^{2}} \hat{r} \\
& \vec{E}=\frac{\vec{F}}{q} \\
& \vec{E}_{Q}=k \frac{Q}{r^{2}} \hat{r} \\
& P E=k \frac{Q_{1} Q_{2}}{r} \\
& V(r)=k \frac{Q}{r} \\
& E_{x}=-\frac{\Delta V}{\Delta x} \\
& W=-q \Delta V
\end{aligned}
$$

Magnetic Forces and Fields
$F=q v B \sin \theta$
$F=I l B \sin \theta$
$\tau=N I A B \sin \theta=\mu B \sin \theta$
$P E=-\mu B \cos \theta$
$B=\frac{\mu_{0} I}{2 \pi r}$
$\varepsilon_{\text {indiced }}=-N \frac{\Delta \phi_{B}}{\Lambda t}=-N \frac{\Delta(B A \cos \theta)}{\Lambda t}$

Constants
$g=9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$
$1 e=1.6 \times 10^{-19} \mathrm{C}$
$k=\frac{1}{4 \pi \varepsilon_{o}}=9 \times 10^{9} \frac{\mathrm{~N} n^{2}}{c^{2}}$
$\varepsilon_{o}=8.85 \times 10^{-12} \frac{\mathrm{c}^{2}}{\mathrm{Nm}^{2}}$
$1 \mathrm{eV}=1.6 \times 10^{-19} \mathrm{~J}$
$\mu_{o}=4 \pi \times 10^{-7} \frac{\mathrm{Tm}}{A}$
$c=3 \times 10^{8} \frac{\mathrm{~m}}{\mathrm{~s}}$
$h=6.63 \times 10^{-34} \mathrm{Js}$
$m_{e}=9.11 \times 10^{-31} \mathrm{~kg}=\frac{0.511 \mathrm{MeV}}{c^{2}}$
$m_{p}=1.67 \times 10^{-27} \mathrm{~kg}=\frac{937.1 \mathrm{MeV}}{c^{2}}$
$m_{n}=1.69 \times 10^{-27} \mathrm{~kg}=\frac{948.3 \mathrm{MeV}}{c^{2}}$
$1 \mathrm{amu}=1.66 \times 10^{-27} \mathrm{~kg}=\frac{931.5 \mathrm{MeV}}{c^{2}}$
$N_{A}=6.02 \times 10^{23}$
$A x^{2}+B x+C=0 \rightarrow x=\frac{-B \pm \sqrt{B^{2}-4 A C}}{2 A}$

## Electric Circuits

$$
\begin{aligned}
& I=\frac{\Delta Q}{\Delta t} \\
& V=I R=I\left(\frac{\rho L}{A}\right) \\
& R_{\text {series }}=\sum_{i=1}^{N} R_{i} \\
& \frac{1}{R_{\text {parallel }}}=\sum_{i=1}^{N} \frac{1}{R_{i}} \\
& P=I V=I^{2} R=\frac{V^{2}}{R} \\
& Q=C V=\left(\frac{\kappa \varepsilon_{0} A}{d}\right) V=\left(\kappa C_{0}\right) V \\
& W=U=\frac{1}{2} Q V=\frac{1}{2} C V^{2}=\frac{Q^{2}}{2 C}
\end{aligned}
$$

$$
Q_{\text {charge }}(t)=Q_{\max }\left(1-e^{-\frac{t}{R C}}\right)
$$

$$
Q_{\text {discharge }}(t)=Q_{\max } e^{-\frac{t}{R C}}
$$

$$
C_{\text {parallel }}=\sum_{i=1}^{N} C_{i}
$$

$$
\frac{1}{C_{\text {series }}}=\sum_{i=1}^{N} \frac{1}{C_{i}}
$$

Light as a Particle \& Relativity

$$
E=h f=\frac{h c}{\lambda}=p c
$$

$$
K E_{\max }=h f-\phi=e V_{\text {stop }}
$$

$$
\Delta \lambda=\frac{h}{m_{e} c}(1-\cos \phi)
$$

$$
\gamma=\frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}}
$$

$$
p=\gamma m v
$$

$$
E_{\text {total }}=K E+E_{\text {rest }}=\gamma m c^{2}
$$

$$
E_{\text {total }}^{2}=p^{2} c^{2}+m^{2} c^{4}
$$

$$
E_{\text {rest }}=m c^{2}
$$

$$
K E=(\gamma-1) m c^{2}
$$

Geometry
Circles: $C=2 \pi r=\pi D \quad A=\pi r^{2}$
Triangles: $\quad A=\frac{1}{2} b h$
Spheres: $A=4 \pi r^{2} \quad V=\frac{4}{3} \pi r^{3}$

Light as a Wave

$$
\begin{aligned}
& c=f \lambda=\frac{1}{\sqrt{\varepsilon_{o} \mu_{o}}} \\
& S(t)=\frac{\text { energy }}{\text { time } \times \text { area }}=c \varepsilon_{o} E^{2}(t)=c \frac{B^{2}(t)}{\mu_{0}} \\
& I=S_{\text {avg }}=\frac{1}{2} c \varepsilon_{o} E_{\max }^{2}=c \frac{B_{\max }^{2}}{2 \mu_{0}} \\
& P=\frac{\text { Force }}{\text { Area }}=\left\{\begin{array}{l}
\frac{2 S}{c} \text { total reflection } \\
\frac{S}{c} \text { total absorption }
\end{array}\right. \\
& S=S_{o} \cos ^{2} \theta \\
& v=\frac{1}{\sqrt{\varepsilon \mu}}=\frac{c}{n} \\
& \theta_{\text {inc }}=\theta_{\text {refl }} \\
& n_{1} \sin \theta_{1}=n_{2} \sin \theta_{2} \\
& \frac{1}{f}=\frac{1}{d_{o}}+\frac{1}{d_{i}} \\
& M=\frac{h_{i}}{h_{o}}=-\frac{d_{i}}{d_{o}} \\
& M_{\text {tooll }}=\prod_{i=1}^{N} M_{i}
\end{aligned}
$$

Nuclear Physics

$$
\begin{aligned}
& E_{\text {binding }}=\left(Z m_{p}+N m_{n}-m_{r ब t}\right) c^{2} \\
& \frac{\Delta N}{\Delta t}=-\lambda N_{o} \rightarrow N(t)=N_{o} e^{-\lambda t} \\
& A(t)=A_{o} e^{-\lambda t} \\
& m(t)=m_{o} e^{-\lambda t} \\
& t_{\frac{1}{2}}=\frac{\ln 2}{\lambda}
\end{aligned}
$$

Misc. Physics 110
Formulae
$\vec{F}=\frac{\Delta \vec{p}}{\Delta t}=\frac{\Delta(m v)}{\Delta t}=m \vec{a}$
$\vec{F}=-k \vec{y}$
$\vec{F}_{C}=m \frac{v^{2}}{R} \hat{r}$
$W=\Delta K E=\frac{1}{2} m\left(v_{f}^{2}-v_{i}^{2}\right)=-\Delta P E$
$P E_{g r a v i t y}=m g y$
$P E_{\text {spring }}=\frac{1}{2} k y^{2}$
$x_{f}=x_{i}+v_{i x} t+\frac{1}{2} a_{x} t^{2}$
$v_{f x}=v_{i x}+a_{x} t$
$v_{f x}^{2}=v_{i x}^{2}+2 a_{x} \Delta x$

