Physics 111

Exam #2

February 9, 2024

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. Erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $\left|\vec{p}\right| \approx m\left|\vec{v}\right| = (5kg)\times\left(2\,\frac{m}{s}\right) = 10\,\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 6 points.

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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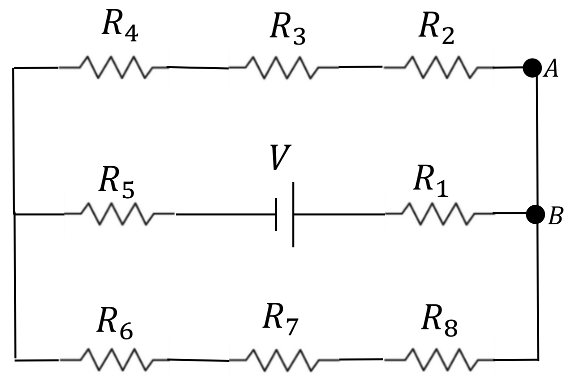
1. Consider the circuit shown below in which some resistors each of resistance $R = 500\Omega$ are wired to a $10V$ battery.



a. How much current is produced by the battery?

R_2, R_3, and R_4 are in series.
$R_{234} = R_2 + R_3 + R_4$
$R_{234} = 500\Omega + 500\Omega + 500\Omega$
$R_{234} = 1500\Omega$

R_6, R_7, and R_8 are in series.
$R_{678} = R_6 + R_7 + R_8$
$R_{678} = 500\Omega + 500\Omega + 500\Omega = 1500\Omega$

R_{234} and R_{678} are in parallel.

$$\frac{1}{R_{234678}} = \frac{1}{R_{234}} + \frac{1}{R_{678}} = \frac{1}{1500\Omega} + \frac{1}{1500\Omega} = \frac{2}{1500\Omega}$$
$$R_{234678} = \frac{1500\Omega}{2} = 750\Omega$$

R_1, R_5, and R_{234678} are in series.

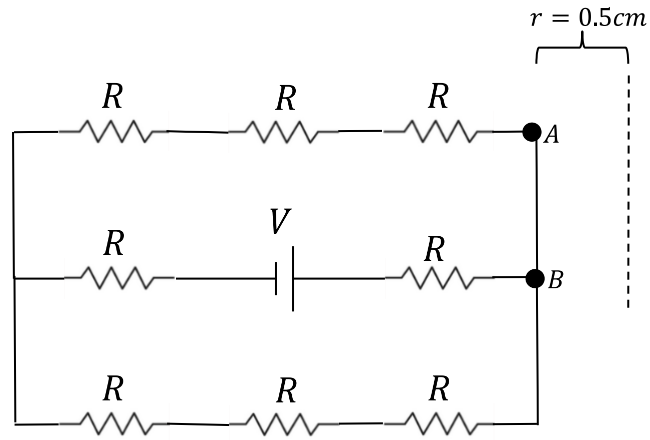
$$R_{eq} = R_{12345678} = R_1 + R_5 + R_{234678} = 500\Omega + 500\Omega + 750\Omega = 1750\Omega$$

$$I_{total} = \frac{V}{R_{eq}} = \frac{10V}{1750\Omega} = 0.0057A = 5.7mA$$

b. What is the energy dissipated across the entire circuit if the circuit is energized for a time $t = 1\ hour$?

$$P = \frac{\Delta E}{\Delta t} \rightarrow \Delta E = P\Delta t = I_{total}^2 R_{eq} \Delta t = (0.0057A)^2 \times 1750\Omega \times 3600s$$
$$\Delta E = 205.7J$$

c. Consider the long straight wire segment labeled by the points A and B in the diagram above, redrawn for convenience below. If the length of the wire segment between points A and B is $L = 20cm$, what is the magnitude and direction of the magnetic field along the dashed line located $0.5cm$ to the right of this segment of wire?

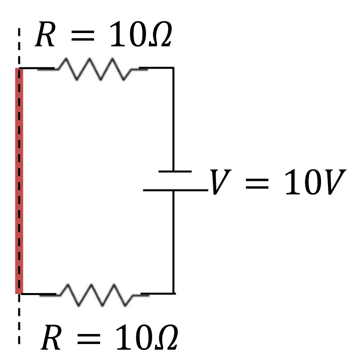


Since the resistance of the upper and lower branches are the same, the total current will split in half, with half going through the upper branch and half going through the lower branch.

$B = \frac{\mu_0 I_{AB}}{2\pi r} = \frac{\mu_0\left(\frac{I_{total}}{2}\right)}{2\pi r} = \frac{\mu_0 I_{total}}{4\pi r} = \frac{4\pi\times10^{-7}\frac{Tm}{A}\times0.0057A}{4\pi\times0.005m} = 1.14\times10^{-7}T$ in magnitude and by the right-hand rule, the direction of the magnetic field is into the page.

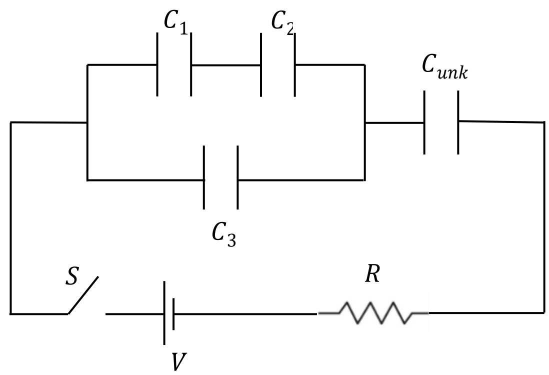
d. Suppose that a second circuit were placed along the dashed line as shown below. This dashed line is the same dashed line from part c. What magnetic force would the orange segment of wire (of length $L = 20cm$) placed on the dashed line feel?

$I = \frac{V}{R_{eq}} = \frac{V}{R+R} = \frac{V}{2R} = \frac{10V}{2\times10\Omega} = 0.5A$ flowing clockwise around the circuit.



$F = ILB = 0.5A \times 0.2m \times 1.14\times10^{-7}T$
$F = 1.14\times10^{-8}N$ in magnitude and by the right-hand rule, the force points to the left.

2. The circuit shown below has some capacitors wired to a switch S, a $10V$ battery and a $25k\Omega$ resistor. Three of the capacitors all have the same capacitance $C_1 = C_2 = C_3 = C = 0.015F$ while one is unknown C_{unk}. When the switch is closed the capacitors in the circuit charge according to $V(t) = 10(1 - e^{-0.0023t})$, for $V(t)$ in Volts and t in seconds.



a. What is the time constant of the circuit and the value of the effective capacitance in the circuit?

$$\frac{1}{\tau} = 0.0023s^{-1} \rightarrow \tau = 434.8s$$

$$\tau = RC_{eq} \rightarrow C_{eq} = \frac{\tau}{R} = \frac{434.8s}{25000\Omega} = 0.0174F$$

b. What is the value of the unknown capacitance in the system, C_{unk}?

C_1 and C_2 are in series.

$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C} \rightarrow C_{12} = \frac{C}{2} = \frac{0.015F}{2} = 0.0075F$$

C_{12} and C_3 are in parallel.

$$C_{123} = C_{12} + C_3 = 0.0075F + 0.015F = 0.0225F$$

C_{123} and C_{unk} are in series.

$$\frac{1}{C_{eq}} = \frac{1}{C_{123}} + \frac{1}{C_{unk}} \rightarrow \frac{1}{C_{unk}} = \frac{1}{C_{eq}} - \frac{1}{C_{123}}$$

$$C_{unk} = \left(\frac{1}{C_{eq}} - \frac{1}{C_{123}}\right)^{-1} = \left(\frac{1}{0.0174F} - \frac{1}{0.0225F}\right)^{-1} = 0.0768F$$

c. Suppose that the battery is removed and the network of fully charged capacitors is allowed to discharge through the $25k\Omega$ resistor. What is the initial amount of stored energy in the system and what percent of the initially stored energy is left to dissipate after a time $t = 1.5\tau$?

$$U_i = \frac{1}{2}C_{eq}V^2 = \frac{1}{2} \times 0.0174F \times (10V)^2 = 0.87J$$

$$U_f(t) = \frac{1}{2}C_{eq}\left(V(t)\right)^2 = \frac{1}{2}C_{eq}\left(V_{max}e^{-\frac{t}{\tau}}\right)^2 = \frac{1}{2}C_{eq}V_{max}^2e^{-\frac{2t}{\tau}} = U_ie^{-\frac{2t}{\tau}}$$

$$U_f(t = 1.5\tau) = U_ie^{-\frac{2\times1.5\tau}{\tau}} = U_ie^{-3} = 0.0498U_i = 5\%U_i$$

Or,

$$V(t = 1.5\tau) = V_{Max}e^{-\frac{1.5\tau}{\tau}} = V_{Max}e^{-1.5} = 0.223V_{Max}$$
$$U_f(t) = \frac{1}{2}C_{eq}\left(V(t)\right)^2 = \frac{1}{2}C_{eq}(0.233V_{max})^2 = 0.0499 \times U_i = 0.05 \times 0.87J = 0.0433J$$
$\% = \frac{U_f - U_i}{U_i} \times 100\% = \frac{0.0433J - 0.87J}{0.87J} \times 100\% = -95\%$ or 95% lost. Thus, there must be 5% remaining.

d. Suppose that at a distance d to the right of the circuit in part a above, a small N-turn circular coil of wire was placed. The coil of wire has a diameter of D and lies in the plane of the paper. Explain the direction of the current flow, if there is one, induced in the small N-turn circular coil of wire as the circuit of capacitors discharges through the resistor. Be sure to fully explain your answer. You may use complete sentences or equations, or a combination of both sentences and equations.

The current in the RC circuit flows counterclockwise as the capacitors discharge through the resistor and the current is decreasing in time according to $I(t) = I_{max}e^{-\frac{t}{\tau}}$. The magnetic field at the small coil of wire's location is pointing out of the page due to the counterclockwise current flowing in the RC circuit and is decreasing with increasing time. Thus, the magnetic flux is decreasing through the small coil of wire. To undo the decrease in magnetic flux, the small coil of wire will develop a counterclockwise current.

3. A proton is accelerated from rest through a potential difference $\Delta V = -1200V$ and eneters a uniform magnetic field of strength B at an angle θ, measured with respect to the magnetic field. Perpendicular to the magnetic field the proton's motion is a circle of radius $R = 8.5cm$ while parallel to the magnetic field the pitch of the proton is $L = 38.7cm$.

   a. What is the speed of the proton when it leaves the accelerator?

$$W = -q\Delta V = \Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}mv_f^2 \rightarrow v_f = \sqrt{-\frac{2q\Delta V}{m}}$$

$$v_f = \sqrt{-\frac{2 \times 1.6 \times 10^{-19}C \times (-1200V)}{1.67 \times 10^{-27}kg}} = 4.8 \times 10^5 \tfrac{m}{s}$$

   b. At what angle θ was the proton's velocity directed with respect to the magnetic field?

$$v_{\parallel} = v\cos\theta = \frac{L}{T}$$

$$v_{\perp} = v\sin\theta = \frac{2\pi R}{T}$$

$$T = \frac{L}{v\cos\theta} = \frac{2\pi R}{v\sin\theta} \rightarrow \frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{2\pi R}{L} = \frac{2\pi \times 8.5cm}{38.7cm} = 1.38$$

$$\rightarrow \theta = 54^0$$

c. What is the strength of the magnetic field, B?

$$F = qv_{\perp}B = m\frac{v_{\perp}^2}{R} \to B = \frac{mv_{\perp}}{qR} = \frac{1.67 \times 10^{-27}kg \times 4.8 \times 10^5\frac{m}{s} \times \sin 54}{1.6 \times 10^{-19}C \times 0.085m}$$

$$B = 0.0476T = 47.6mT$$

d. In one orbit about the magnetic field, how much work was done on the proton by the magnetic field? To earn full credit, be sure to fully explain your answer in either complete sentences and/or by using equations.

Method 1: Since the magnetic force is always perpendicular to the velocity, there is, then, no component of the magnetic force that can be used to change the speed, or magnitude, of the velocity. This means that the change in kinetic energy is zero and thus the work done is zero.

Method 2: $W = \vec{F} \cdot \overrightarrow{\Delta x} = F\Delta x \cos 90 = 0$ where the angle between the magnetic force and the displacement vector (directed along the velocity) is $\theta = 90^0$.

Method 3: Since the displacement in one orbit of the proton around the magnetic field is zero, $\Delta x = 0$, $W = \vec{F} \cdot \overrightarrow{\Delta x} = 0$.

Physics 111 Formula Sheet

Electrostatics

$F = k\frac{q_1 q_2}{r^2}$

$\vec{F} = q\vec{E}; \quad E_{pc} = k\frac{q}{r^2}; \quad E_{plate} = \frac{q}{\epsilon_0 A}$

$E = -\frac{\Delta V}{\Delta x}$

$V_{pc} = k\frac{q}{r}$

$U_e = k\frac{q_1 q_2}{r} = qV$

$W = -q\Delta V = -\Delta U_e = \Delta K$

Magnetism

$\vec{F} = q\vec{v} \times \vec{B} \rightarrow F = qvB\sin\theta$

$\vec{F} = I\vec{L} \times \vec{B} \rightarrow F = ILB\sin\theta$

$V_{Hall} = wv_dB$

$B = \frac{\mu_0 I}{2\pi r}$

$\varepsilon = \Delta V = -N\frac{\Delta\phi_B}{\Delta t}$

$\phi_B = BA\cos\theta$

Electric Circuits - Capacitors

$Q = CV; \quad C = \frac{\kappa\epsilon_0 A}{d}$

$C_{parallel} = \sum_{i=1}^{N} C_i$

$\frac{1}{C_{series}} = \sum_{i=1}^{N}\frac{1}{C_i}$

$Q_{charging}(t) = Q_{max}\left(1 - e^{-\frac{t}{\tau}}\right)$

$Q_{discharging}(t) = Q_{max}e^{-\frac{t}{\tau}}$

$I(t) = I_{max}e^{-\frac{t}{\tau}} = \frac{Q_{max}}{\tau}e^{-\frac{t}{\tau}}$

$\tau = RC$

$U_C = \frac{1}{2}qV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$

Electric Circuits - Resistors

$I = \frac{\Delta Q}{\Delta t}$

$I = neAv_d; \quad n = \frac{\rho N_A}{m}$

$V = IR$

$R = \frac{\rho L}{A}$

$R_{series} = \sum_{i=1}^{N} R_i$

$\frac{1}{R_{parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$

$P = \frac{\Delta E}{\Delta t} = IV = I^2R = \frac{V^2}{R}$

Light as a Wave

$c = f\lambda$

$S(t) = \frac{\text{Energy}}{\text{time}\times\text{Area}} = c\epsilon_0 E^2(t) = c\frac{B^2(t)}{\mu_0}$

$I = S_{avg} = \frac{1}{2}c\epsilon_0 E_{max}^2 = c\frac{B_{max}^2}{2\mu_0}$

$P = \begin{cases} \frac{S}{c}; & \text{absorbed} \\ \frac{2S}{c}; & \text{reflected} \end{cases}$

$S = S_0\cos^2\theta$

$v = \frac{c}{n}$

$\theta_{\text{incident}} = \theta_{\text{reflected}}$

$n_1\sin\theta_1 = n_2\sin\theta_2$

$P = \frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i}$

$M = \frac{d_i}{d_0}; \quad |M| = \frac{h_i}{h_0}$

Light as a Particle/Relativity

$E = hf = \frac{hc}{\lambda}$

$K_{max} = hf - \phi$

$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$

$\frac{1}{E'} = \frac{1}{E} + \frac{(1-\cos\phi)}{E_{rest}}; \quad E_{rest} = mc^2$

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$p = \gamma mv$

$E_{total} = E_{rest} + K = \gamma mc^2$

$K = (\gamma - 1)mc^2$

$E_{total}^2 = p^2c^2 + m^2c^4$

Nuclear Physics

$N = N_0 e^{-\lambda t}$

$m = m_0 e^{-\lambda t}$

$A = A_0 e^{-\lambda t}$

$A = \lambda N$

$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

Constants

$g = 9.8 \frac{m}{s^2}$

$1e = 1.6 \times 10^{-19} C$

$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$

$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$

$1eV = 1.6 \times 10^{-19} J$

$\mu_0 = 4\pi \times 10^{-7} \frac{Tm}{A}$

$c = 3 \times 10^8 \frac{m}{s}$

$h = 6.63 \times 10^{-34} Js = 4.14 \times 10^{-15} eVs$

$N_A = 6.02 \times 10^{23}$

$1u = 1.66 \times 10^{-27} kg = 931.5 \frac{MeV}{c^2}$

$m_p = 1.67 \times 10^{-27} kg = 937.1 \frac{MeV}{c^2}$

$m_n = 1.69 \times 10^{-27} kg = 948.3 \frac{MeV}{c^2}$

$m_e = 9.11 \times 10^{-31} kg = 0.511 \frac{MeV}{c^2}$

Physics 110 Formulas

$\vec{F} = m\vec{a}; \quad F_G = \frac{GM_1 m_2}{r^2}; \quad F_s = -ky; \quad a_c = \frac{v^2}{r}$

$W = -\Delta U_g - \Delta U_s = \Delta K$

$U_g = mgy$

$U_s = \frac{1}{2} k y^2$

$K = \frac{1}{2} m v^2$

$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2} \vec{a} t^2$

$\vec{v}_f = \vec{v}_i + \vec{a} t$

$v_f^2 = v_i^2 + 2 a_r \Delta r$

Common Metric Units

$nano\ (n) = 10^{-9}$

$micro\ (\mu) = 10^{-6}$

$milli\ (m) = 10^{-3}$

$centi\ (c) = 10^{-2}$

$kilo\ (k) = 10^{3}$

$mega\ (M) = 10^{6}$

Geometry/Algebra

Circles:	$A = \pi r^2$	$C = 2\pi r = \pi$
Spheres:	$A = 4\pi r^2$	$V = \frac{4}{3} \pi r^3$
Triangles:	$A = \frac{1}{2} bh$	
Quadratics:	$ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$