# Physics 111 

## Exam \#2

February 23, 2018

Name $\qquad$

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}|=(5 \mathrm{~kg}) \times\left(2 \frac{\mathrm{~m}}{\mathrm{~s}}\right)=10 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 3 points and each free-response part is worth 9 points

| Problem \#1 | $/ 24$ |
| :---: | :---: |
| Problem \#2 | $/ 24$ |
| Problem \#3 | $/ 24$ |
| Total | $/ 72$ |

I affirm that I have carried out my academic endeavors with full academic honesty.

1. A mass spectrometer is an analytical instrument used to identify the various molecules in a sample by measuring their charge-to-mass ratio $q / m$. The sample is ionized and the positive ions are accelerated through a potential difference $\Delta V$, and then enter a region of uniform magnetic field. The magnetic filed bends the ions in to circular trajectories, but after just half a circle they either strike the wall or pass through a small opening to a detector. As the accelerating voltage is slowly increased, different ions reach the detector and are measured. Consider the mass spectrometer shown below with a magnetic field $B=200 \mathrm{mT}$ and a $d=8.00 \mathrm{~cm}$ spacing
 between the entrance and exit holes.
a. What accelerating potential difference is required to detect $\mathrm{CO}^{+}$ions? Some atomic masses are shown in the table on the right.

| Element | Atomic mass <br> $(\mathrm{amu})$ |
| :---: | :---: |
| C | 12.000 |
| N | 14.003 |
| O | 15.995 |

$$
\begin{aligned}
& F_{b}=q v_{\perp} B=\frac{m v_{\perp}^{2}}{R} \rightarrow v_{\perp}=\frac{q R B}{m}=\frac{e d B}{2 m} \\
& W=-q \Delta V=-e \Delta V=\frac{1}{2} m v_{f}^{2} \rightarrow \Delta V=-\frac{m}{2 e} v_{f}^{2}=-\frac{m}{2 e}\left(\frac{e d B}{2 m}\right)^{2}=-\frac{e d^{2} B^{2}}{8 m} \\
& \therefore \Delta V=-\frac{e d^{2} B^{2}}{8 m}=-\frac{1.6 \times 10^{-19} C \times(0.08 m)^{2} \times\left(200 \times 10^{-3} T\right)^{2}}{8\left(27.995 u \times \frac{1.66 \times 10^{-27} \mathrm{~kg}}{1 u}\right)}=-110.2 \mathrm{~V}
\end{aligned}
$$

b. Suppose that you wanted to see $N_{2}^{+}$ions rather than the $\mathrm{CO}^{+}$ions. $N_{2}^{+}$has nominally the same mass as a $\mathrm{CO}^{+}$ion but because of small but measurable differences in the accelerating voltages the two ion species are easily separable.
To see the $N_{2}^{+}$ions the acceleration voltage needed would
(1.) need to decrease from the value used for the $\mathrm{CO}^{+}$ions.
2. remain unchanged from the value used for the $\mathrm{CO}^{+}$ions.
3. need to increase from the value used for the $\mathrm{CO}^{+}$ions.
4. be unable to be determined from the information given in the problem.
c. Suppose that instead of the mass spectrometer setup you had a uniform magnetic field that points along the z -axis with a value of $|\vec{B}|=30 \mathrm{mT}$. An electron enters the magnetic filed with a speed $|\vec{v}|=5 \times 10^{6} \frac{\mathrm{~m}}{\mathrm{~s}}$ at an angle $30^{\circ}$ above the $x-y$ plane as shown in the figure on the right. What is the radius of the circular orbit about the magnetic helix, called the pitch, $p$ ?


$$
\begin{aligned}
& F_{b}=q v_{\perp} B=\frac{m v_{\perp}^{2}}{R} \rightarrow R=\frac{m v_{\perp}}{e B}=\frac{9.11 \times 10^{-31} \mathrm{~kg} \times 5 \times 10^{6} \frac{\mathrm{~m}}{s} \sin 60}{1.6 \times 10^{-19} \mathrm{C} \times 30 \times 10^{-3} \mathrm{~T}}=8.22 \times 10^{-4} \mathrm{~m} \\
& v_{\|}=\frac{p}{T} \rightarrow p=v_{\|} T=v_{\|}\left(\frac{2 \pi R}{v_{\perp}}\right)=\left(\frac{v \cos \phi}{v \sin \phi}\right) 2 \pi R=\frac{2 \pi R}{\tan \phi}=\frac{2 \pi \times 8.22 \times 10^{-4} \mathrm{~m}}{\tan 60}=2.98 \times 10^{-3} \mathrm{~m}
\end{aligned}
$$

d. Suppose instead of electrons in an external field you had the following setup in which a wire of constant linear mass density $\lambda=\frac{\text { mass }}{\text { length }}$ is held between the poles of a magnet in the plane of the page. The magnets are both circular with radius $r$. If an ideal battery provided the current in the circuit, what is the initial acceleration of the wire?

1. $|\vec{a}|=\frac{R \lambda}{V B}$ into the page.
2. $|\vec{a}|=\frac{R \lambda}{V B}$ out of the page.
(3.) $|\vec{a}|=\frac{V B}{R \lambda}$ into the page.

3. $|\vec{a}|=\frac{V B}{R \lambda}$ out of the page.
4. Of magnetic fields, airplanes and helicopters
a. Suppose you had the two circuits shown on the right, where circuit \#1 is on the left and circuit \#2 is on the right. A distance $d$ separates the right and left sides of circuits \#1 and \#2. At the midpoint between the two circuits, which of the following gives the net magnitude of the magnetic field? Assume that up the page, out of the page and to the right are the positive directions.

5. $\left|\vec{B}_{n e t}\right|=0$.
6. $\left|\vec{B}_{n e t}\right|=\frac{\mu_{o}}{\pi d}\left(\frac{V_{1}}{R_{1}}+\frac{V_{2}}{R_{2}}\right)$.
(3.) $\left|\vec{B}_{\text {net }}\right|=\frac{\mu_{o}}{\pi d}\left(\frac{V_{1}}{R_{1}}-\frac{V_{2}}{R_{2}}\right)$.
7. $\left|\vec{B}_{n e t}\right|=\frac{\mu_{o}}{\pi d}\left(\frac{V_{2}}{R_{2}}-\frac{V_{1}}{R_{1}}\right)$.
b. Suppose you had two circular coils each of radius $r$ separated by a distance $d$. They are held in place so that their faces are parallel to each other. Their centers lay on a line that is perpendicular to both faces. If each has a current $I$ flowing in the directions shown, the magnetic force on the upper loop due to the lower loop tries to make
8. the upper loop smaller and pull it downward.
9. the upper loop smaller and push it upward.
10. the upper loop larger and pull it downward.
11. the upper loop larger and push it upward.

c. The Earth's magnetic field has components that point north parallel to the ground ( $B_{\|}=11.1 \mu T$ ) and vertically down into the ground ( $B_{\perp}=51.6 \mu T$ ). A Boeing 737 airplane has a wingspan $40 \mathrm{~m} \sim 120$ feet and is flying parallel to the ground at a speed $v=235.5 \frac{\mathrm{~m}}{\mathrm{~s}} \sim 500 \frac{\mathrm{mi}}{\mathrm{hr}}$. As the plane flies north, what potential difference is induced across the wingtips and which wingtip becomes negatively charged?
$\varepsilon=B l v=51.6 \times 10^{-6} T \times 40 \mathrm{~m} \times 235.5 \frac{\mathrm{~m}}{\mathrm{~s}}=0.49 \mathrm{~V}$ and as the plane flies north, the magnetic force on the charges pushes the electrons to the right. So the right wingtip becomes negatively charged.
d. Suppose that you have a metal bar of length $L=3 m$ rotating about one end with angular velocity $\omega=\frac{\Delta \theta}{\Delta t}=47 \frac{\mathrm{radians}}{\mathrm{s}}$ through a magnetic field $B_{\perp}=51.6 \mu \mathrm{~T}$ perpendicular to the face of the bar pointing into the ground. What potential difference is induced across the bar? Hint: As the bar rotates it sweeps out an angle $\Delta \theta$. Out of $2 \pi$ radians, what fraction of the area of the circle is swept out in a time $\Delta t$ by the bar?

12. The study of light is fundamental to the workings of many devices, such as lenses.
a. Suppose that light strikes a plane boundary between two materials with different refractive indices. Which of the following cannot occur?
13. There is a reflected beam, but no transmitted beam.
(2.) There is a transmitted beam, but not reflected beam.
14. There are both transmitted and reflected beams.
15. The speed of light increases when the light enters the second material from the first.
16. The speed of light decreases when the light enters the second material from the first.
b. Consider the 2 cm thick slab of material below that has an unknown index of refraction. Light from a 0.5 mW laser pointer is incident at angle $\theta_{1}=45^{\circ}$ with respect to the normal to the side surface. The light propagates through the material and exits at an angle $\theta_{2}=76^{\circ}$ with respect to the normal to the bottom surface. What is the index of refraction of the material? Hint: $\sin (90-\phi)=\cos \phi$.


Left surface: $n_{1} \sin \theta_{1}=n_{m} \sin \alpha$
Bottom surface: $n_{m} \sin \beta=n_{m} \sin (90-\alpha)=n_{m} \cos \alpha=n_{1} \sin \theta_{2}$.
Dividing the two expressions:

$$
\begin{aligned}
& \frac{n_{m} \sin \alpha}{n_{m} \cos \alpha}=\frac{n_{1} \sin \theta_{1}}{n_{1} \sin \theta_{2}} \rightarrow \tan \alpha=\frac{\sin \theta_{1}}{\sin \theta_{2}}=\frac{\sin 45}{\sin 76} \rightarrow \alpha=36.1^{0} \\
& \therefore n_{m} \sin \alpha=n_{1} \sin \theta_{1} \rightarrow n_{m}=\frac{n_{1} \sin \theta_{1}}{\sin \alpha}=\frac{\sin 45}{\sin 36.1}=1.2
\end{aligned}
$$

c. Suppose that you have an arrangement of lenses placed such that the lens on the left has a focal length of 127 mm while 252 mm to the right of this lens a second lens with focal length 48 mm is placed. This combination of lenses is used to image the motion of an object moving at $0.27 \frac{\mathrm{~mm}}{\mathrm{~s}}$. If the image of the object is projected onto a screen located 1.27 m to the right of the second lends, what would be the velocity in $\frac{m m}{s}$ of the object on the screen?

not to scale
Object distance for lens 2:
$\frac{1}{d_{02}}+\frac{1}{d_{i 2}}=\frac{1}{f_{2}} \rightarrow \frac{1}{d_{02}}=\frac{1}{f_{2}}-\frac{1}{d_{i 2}}=\frac{1}{48 \mathrm{~mm}}-\frac{1}{1270 \mathrm{~mm}} \rightarrow d_{02}=49.9 \mathrm{~mm}$
Image distance for lens 1 :
$d_{02}+d_{i 1}=D \rightarrow d_{i 1}=D-d_{02}=252 \mathrm{~mm}-49.9 \mathrm{~mm}-202.1 \mathrm{~mm}$
Object distance for lens 1 :
$\frac{1}{d_{01}}+\frac{1}{d_{i 1}}=\frac{1}{f_{1}} \rightarrow \frac{1}{d_{01}}=\frac{1}{f_{1}}-\frac{1}{d_{i 1}}=\frac{1}{127 \mathrm{~mm}}-\frac{1}{202.1 \mathrm{~mm}} \rightarrow d_{01}=341.2 \mathrm{~mm}$
The speed is given by the magnification:

$$
v_{\text {screen }}=M_{1} M_{2} v_{0}=\left(-\frac{d_{i 1}}{d_{01}}\right)\left(-\frac{d_{i 2}}{d_{02}}\right) v_{0}=\left(-\frac{202.1 \mathrm{~mm}}{341.8 \mathrm{~mm}}\right)\left(-\frac{1270 \mathrm{~mm}}{49.9 \mathrm{~mm}}\right) 0.27 \frac{\mathrm{~mm}}{\mathrm{~s}}=4.1 \frac{\mathrm{~mm}}{\mathrm{~s}}
$$

d. Suppose that you have a converging lens and the arrangement of colored objects shown below. Which of the following are true? There may be more than one correct answer.


1. Only a real image of the object will be produced.
2. Only a virtual image of the object will be produced.
3. Both real and virtual images will be produced.
4. Whether real or virtual images are produced, the magnification will be greater than unity.
5. Whether real or virtual images are produced, the magnification will be less than unity.
6. There is not enough information on the size of or locations of the objects to answer this question.

## Physics 111 Equation Sheet

Electric Forces, Fields and Potentials

$$
\begin{aligned}
& \vec{F}=k \frac{Q_{1} Q_{2}}{r^{2}} \hat{r} \\
& \vec{E}=\frac{\vec{F}}{q} \\
& \vec{E}_{Q}=k \frac{Q}{r^{2}} \hat{r} \\
& P E=k \frac{Q_{1} Q_{2}}{r} \\
& V(r)=k \frac{Q}{r} \\
& E_{x}=-\frac{\Delta V}{\Delta x} \\
& W=-q \Delta V=-q\left[V_{f}-V_{i}\right]
\end{aligned}
$$

Magnetic Forces and Fields
$F=q v B \sin \theta$
$F=I l B \sin \theta$
$B=\frac{\mu_{0} I}{2 \pi r}$
$\varepsilon_{\text {induced }}=\left\{\begin{array}{c}-N \frac{\Delta \phi_{B}}{\Delta t}=-N \frac{\Delta(B A \cos \theta)}{\Delta t} \\ B l v\end{array}\right.$

## Constants

$g=9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$
$1 e=1.6 \times 10^{-19} \mathrm{C}$
$k=\frac{1}{4 \pi \varepsilon_{o}}=9 \times 10^{9} \frac{\mathrm{~N} n^{2}}{c^{2}}$
$\varepsilon_{o}=8.85 \times 10^{-12} \frac{\mathrm{C}^{2}}{\mathrm{Nm}{ }^{2}}$
$1 \mathrm{eV}=1.6 \times 10^{-19} \mathrm{~J}$
$\mu_{o}=4 \pi \times 10^{-7} \frac{7 m}{A}$
$c=3 \times 10^{8} \frac{\mathrm{~m}}{\mathrm{~s}}$
$h=6.63 \times 10^{-34} \mathrm{Js}$
$m_{e}=9.11 \times 10^{-31} \mathrm{~kg}=\frac{0.511 \mathrm{MeV}}{c^{2}}$
$m_{p}=1.67 \times 10^{-27} \mathrm{~kg}=\frac{937.1 \mathrm{MeV}}{c^{2}}$
$m_{n}=1.69 \times 10^{-27} \mathrm{~kg}=\frac{948.3 \mathrm{MeV}}{c^{2}}$
$1 \mathrm{amu}=1.66 \times 10^{-27} \mathrm{~kg}=\frac{931.5 \mathrm{MeV}}{c^{2}}$
$N_{A}=6.02 \times 10^{23}$
$A x^{2}+B x+C=0 \rightarrow x=\frac{-B \pm \sqrt{B^{2}-4 A C}}{2 A}$

$$
\begin{aligned}
& I=\frac{\Delta Q}{\Delta t}=n e A v_{d} ; n=\frac{\rho N_{A}}{M} \\
& V=I R=I\left(\frac{\rho L}{A}\right) \\
& R_{\text {series }}=\sum_{i=1}^{N} R_{i} \\
& \frac{1}{R_{\text {parallel }}}=\sum_{i=1}^{N} \frac{1}{R_{i}} \\
& P=I V=I^{2} R=\frac{V^{2}}{R} \\
& Q=C V=\left(\frac{\kappa \varepsilon_{0} A}{d}\right) V=\left(\kappa C_{0}\right) V \\
& P E=\frac{1}{2} Q V=\frac{1}{2} C V^{2}=\frac{Q^{2}}{2 C} \\
& Q_{\text {charge }}(t)=Q_{\max }\left(1-e^{-\frac{t}{R C}}\right) \\
& Q_{\text {discharge }}(t)=Q_{\max } e^{-\frac{t}{R C}} \\
& C_{\text {parallel }}=\sum_{i=1}^{N} C_{i} \\
& \frac{1}{C_{\text {series }}}=\sum_{i=1}^{N} \frac{1}{C_{i}}
\end{aligned}
$$

Light as a Particle \& Relativity

$$
\begin{aligned}
& E=h f=\frac{h c}{\lambda}=p c \\
& K E_{\max }=h f-\phi=e V_{\text {stop }} \\
& \Delta \lambda=\frac{h}{m_{e} c}(1-\cos \phi) \\
& \gamma=\frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}} \\
& p=\gamma m v \\
& E_{\text {total }}=K E+E_{\text {rest }}=\gamma m c^{2} \\
& E_{\text {total }}^{2}=p^{2} c^{2}+m^{2} c^{4} \\
& E_{\text {rest }}=m c^{2} \\
& \text { KE }=(\gamma-1) m c^{2} \\
& \text { Geometry } \\
& \text { Circles: } C=2 \pi r=\pi D \quad A=\pi r^{2} \\
& \text { Triangles: } A=\frac{1}{2} b h \\
& \text { Spheres: } A=4 \pi r^{2} \quad V=\frac{4}{3} \pi r^{3}
\end{aligned}
$$

Nuclear Physics

$$
\begin{aligned}
& E_{\text {binding }}=\left(Z m_{p}+N m_{n}-m_{r \varepsilon t}\right) c^{2} \\
& \frac{\Delta N}{\Delta t}=-\lambda N_{o} \rightarrow N(t)=N_{o} e^{-\lambda t} \\
& A(t)=A_{o} e^{-\lambda t} \\
& m(t)=m_{o} e^{-\lambda t} \\
& t_{\frac{1}{2}}=\frac{\ln 2}{\lambda}
\end{aligned}
$$

Misc. Physics 110
Formulae
$\vec{F}=\frac{\Delta \vec{p}}{\Delta t}=\frac{\Delta(m v)}{\Delta t}=m \vec{a}$
$\vec{F}=-k \vec{y}$
$\vec{F}_{C}=m \frac{v^{2}}{R} \hat{r}$
$W=\Delta K E=\frac{1}{2} m\left(v_{f}^{2}-v_{i}^{2}\right)=-\Delta P E$
$P E_{\text {gravity }}=m g y$
$P E_{\text {spring }}=\frac{1}{2} k y^{2}$
$x_{f}=x_{i}+v_{i x} t+\frac{1}{2} a_{x} t^{2}$
$v_{f x}=v_{i x}+a_{x} t$
$v_{f x}^{2}=v_{i x}^{2}+2 a_{x} \Delta x$

$$
\begin{aligned}
& c=f \lambda=\frac{1}{\sqrt{\varepsilon_{o} \mu_{o}}} \\
& S(t)=\frac{\text { energy }}{\text { time } \times \text { area }}=c \varepsilon_{o} E^{2}(t)=c \frac{B^{2}(t)}{\mu_{0}} \\
& I=S_{\text {avg }}=\frac{1}{2} c \varepsilon_{o} E_{\max }^{2}=c \frac{B_{\max }^{2}}{2 \mu_{0}} \\
& P=\frac{\text { Force }}{\text { Area }}=\left\{\begin{array}{l}
\frac{S}{c} \\
\frac{2 S}{c}
\end{array}\right. \\
& S=S_{o} \cos ^{2} \theta \\
& v=\frac{1}{\sqrt{\varepsilon \mu}}=\frac{c}{n} \\
& \theta_{\text {inc }}=\theta_{\text {ref }} \\
& n_{1} \sin \theta_{1}=n_{2} \sin \theta_{2} \\
& \frac{1}{f}=\frac{1}{d_{o}}+\frac{1}{d_{i}} \\
& M=\frac{h_{i}}{h_{o}}=-\frac{d_{i}}{d_{o}} \\
& M_{\text {total }}=\prod_{i=1}^{N} M_{i}
\end{aligned}
$$

Light as a Wave

