## Physics 111

## Exam #2

# March 5, 2021

Name\_\_\_\_\_

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example,  $|\vec{p}| \approx m |\vec{v}| = (5kg) \times (2\frac{m}{s}) = 10\frac{kg \cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple-choice questions are worth 3 points and each free-response part is worth 7 points.

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

1. Motion of a charge in a magnetic field

An electron is accelerated, from rest, through a potential difference of 250V, at which point it enters a set of parallel horizontal metal plates that contain crossed electric and magnetic fields. The magnetic field points into the plane of the paper and has magnitude  $|\vec{B}| = B = 0.7611T$  as shown below.

- a. Which of the following below gives the direction of the electric field between the horizontal metal plates so that the electron goes undeflected ( $\vec{F}_{net} = 0$ ) between the plates?
  - 1. The electric field points to the right.
  - 2. The electric field points to the left.
  - 3. The electric field points up the plane of the paper.
  - 4.) The electric field points down the plane of the paper.
- b. What is the direction of the electron's motion through the field? That is, which way does the electron initially move when it leaves the horizontal set of capacitor plates and what is the diameter of the electron's motion through the field? Assume that the velocity is perpendicular to the magnetic field.

$$W = -q\Delta V = -(-e)V = 250eV$$

$$W = \Delta K = \frac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2W}{m}}$$

$$v = \sqrt{\frac{2 \times 250eV \times \frac{1.6 \times 10^{-19}J}{1eV}}{9.11 \times 10^{-31}kg}}$$

$$X = X \times X \times X$$

$$X \times X \times X \times X$$

$$v = 9.4 \times 10^6 \frac{m}{s}$$

 $F = qvB = ma = m\frac{v^2}{R} \rightarrow D = 2R = \frac{2mv}{qB} = \frac{2\times9.11\times10^{-31}kg\times9.4\times10^{6}\frac{m}{s}}{1.6\times10^{-19}C\times0.7611T} = 1.4\times10^{-4}m$ The direction of the magnetic force is down the plane of the page. c. Derive an expression for the orbital period of the election's motion about the magnetic field and show that the motion is independent of the orbital radius

$$v_{\perp} = \frac{2\pi R}{T} = \frac{qRB}{m} \rightarrow T = \frac{2\pi m}{qB}$$
 independent of the radius of the charged particle.

Where 
$$v_{\perp}$$
 is obtained from:  $F = qv_{\perp}B = m\frac{v_{\perp}^2}{R} \rightarrow v_{\perp} = \frac{qRB}{m}$ .

d. Suppose that the set of horizontal capacitor plates were tipped into the plane of the page by an angle of  $\theta = 12^{\circ}$ . What are the orbital radius and pitch of the elections in the magnetic field?

$$v_{\parallel} = v \cos \theta = \frac{l}{T} \to l = (v \cos \theta)T = (v \cos \theta)\frac{2\pi m}{qB}$$
$$l = 9.4 \times 10^{6} \frac{m}{s} (\cos 78) \frac{2\pi \times 9.11 \times 10^{-31} kg}{1.6 \times 10^{-19} C \times 0.7611T} = 9.2 \times 10^{-5} m$$

$$v_{\perp} = v \sin \theta = \frac{2\pi R}{T} \to R = \frac{(v \sin \theta)T}{2\pi} = \frac{(v \sin \theta)\frac{2\pi m}{qB}}{2\pi} = \frac{mv}{qB} \sin \theta$$
$$R = \frac{9.11 \times 10^{-31} kg \times 9.4 \times 10^{6} \frac{m}{s}}{1.6 \times 10^{-19} C \times 0.7611T} \sin 78 = 6.9 \times 10^{-5} m$$

## 2. Faraday's law

Suppose that a wire loop with dimensions shown in the figure on the right, was placed in the plane of the page below a beam of protons. The proton beam is directed to the right across the plane of the page at a rate of  $1.25 \times 10^{17} \frac{protons}{second}$ 



a. What are the net magnetic fields, due to the beam of protons, at the location of the *upper* wire of the loop and the *lower wire of the loop*?

$$B_{u} = \frac{\mu_{0} I_{u}}{2\pi r_{u}} = \frac{4\pi \times 10^{-7} \frac{Tm}{A} \times \left(1.25 \times 10^{17} \frac{protons}{s} \times \frac{1.6 \times 10^{-19} C}{proton}\right)}{2\pi \times 0.1m} = 4 \times 10^{-8} T \text{ into the page.}$$
$$B_{l} = \frac{\mu_{0} I_{l}}{2\pi r_{l}} = \frac{4\pi \times 10^{-7} \frac{Tm}{A} \times \left(1.25 \times 10^{17} \frac{protons}{s} \times \frac{1.6 \times 10^{-19} C}{proton}\right)}{2\pi \times 0.2m} = 2 \times 10^{-8} T \text{ into the page.}$$

b. What is the net magnetic force on the wire loop if a counterclockwise current of  $I = 12\mu A$  flows in the wire loop? Note, the current in the wire loop is due to a battery and resistor, which are not shown in the picture.

 $F_{B,u} = I_u L B_{u,l} = 12 \times 10^{-6} A \times 0.1 m \times 4 \times 10^{-8} T$  $F_{B,u} = 4.8 \times 10^{-14} N$  down the plane of the page.

 $F_{B,l} = I_l L B_{l,u} = 12 \times 10^{-6} A \times 0.1 m \times 2 \times 10^{-8} T$  $F_{B,u} = 2.4 \times 10^{-14} N$  up the plane of the page.

 $F_{net} = F_l - F_u = 4.8 \times 10^{-14} N - 2.4 \times 10^{-14} N = 2.4 \times 10^{-14} N$  down the plane of the page.

c. Suppose that the battery is removed from the wire loop and that the beam of protons is turned off over a time of  $3\mu s$ . What are the magnitude and direction of the induced current in the loop of wire due to the changing magnetic field if the resistance of the wire loop is  $1\mu\Omega$ ? Assume that the magnetic field at any point in time is uniform over the wire loop and is given as the value of the magnetic field at the midpoint of the loop.

$$B_{mp} = \frac{\mu_0 I_{protons}}{2\pi r_{mp}}$$
  

$$B_{mp} = \frac{4\pi \times 10^{-7} \frac{Tm}{A} \times \left(1.25 \times 10^{17} \frac{protons}{s} \times \frac{1.6 \times 10^{-19} C}{proton}\right)}{2\pi \times 0.15m} = 2.7 \times 10^{-8} T \text{ into the plane of the page.}$$

The induced emf: 
$$\varepsilon = |-N| \frac{\Delta \phi_B}{\Delta t} = NA \cos \phi \frac{\Delta B}{\Delta t}$$
.

$$\varepsilon = \left| -NA \frac{\Delta \phi_B}{\Delta t} \right| = \left| l^2 \cos \phi \frac{\Delta B}{\Delta t} \right| = \left| l^2 \frac{\Delta B}{\Delta t} \right|$$
$$\varepsilon = \left| (0.1m)^2 \left[ \frac{0 - 2.7 \times 10^{-8}T}{3 \times 10^{-6}s} \right] \right| = 8.9 \times 10^{-5} V$$

The current:  $I = \frac{\varepsilon}{R} = \frac{8.9 \times 10^{-5} V}{1 \times 10^{-6} \Omega} = 89A$ Since the magnetic flux is decreasing the induced current would be clockwise to oppose the decrease.

- d. Suppose that you had the following situation, in which the proton beam is directed along the normal to the loop of wire as shown below. The proton beam decreases over a time of  $3\mu s$  and after  $3\mu s$ , becomes zero. Which of the following gives the current induced in the wire loop?
  - 1. As the beam of protons disappears, a counterclockwise current in the wire.
  - 2. As the beam of protons disappears, a clockwise current in the wire.
  - 3.) As the beam of protons disappears, zero induced current in the wire.
  - 4. As the beam of protons disappears, a current is produced but the direction is unable to be determined.



0.1m

#### 3. Lenses

Consider the two lenses shown below in which the lens on the left  $(L_1)$  has a power of +2.5D, where the power is given as  $\frac{1}{f}$  in meters. The lens on the right  $(L_2)$  has an unknown focal length. An object is placed at a distance of 32cm to the left of  $L_1$ , and the lenses are separated by a distance of 1.5cm.

a. What is the location of the image of the object with respect to  $L_1$  and what is the focal length of the second lens,  $L_2$ , if the lens to screen distance is 2.5*cm* and a sharp image is produced on the screen?

$$\frac{1}{d_{01}} + \frac{1}{d_{11}} = \frac{1}{f_1} \rightarrow \frac{1}{d_{11}} = \frac{1}{f_1} - \frac{1}{d_{01}} = 2.5m^{-1} - \frac{1}{0.32m}$$
 object  

$$d_{i1} = -1.6m$$

$$d_{02} = D + d_{i1} = 0.015m + 1.6m = 1.615m$$

$$\frac{1}{d_{02}} + \frac{1}{d_{i2}} = \frac{1}{f_2} \rightarrow \frac{1}{f_2} = \frac{1}{1.615m} + \frac{1}{0.025m}$$

$$L_1$$

$$L_2$$
Screen  
Diagram not drawn to scale  

$$f_2 = 0.0246m = 2.46cm$$

b. What is the total magnification of the object on the screen? Is the final image on the screen upright or inverted with respect to the original object?

$$M_T = M_1 M_2 = \left(\frac{d_{i1}}{d_{01}}\right) \left(\frac{d_{i2}}{d_{02}}\right) = \left(\frac{1.6m}{0.32m}\right) \left(\frac{0.025m}{1.615m}\right) = 0.078$$

The final image is inverted with respect to the original object.

c. Suppose that we replace the screen with a piece of material with index of refraction  $n_m = 1.40$ . This piece of material is surrounded on all sides by air. At what angle of incidence  $\theta$  on the front surface of the material would be needed to guarantee total internal reflection of the light in the material? Is  $\theta$  a maximum or a minimum angle?

Upper surface:  $n_m \sin \theta_c = n_{air} \sin 90 = 1$  $\sin \theta_c = \frac{1.0}{1.4} \rightarrow \theta_c = 45.6^0$ 

$$\theta$$
  $n_m = 1.40$ 

Front surface:

 $n_{air}\sin\theta = n_m\sin\theta_m = n_m\sin(90 - \theta_c) \rightarrow \sin\theta = \frac{n_m}{n_{air}}\sin(90 - \theta_c)$  $\sin\theta = \frac{1.4}{1.0}\sin(90 - 45.6) = 0.98 \rightarrow \theta = 78.4^0$ 

This is a minimum angle of incidence that the light can have on the front surface. If this angle is greater than  $78.4^{\circ}$ , the light will be totally internally reflected.

- d. Suppose that the light from a green laser pointer is aimed onto the face of a transparent plastic cube ( $n_{cube} = 1.425$ ) as shown below. If the plastic cube were surrounded on all sides by oil ( $n_{oil} = 1.516$ ), which of the following best illustrates the direction of the beam after it emerged from the plastic cube?
  - 1. 1.
  - 2) 2.
  - 3. 3.
  - 4. 4.
  - 5. None of these gives the correct direction of the beam after it emerges from the glass cube.



# Physics 111 Equation Sheet

**Electric Forces, Fields and Potentials** 

$$\vec{F} = k \frac{Q_1 Q_2}{r^2} \hat{r}$$
$$\vec{E} = \frac{\vec{F}}{q}$$
$$\vec{E}_Q = k \frac{Q}{r^2} \hat{r}$$
$$PE = k \frac{Q_1 Q_2}{r}$$
$$V(r) = k \frac{Q}{r}$$
$$E_x = -\frac{\Delta V}{\Delta x}$$
$$W = -q \Delta V = -q \left[ V_f - V_i \right]$$

**Magnetic Forces and Fields**  $F = qvB\sin\theta$  $F = IlB\sin\theta$  $\tau = NIAB\sin\theta = \mu B\sin\theta$  $PE = -\mu B\cos\theta$  $B = \frac{\mu_0 I}{2\pi r}$  $\varepsilon_{induced} = -N \frac{\Delta \phi_B}{\Delta t} = -N \frac{\Delta (BA \cos \theta)}{\Delta t}$ 

**Constants** 

 $g = 9.8 \frac{m}{r^2}$  $1e = 1.6 \times 10^{-19}C$  $k = \frac{1}{4\rho e_{-}} = 9 \times 10^9 \frac{Nm^2}{C^2}$  $e_{0} = 8.85 \times 10^{-12} \frac{C^{2}}{V_{0}^{2}}$  $1eV = 1.6 \times 10^{-19} J$  $M_{o} = 4 p \times 10^{-7} \frac{Tm}{4}$  $c = 3 \times 10^8 \frac{m}{r}$  $h = 6.63 \times 10^{-34} Js$  $m_e = 9.11 \times 10^{-31} kg = \frac{0.511 MeV}{c^2}$  $m_p = 1.67 \times 10^{-27} kg = \frac{937.1 MeV}{c^2}$  $m_n = 1.69 \times 10^{-27} kg = \frac{948.3 MeV}{c^2}$  $1amu = 1.66 \times 10^{-27} kg = \frac{931.5 MeV}{c^2}$  $N_A = 6.02 \times 10^{23}$  $Ax^{2} + Bx + C = 0 \rightarrow x = \frac{-B \pm \sqrt{B^{2} - 4AC}}{2A}$ 

Lifetite circuits  

$$I = \frac{DQ}{Dt} = NeAv_d$$

$$C = f\lambda = \frac{1}{\sqrt{\varepsilon_o \mu_o}}$$

$$V = IR = I\left(\frac{rL}{A}\right)$$

$$S(t) = \frac{energy}{time \times area} = c\varepsilon_o E^2(t) = c\frac{B^2(t)}{\mu_0}$$

$$R_{series} = \sum_{i=1}^{N} R_i$$

$$I = S_{avg} = \frac{1}{2} c\varepsilon_o E_{max}^2 = c\frac{B_{max}^2}{2\mu_0}$$

$$\frac{1}{R_{parallel}} = \sum_{i=1}^{N} \frac{1}{R_i}$$

$$P = \frac{S}{c} = \frac{Force}{Area}$$

$$P = IV = I^2 R = \frac{V^2}{R}$$

$$S = S_o \cos^2 \theta$$

$$V = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{n}$$

$$\theta_{inc} = \theta_{rel}$$

$$W = U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$Q_{charge}(t) = Q_{max}\left(1 - e^{-\frac{t}{RC}}\right)$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$Q_{discharge}(t) = Q_{max}e^{-\frac{t}{RC}}$$

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$C_{parallel} = \sum_{i=1}^{N} \frac{1}{C_i}$$

$$M = m\lambda \text{ or } (m + \frac{1}{2})\lambda$$

$$a \sin \phi = m'\lambda$$

Light as a Particle & Relativity  $E = hf = \frac{hc}{\lambda} = pc$  $KE_{max} = hf - \phi = eV_{stop}$ 

$$\Delta \lambda = \frac{h}{m_e c} (1 - \cos \phi)$$
  

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$
  

$$p = \gamma m v$$
  

$$E_{total} = KE + E_{rest} = \gamma m c^2$$
  

$$E_{total}^2 = p^2 c^2 + m^2 c^4$$
  

$$E_{rest} = m c^2$$
  

$$KE = (\gamma - 1)mc^2$$

### Geometry

Circles:  $C = 2\rho r = \rho D$   $A = \rho r^2$ *Triangles*:  $A = \frac{1}{2}bh$ Spheres:  $A = 4\rho r^2$   $V = \frac{4}{3}\rho r^3$ 

**Nuclear Physics**  $E_{binding} = \left(Zm_p + Nm_n - m_{rest}\right)c^2$  $\frac{\Delta N}{\Delta t} = -\lambda N_o \rightarrow N(t) = N_o e^{-\lambda t}$  $A(t) = A_o e^{-\lambda t}$  $m(t) = m_0 e^{-\lambda t}$  $t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$ 

or  $\left(m + \frac{1}{2}\right)\lambda$ 

**Misc. Physics 110** Formulae

$$\vec{F} = \frac{\Delta \vec{p}}{\Delta t} = \frac{\Delta(mv)}{\Delta t} = m\vec{a}$$
  

$$\vec{F} = -k\vec{y}$$
  

$$\vec{F}_{c} = m\frac{v^{2}}{R}\hat{r}$$
  

$$W = \Delta KE = \frac{1}{2}m(v_{f}^{2} - v_{i}^{2}) = -\Delta PE$$
  

$$PE_{gravity} = mgy$$
  

$$PE_{spring} = \frac{1}{2}ky^{2}$$
  

$$x_{f} = x_{i} + v_{ix}t + \frac{1}{2}a_{x}t^{2}$$
  

$$v_{fx} = v_{ix} + a_{x}t$$
  

$$v_{fx}^{2} = v_{ix}^{2} + 2a_{x}\Delta x$$

Light as a Wave