Physics 111

Exam #2

February 25, 2022

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $\left|\vec{p}\right| \approx m\left|\vec{v}\right| = (5kg)\times\left(2\,\tfrac{m}{s}\right) = 10\,\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 6 points.

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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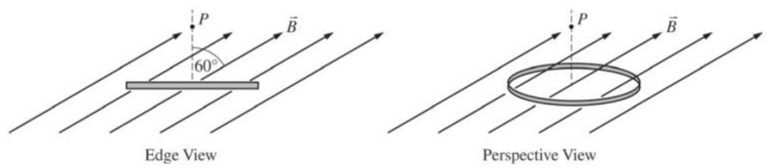
1. A circular loop of wire with radius $r = 0.1m$ and resistance $R = 10\Omega$ is held in place horizontally in a magnetic field $\vec{B}$ directed upward at an angle of 60^0 with respect to the vertical, as shown below. The magnetic field in varies in time according to $B(t) = a(1 - bt)$, where $a = 4T$ and $b = 0.1s^{-1}$for $0 \leq t \leq 10s$.



a. What is the magnitude and direction of the induced current in the wire loop at a time $t_f = 10s$?

The magnetic flux is decreasing with time, since $\vec{B}$ is decreasing with time. To undo the decrease a current flows CCW in the loop, by the right-hand rule.

$B(t) = a(1 - bt) = 4 - 0.4t$
$\Delta B = B_f - B_i = (4 - 0.4 \times 10) - (4 - 0.4 \times 0) = -4T$
$I = \left|\frac{\varepsilon}{R}\right| = \left|\frac{1}{R}\frac{\Delta\phi_B}{\Delta t}\right| = \left|\frac{A}{R}\frac{\Delta B}{\Delta t}\cos\theta\right| = \left|\frac{\pi r^2}{R}\frac{\Delta B}{\Delta t}\cos\theta\right| = \left|\frac{\pi(0.1m)^2}{10\Omega} \times \frac{-4T}{10s}\cos 60\right|$
$I = 6.2 \times 10^{-4}A = 0.62mA$

b. What is the energy dissipated in the loop after a time $t_f = 10s$?

$P = I^2R = \frac{\Delta E}{\Delta t} \rightarrow \Delta E = P\Delta t = I^2R\Delta t = (6.2 \times 10^{-4}A)^2 \times 10\Omega \times 10s$
$\Delta E = 3.9 \times 10^{-5}J$

c. What is the magnitude and direction of the electric field induced in the loop at $t_f = 10s$?

$$E = \left| -\frac{\Delta V}{\Delta x} \right| = \frac{IR}{2\pi r} = \frac{6.3 \times 10^{-4} A \times 10\Omega}{2\pi \times 0.1m} = 1 \times 10^{-2} \frac{V}{m} = 0.01 \frac{V}{m}$$

In the direction of the current flow CCW around the loop.

d. At at time $t_f = 10s$ what are the expressions for the magnitudes of the forces and the directions of forces on the right side of the loop, the left side of the loop, and the net force on the loop? Consider a small length $L = 0.2cm$ on each side of the wire in your calculations.

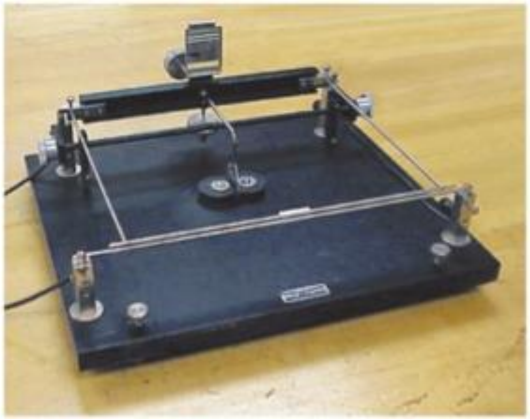
$$B(t) = 4 - 0.4t \rightarrow B(10s) = 4T - 0.4\frac{T}{s} \times 10s = 0T$$

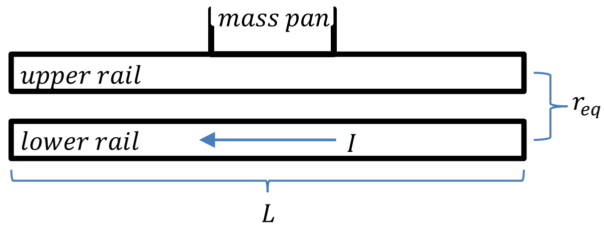
On the right side of the loop:
$F_R = ILB = 6.3 \times 10^{-4} A \times 0.2 \times 10^{-2} m \times 0T = 0N$ in magnitude.

On the left side of the loop:
$F_L = ILB = 1.3 \times 10^{-3} A \times 0.2 \times 10^{-2} m \times 0T = 0N$ in magnitude.

Thus, the net force on the loop is zero.

2. A current balance is a device for investigating magnetic force on wires and is also used to measure the permeability of free space, μ_0. In this system, the lower rail is fixed and cannot move, while the upper rail (with attached mass pan) can move. Current I is flowed through the system and the upper rail moves up away from the lower rail. The experimenter must add mass to the mass pan to return the rails to their original, equilibrium, separation, r_{eq}.





a. What is the direction of the current flow in the upper rail so that the upper rail will move up away from the lower rail? Explain your answer fully to earn full credit.

The upper rail is in a magnetic field created by the current flowing in the lower rail. This magnetic field points into the page and the upper wire will feel an upwards magnetic force if the current, by the right-hand rule, is flowing to the right.

b. Suppose that $I = 12A$ current is turned on and flows through the rails. If the centers of the rails are separated by a distance of $r = 1cm$, what is the net magnetic field at the midpoint between the rails? Take into the page as the positive direction for the magnetic field.

$$\vec{B}_{net} = \vec{B}_L + \vec{B}_u \rightarrow B_{net} = +B_L + B_u = +\frac{\mu_0 I}{2\pi r_{mp}} + \frac{\mu_0 I}{2\pi r_{mp}} = \frac{\mu_0 I}{\pi r_{mp}}$$

$$B_{net} = \frac{4\pi \times 10^{-7}\frac{Tm}{A} \times 12A}{\pi(0.05m)} = 9.6 \times 10^{-5} T \text{ into the page.}$$

c. Suppose that same $I = 12A$ current is on and flows through the rails. How much mass would have to be added to the mass pan to return the system to an equilibrium separation of $r_{eq} = 0.5mm$? Assume that the upper/lower rails have a radius of $r_{rail} = 0.25mm$, a length $L = 30cm$, and are made of aluminum $\rho_{Al} = 2700\frac{kg}{m^3}$.

$$\rho_{Al} = \frac{m_{rail}}{V} \rightarrow m_{rail} = \rho_{Al}V = \rho_{Al}\pi r_{rail}^2 L = 2700\frac{kg}{m^3} \times \pi\left(0.25 \times 10^{-3}m\right)^2 \times 0.3m$$
$$m_{rail} = 0.00016kg = 1.6 \times 10^{-4}kg$$

$$F_{net,y} = F_B - F_{W_{rail}} - F_{W_{added}} = ma_y = 0$$
$$F_{W_{added}} = F_B - F_{W_{rail}} \rightarrow m_{added}\,g = ILB - m_{rail}g \rightarrow m_{added} = \frac{ILB}{g} - m_{rail}$$
$$m_{added} = \frac{IL}{g}\left(\frac{\mu_0 I}{2\pi r_{eq}}\right) - m_{rail} = \frac{\mu_0 I^2 L}{2\pi r_{eq} g} - m_{raiil}$$
$$m_{added} = \frac{4\pi\times10^{-7}\frac{Tm}{A}(12A)^2\times0.3m}{2\pi\times0.5\times10^{-3}m\times9.8\frac{m}{s^2}} - 1.6 \times 10^{-4}kg = 1.6 \times 10^{-3}kg = 1.6g$$

d. This experiment is designed to determine a value for the permeability of free space, μ_0. To do this data are taken on the mass added to the pan as a function of the current in the rails. From these data, a plot is made of the *mass added to the pan versus the square of the current* flowing in the rails. What would be the slope and y-intercept of the plot and explain how you would use this plot to extract a value for the permeability of free space.

From part c:

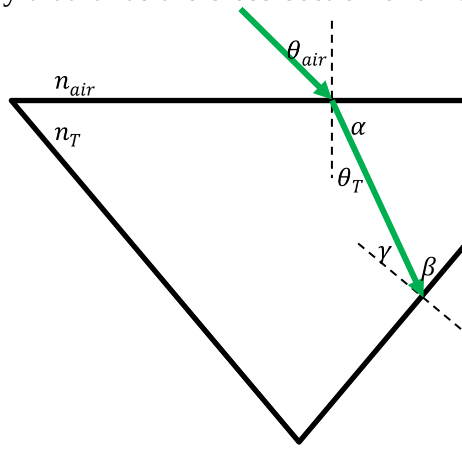
$m_{added} = \frac{\mu_0 I^2 L}{2\pi r_{eq} g} - m_{rail}$ which is a straight line.

The intercept is the mass of the rail, m_{rail}.

The slope can be used to determine the permeability of free space, μ_0.

$$slope = \frac{\mu_0 L}{2\pi r_{eq} g} \rightarrow \mu_0 = \frac{2\pi r_{eq} g \times slope}{L}$$

3. Topaz is a gemstone with an index of refraction $n_T = 1.6$. Suppose a topaz gem were cut in such a way that it has the cross-section of an equilateral triangle shown below.



a. What is the critical angle for the topaz/air interface on the right and what is the speed of light in topaz?

$$n_T \sin\theta_c = n_{air} \sin 90 = 1 \rightarrow \theta_c = \sin^{-1}\left(\frac{1}{n_T}\right) = \sin^{-1}\left(\frac{1}{1.6}\right) = 38.7^0$$

$$v_T = \frac{c}{n_T} = \frac{3\times10^8 \frac{m}{s}}{1.6} = 1.88 \times 10^8 \frac{m}{s}$$

b. If light is incident in air onto the upper surface of the topaz at an angle $\theta_{air} = 25^0$, will the light be internally reflected from in the gem? If it will not be internally reflected, at what angle will it re-enter the air?

$$n_{air} \sin\theta_{air} = n_T \sin\theta_T \rightarrow \sin\theta_T = \frac{n_{air}}{n_T} \sin\theta_{air} = \frac{1.0}{1.6} \sin 25 = 0.2641$$
$$\theta_T = 15.3^0$$

From the geometry:
$90^0 = \theta_T + \alpha \rightarrow \alpha = 90^0 - 15.3^0 = 74.7^0$
$\alpha + \beta + 60^0 = 180^0 \rightarrow \beta = 120^0 - \alpha = 120^0 - 74.7^0 = 45.3^0$
$90^0 = \beta + \gamma \rightarrow \gamma = 90^0 - \beta = 90^0 - 45.3^0 = 44.7^0$

Since $\gamma = 44.7^0$is greater than the critical angle for topaz/air $\theta_c = 38.7^0$, the light will be internally reflected. In fact, for most angles of incidence on the upper air/topaz surface, the light is internally reflected in the gem. This light emerges out of the upper surface of the gem toward the viewer and is what gives the gem its sparkle.

c. Albeit expensive, suppose that topaz was used to make two thin lenses. Lens #1 has an unknown focal length f_1 and is placed to the left of lens #2 (with focal length $f_2 = 48mm$) by an amount $D = 61mm$. A $2cm$ tall object is placed $36mm$ to the left of lens #1 and a real image is seen on a screen $125mm$ to the right of lens #2. What type of lens is lens #1 and what is its focal length, f_1?

For lens #2:

$$\frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f_2} \to \frac{1}{d_{o2}} = \frac{1}{f_2} - \frac{1}{d_{i2}} \to d_{02} = \left(\frac{1}{48mm} - \frac{1}{125mm}\right)^{-1} = 77.9mm$$

$$d_{o2} = D + d_{i1} \to d_{i1} = d_{o2} - D = 77.9mm - 61mm = 16.9mm$$

For lens #1:

$$\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{f_1} \to \frac{1}{f_1} = \frac{1}{d_{o1}} - \frac{1}{d_{i1}} \to f_1 = \left(\frac{1}{36mm} - \frac{1}{16.9mm}\right)^{-1} = -31.9mm$$

Since f_1 is negative, lens #1 is a diverging lens.

d. What is the size of the image on the screen?

$$\frac{h_{if}}{h_o} = M_1 M_2 \to h_{if} = M_1 M_2 h_o = \left|-\frac{d_{i1}}{d_{o1}}\right|\left|-\frac{d_{i2}}{d_{o2}}\right| h_o$$

$$h_{if} = \left(\frac{16.9mm}{36mm}\right)\left(\frac{125mm}{77.9mm}\right) \times 2cm = 1.5cm$$

Physics 111 Formula Sheet

Electrostatics

$F = k\frac{q_1 q_2}{r^2}$

$\vec{F} = q\vec{E};\quad E_{pc} = k\frac{q}{r^2};\quad E_{plate} = \frac{q}{\epsilon_0 A}$

$E = -\frac{\Delta V}{\Delta x}$

$V = k\frac{q}{r}$

$U_e = k\frac{q_1 q_2}{r} = qV$

$W = -q\Delta V = -\Delta U_e = \Delta K$

Magnetism

$\vec{F} = q\vec{v} \times \vec{B} \rightarrow F = qvB\sin\theta$

$\vec{F} = I\vec{L} \times \vec{B} \rightarrow F = ILB\sin\theta$

$B = \frac{\mu_0 I}{2\pi r}$

$\varepsilon = \Delta V = -N\frac{\Delta\phi_B}{\Delta t}$

$\phi_B = BA\cos\theta$

Electric Circuits - Capacitors

$Q = CV;\quad C = \frac{\kappa\epsilon_0 A}{d}$

$C_{parallel} = \sum_{i=1}^{N} C_i$

$\frac{1}{C_{series}} = \sum_{i=1}^{N}\frac{1}{C_i}$

$Q_{charging}(t) = Q_{max}\left(1 - e^{-\frac{t}{\tau}}\right)$

$Q_{discharging}(t) = Q_{max}e^{-\frac{t}{\tau}}$

$I(t) = I_{max}e^{-\frac{t}{\tau}} = \frac{Q_{max}}{\tau}e^{-\frac{t}{\tau}}$

$\tau = RC$

$U_C = \frac{1}{2}qV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$

Electric Circuits - Resistors

$I = \frac{\Delta Q}{\Delta t}$

$I = neAv_d;\quad n = \frac{\rho N_A}{m}$

$V = IR$

$R = \frac{\rho L}{A}$

$R_{series} = \sum_{i=1}^{N} R_i$

$\frac{1}{R_{parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$

$P = \frac{\Delta E}{\Delta t} = IV = I^2 R = \frac{V^2}{R}$

Light as a Wave

$c = f\lambda$

$S(t) = \frac{\text{Energy}}{\text{time}\times\text{Area}} = c\epsilon_0 E^2(t) = c\frac{B^2(t)}{\mu_0}$

$I = S_{avg} = \frac{1}{2}c\epsilon_0 E_{max}^2 = c\frac{B_{max}^2}{2\mu_0}$

$P = \begin{cases} \frac{S}{c}; & \text{absorbed} \\ \frac{2S}{c}; & \text{reflected} \end{cases}$

$S = S_0\cos^2\theta$

$v = \frac{c}{n}$

$\theta_{\text{incident}} = \theta_{\text{reflected}}$

$n_1\sin\theta_1 = n_2\sin\theta_2$

$P = \frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i}$

$M = -\frac{d_i}{d_0};\quad |M| = \frac{h_i}{h_0}$

Light as a Particle/Relativity

$E = hf = \frac{hc}{\lambda}$

$K_{max} = hf - \phi$

$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$

$\frac{1}{E'} = \frac{1}{E} + \frac{(1-\cos\phi)}{E_{rest}};\quad E_{rest} = mc^2$

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$p = \gamma mv$

$E_{total} = E_{rest} + K = \gamma mc^2$

$K = (\gamma - 1)mc^2$

$E_{total}^2 = p^2c^2 + m^2c^4$

Nuclear Physics

$N = N_0 e^{-\lambda t}$

$m = m_0 e^{-\lambda t}$

$A = A_0 e^{-\lambda t}$

$A = \lambda N$

$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

Constants

$g = 9.8 \frac{m}{s^2}$

$1e = 1.6 \times 10^{-19} C$

$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$

$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$

$1eV = 1.6 \times 10^{-19} J$

$\mu_0 = 4\pi \times 10^{-7} \frac{Tm}{A}$

$c = 3 \times 10^8 \frac{m}{s}$

$h = 6.63 \times 10^{-34} Js = 4.14 \times 10^{-15} eVs$

$N_A = 6.02 \times 10^{23}$

$1u = 1.66 \times 10^{-27} kg = 931.5 \frac{MeV}{c^2}$

$m_p = 1.67 \times 10^{-27} kg = 937.1 \frac{MeV}{c^2}$

$m_p = 1.67 \times 10^{-27} kg = 937.1 \frac{MeV}{c^2}$

$m_n = 1.69 \times 10^{-27} kg = 948.3 \frac{MeV}{c^2}$

$m_e = 9.11 \times 10^{-31} kg = 0.511 \frac{MeV}{c^2}$

$m_e = 9.11 \times 10^{-31} kg = 0.511 \frac{MeV}{c^2}$

Physics 110 Formulas

$\vec{F} = m\vec{a}; \quad F_G = \frac{GM_1 m_2}{r^2}; \quad F_s = -ky; \quad a_c = \frac{v^2}{r}$

$W = -\Delta U_g - \Delta U_s = \Delta K$

$U_g = mgy$

$U_s = \frac{1}{2} k y^2$

$K = \frac{1}{2} m v^2$

$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2} \vec{a} t^2$

$\vec{v}_f = \vec{v}_i + \vec{a} t$

$v_f^2 = v_i^2 + 2 a_r \Delta r$

Common Metric Units

$nano\ (n) = 10^{-9}$

$micro\ (\mu) = 10^{-6}$

$milli\ (m) = 10^{-3}$

$centi\ (c) = 10^{-2}$

$kilo\ (k) = 10^3$

$mega\ (M) = 10^6$

Geometry/Algebra

Circles:	$A = \pi r^2$	$C = 2\pi r = \pi$
Spheres:	$A = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$
Triangles:	$A = \frac{1}{2} bh$	
Quadratics:	$ax^2 + bx + c = 0 \rightarrow x =$	

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$