Physics 111

Exam #2

February 28, 2025

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So, erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}| = (5kg)\times\left(2\,\frac{m}{s}\right) = 10\,\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 6 points.

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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1. Potassium has three isotopes (elements with the same number of protons but different numbers of neutrons in their nucleus), ${}^{39}_{19}K$, ${}^{40}_{19}K$, and ${}^{41}_{19}K$. All three can form singly charged positive ions ($q = +e$) and all three charges isotopes of potassium are accelerated from rest through a potential difference V_{acc} where they acquire individual speeds $v_{{}^{39}_{19}K}$, $v_{{}^{40}_{19}K}$, and $v_{{}^{41}_{19}K}$. The charges then enter a region of space, shown in Figure 1A below, where there are crossed electric and magnetic fields. The magnetic field in this region of space is uniform with magnitude $B = 2T$, while the electric field can be varied. As the charges pass through this region of crossed electric and magnetic fields, the electric field can vary so that each charge emerges from the right end of the plates with a constant velocity. The charges that leave the right end of the plates enter a region of space where there is only the $B = 2T$ uniform magnetic field.

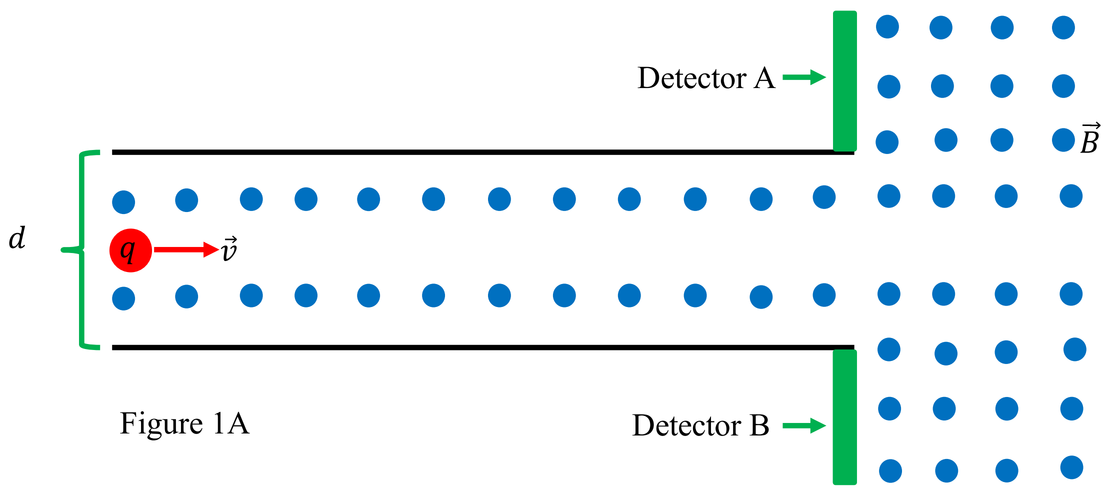

Figure 1A

a. Suppose the distance between the upper and lower plates in Figure 1A is $d = 1cm$. What is(are) the potential difference(s) that would be needed across the plates so that each charge leaves the right end of the plates with a speed $v = v_{{}^{39}_{19}K} = v_{{}^{40}_{19}K} = v_{{}^{41}_{19}K} = 3 \times 10^5 \frac{m}{s}$.

$$F_E - F_B = ma_y = 0 \rightarrow F_E = F_B \rightarrow qE = qvB \rightarrow E = -\frac{\Delta V}{\Delta y} = vB$$

$$\Delta V = -vdB = 3 \times 10^5 \frac{m}{s} \times 0.01m \times 2T = -6000V = -6kV$$

b. What is the ratio of the orbital radii of potassium-39 to potassium-40, $\frac{r_{{}^{39}_{19}K}}{r_{{}^{40}_{19}K}}$ and which detector (A or B) will these charges strike? Be sure to explain your choice for which detector. Hint: The masses of potassium-39 and potassium-40 are $38.963707u$ and $39.963998u$ respectively.

$$F_B = qvB = m\frac{v^2}{r} \rightarrow r = \frac{mv}{qB}$$

$$\frac{r_{{}^{39}_{19}K}}{r_{{}^{40}_{19}K}} = \frac{eBm_{{}^{39}_{19}K}v_{{}^{39}_{19}K}}{eBm_{{}^{40}_{19}K}v_{{}^{40}_{19}K}} = \frac{m_{{}^{39}_{19}K}}{m_{{}^{40}_{19}K}} = \frac{38.963707u}{39.963998u} = 0.975$$

By the right-hand rule, the charges will strike the lower detector, detector B.

c. Suppose that a charge of unknown mass and charge (both in magnitude and sign) was incident on the left end of the plates. This charge is directed between the plates by varying the electric field across the plates in such a way that the unknown charge is seen to strike detector A and a distance $D = 25mm$ from where it exits the right end of the plates. What is the sign of the unknown charge (explain your choice) and what is the charge to mass ratio for this moving charge if the electric field between the plates has a magnitude $E = 4 \times 10^5 \frac{N}{C}$.

By the right-hand rule, since the charges strike the upper detector, detector A, the charge must be negative.

$$F_B = qvB = m\frac{v^2}{r} \rightarrow qB = \frac{mv}{r} \rightarrow \frac{q}{m} = \frac{v}{rB} = \frac{E}{rB^2} = \frac{4 \times 10^5 \frac{N}{C}}{12.5 \times 10^{-3} m \times (2T)^2}$$

$$\frac{q}{m} = 8 \times 10^6 \frac{C}{kg}$$

d. Suppose instead that you have a wire with current I flowing as shown below in Figure 1B. A charge q is thrown from point P with a velocity $\vec{v}$ in the plane of the page as shown in Figure 1B. The resulting motion of the charge is plotted in Figure 1B as the dashed line. The motion is not helical and takes place entirely in the plane of the page. Explain the sign of the charge and how/why the motion of the charge occurs as it does.

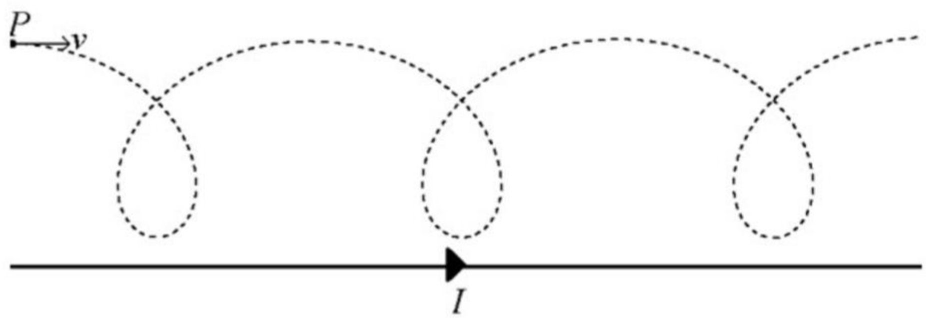

Figure 1B

The current in the wire flowing to the right produces a magnetic field above the wire that points out of the page. The charges velocity is initially to the right and since it moves towards the wire, the force on it must be down. By the right-hand rule this makes the charge positive.

A charge q in a magnetic field $\vec{B}$ with its velocity $\vec{v}$ perpendicular to the magnetic field feels a magnetic force $\vec{F}$ and the force will cause the charge to move in a circle of radius R since the force is perpendicular to the charge's velocity. No work is done by the magnetic force, so the speed of the charge does not change. This radius R is constant if the magnetic field everywhere is constant.

However, we notice that the magnetic field produced by the wire is not constant and varies as r^{-1}. AS the charge gets closer to the wire the magnetic force gets larger (as the magnitude of B increases) and the orbital radii of the charge are still "circles" but of ever decreasing radii since $R \propto F^{-1}$. This produces the right-hand motion of all the teardrops. To get the left-hand motion we note that the velocity vector is changing direction as the charge approaches the wire. When the charge is nearest to the wire, its velocity vector points opposite to its initial motion. This changes the direction of the force on the charge from towards the wire to away. Now the magnetic field is decreasing as the charge moves away from the wire and the "circles" that describe the motion are getting everywhere larger which completes the left-hand side of the teardrop.

*** Note: The problems which formed this question came from MPHW10 (24.67 – 24.70) and MPHW12 (24.59 – 24.62) ***

2. A sideview of a bar of mass $m = 500g$, length $L = 10cm$, and resistance $R = 10\mu\Omega$ is pushed up a set of rails inclined at an angle $\theta = 10^0$by an outside force F as shown in Figure 2A below at a constant speed $v = 2\frac{m}{s}$. A uniform $B = 50mT$ magnetic field points through the ramp and is directed along the normal to the ramp. Friction exists between the rails and the bar with coefficient of friction $\mu = 0.1$. Figure 2B shows a face-on view of the bar and rails.

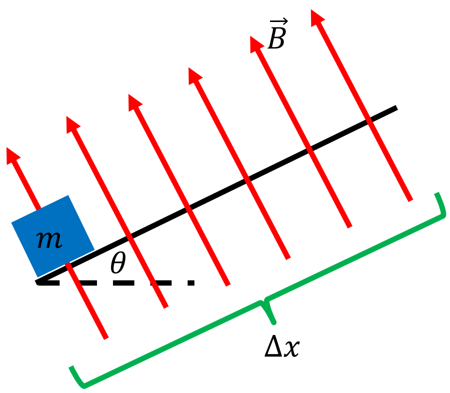

Figure 2A

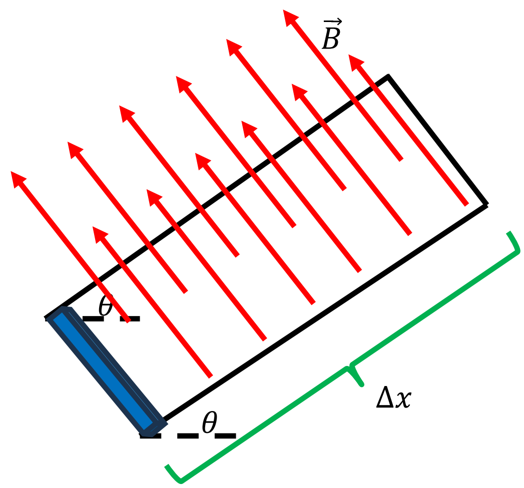

Figure 2B

a. What is the magnitude of the external force F?

Parallel to the ramp and taking up the ramp as the positive x-direction we have:

$$F - F_{fr} - F_{w,x} - F_B = ma_x = 0 \rightarrow F = F_{fr} + F_{w,x} + F_B$$

$$F = \mu F_N + F_w \sin\theta + ILB = \mu mg\cos\theta + mg\sin\theta + \left(\frac{BLv\cos\phi}{R}\right)LB$$

$$F = 0.5kg \times 9.8\frac{m}{s^2}(0.1\cos 10 + \sin 10) + \frac{(50\times 10^{-3}T)^2(0.1m)^2 \times 2\frac{m}{s}}{10\times 10^{-6}\Omega} = 6.33N$$

Perpendicular to the ramp we have:

$$F_N - F_{w,y} = ma_y = 0 \rightarrow F_N = F_w\cos\theta = mg\cos\theta$$

b. What is the magnitude and direction of the current that flows in the bar? Be sure to explain your choice for the direction.

$$I = \frac{\epsilon}{R} = \frac{BLv\cos\phi}{R} = \frac{50\times 10^{-3}T \times 0.1m \times 2\frac{m}{s}\cos 0}{10\times 10^{-6}\Omega} = 1000A = 1kA$$

This is probably unreasonable for a current, but the number is correct.

The direction of the current flow is to undo the change in magnetic flux through the loop. As the bar is pushed up the ramp, the magnetic flux is decreasing. To undo the decrease, a counterclockwise current would have to be produced.

c. If the bar is pushed from the bottom of the ramp to its top over a distance $\Delta x = 1.5m$ at $v = 2\frac{m}{s}$, how much energy is dissipated?

$$P = \frac{\Delta E}{\Delta t} = I^2R \rightarrow \Delta E = I^2R\Delta t = (1000A)^2 \times 10\times 10^{-6}\Omega \times 0.75s = 7.5J$$

Where the time to reach the top of the ramp from the bottom is given by:

$$x_f = x_i + v_{ix}t + \tfrac{1}{2}a_xt^2 \rightarrow \Delta x = v_{ix}t \rightarrow t = \frac{\Delta x}{v} = \frac{1.5m}{2\frac{m}{s}} = 0.75s$$

d. At the very top of the ramp, the bar is released from rest and is allowed to slide down the ramp. What is the terminal speed of the bar down the ramp and what current (magnitude and direction) would flow?

Parallel to the ramp and taking down the ramp as the positive x-direction we have:

$$-F_{fr} + F_{w,x} - F_B = ma_x = 0 \rightarrow F_B = F_{wx} - F_{fr} = mg\sin\theta - \mu mg\cos\theta$$

$$F_B = ILB = mg(\sin\theta - \mu\cos\theta) = 0.5kg \times 9.8\frac{m}{s^2}(\sin 10 - 0.1\cos 10) = 0.368N$$
$$\rightarrow I = \frac{F_B}{LB} = \frac{0.368N}{0.1m\times 50\times 10^{-3}T} = 73.7A$$

As the bar slides down the ramp, the magnetic flux is increasing through the loop. To undo the increase in magnetic flux a clockwise current must flow.

$$I = \frac{\epsilon}{R} = \frac{BLv}{R} \rightarrow v_{term} = \frac{IR}{BL} = \frac{73.7A\times 10\times 10^{-6}\Omega}{50\times 10^{-3}T\times 0.1m} = 0.15\frac{m}{s}$$

*** Note: The problems which formed this question came from MPHW13 (Concept Q 25.3 and Problem 25.65), MPHW14 (21.17 & 21.82) as well as Ch. 4 & Ch. 5 KJF ***

3. An endoscope is a narrow bundle of optical fibers that can be inserted through a small incision in the body or passed through an opening in the body to view the body or perform a surgical procedure. Figures 3A and 3B below shows an optical fiber scope, and a fiber scope used in an arthroscopic surgery of the knee. The objective lens (inside of the patient) focuses a real image on the entrance of the fiber scope. Individual fibers use total internal reflection to transport the light from the front surface of the scope to the exit surface. The light that exits the exit face from all the fibers produces a magnified image and the doctor observes the magnified image through the eyepiece lens.

Figure 3A

Figure 3B

a. Figure 3C shows the basic setup of a fiber optic scope with its lensing system while Figure 3D show a single fiber of the bundle that makes up the endoscope. The core of the fiber and the material that surrounds the core (the cladding) have indices of refraction $n_{core}1.62$ and $n_{claddiing} = 1.50$ respectively. At what angle of incidence on the front surface of the fiber, θ_{max} is needed so that the light is totally internally reflected in core.

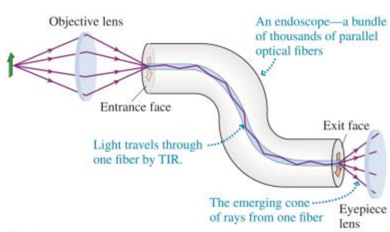

Figure 3C

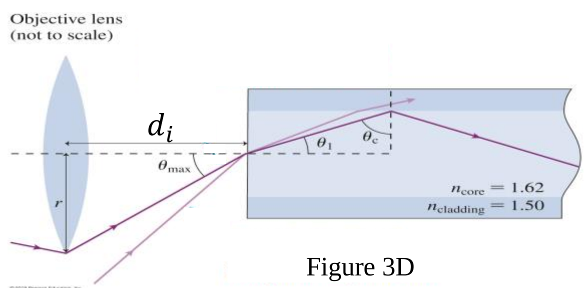

Figure 3D

At the core/cladding interface:

$$n_{core} \sin\theta_{core} = n_{cladding} \sin\theta_{cladding} \rightarrow n_{core} \sin\theta_c = n_{cladding} \sin 90$$

$$\theta_c = \sin^{-1}\left(\frac{n_{cladding}}{n_{core}}\right) = \sin^{-1}\left(\frac{1.50}{1.62}\right) = 67.8^0$$

From the geometry in the problem:

$$\theta_1 + \theta_C = 90^0 \rightarrow \theta_1 = 90^0 - \theta_C = 90^0 - 67.8^0 = 22.2^0$$

At the air/core interface:

$$n_{air} \sin\theta_{air} = n_{core} \sin\theta_{core} \rightarrow n_{air} \sin\theta_{max} = n_{core} \sin\theta_1$$

$$\theta_{max} = \sin^{-1}\left(\frac{n_{core}}{n_{air}} \sin\theta_1\right) = \sin^{-1}\left(\frac{1.62}{1.00} \sin 22.2\right) = 37.7^0$$

b. A typical objective lens is $3mm$ in diameter and can focus on an object $3mm$ in front of it. What power should the objective lens have so that rays from its lower edge enter the fiber at θ_{max}?

$$P = \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{3 \times 10^{-3}m} + \frac{1}{1.94 \times 10^{-3}m} = 848.8D$$

Where the image distance is determined from:

$$\tan\theta_{max} = \frac{r}{d_i} \rightarrow d_i = \frac{r}{\tan\theta_{max}} = \frac{1.5mm}{\tan 37.7} = 1.94mm$$

c. What is the magnification of the objective lens and the height of the object in the body if the image on the screen was $10cm$ tall?

$$M = \frac{d_i}{d_0} = \frac{1.94mm}{3mm} = 0.65 \rightarrow M = \frac{h_i}{h_o} \rightarrow h_o = \frac{h_i}{M} = \frac{10cm}{0.65} = 15.5cm$$

d. The objective lens of an endoscope must be carefully matched to the optical fiber bundle. This means that ideally the diameter of the lens is such that the rays from the edge of the lens enter the fiber at θ_{max}. Explain what would happen in the following two cases.

1. The lens diameter is much larger than r.

If the lens diameter is much larger than r, then the light that enters the core of the scope would be at angles larger than θ_{max}. This would in turn make θ_2 larger and the angle that the light would strike the core/cladding interface would decrease and become less than the critical angle and the light would not be totally internally reflected in the scope.

2. The lens diameter is much smaller than r.

If the lens diameter is small than r, t then the light that enters the core of the scope would be at angles smaller than θ_{max}. This would in turn make θ_2 smaller and the angle that the light would strike the core/cladding interface would increase and become greater than the critical angle and the light would be totally internally reflected in the scope. The image formed would be dimmer since we'd gather less light.

*** Note: The problems which formed this question came from Ch. 18 pages 647 – 648, MPHW18 (18.20 & 18372) and MPHW19 (19.16 & 23.47)***

Physics 111 Formula Sheet

Electrostatics

$F = k\frac{q_1 q_2}{r^2}$

$\vec{F} = q\vec{E}$; $\quad E_{pc} = k\frac{q}{r^2}$; $\quad E_{plate} = \frac{q}{\epsilon_0 A}$

$E = -\frac{\Delta V}{\Delta x}$

$V_{pc} = k\frac{q}{r}$

$U_e = k\frac{q_1 q_2}{r} = qV$

$W = -q\Delta V = -\Delta U_e = \Delta K$

Magnetism

$\vec{F} = q\vec{v} \times \vec{B} \rightarrow F = qvB\sin\theta$

$\vec{F} = I\vec{L} \times \vec{B} \rightarrow F = ILB\sin\theta$

$V_{Hall} = wv_d B$

$B = \frac{\mu_0 I}{2\pi r}$

$\varepsilon = \Delta V = -N\frac{\Delta\phi_B}{\Delta t}$

$\phi_B = BA\cos\theta$

Electric Circuits - Capacitors

$Q = CV$; $\quad C = \frac{\kappa\epsilon_0 A}{d}$

$C_{parallel} = \sum_{i=1}^{N} C_i$

$\frac{1}{C_{series}} = \sum_{i=1}^{N}\frac{1}{C_i}$

$Q_{charging}(t) = Q_{max}\left(1 - e^{-\frac{t}{\tau}}\right)$

$Q_{discharging}(t) = Q_{max}e^{-\frac{t}{\tau}}$

$I(t) = I_{max}e^{-\frac{t}{\tau}} = \frac{Q_{max}}{\tau}e^{-\frac{t}{\tau}}$

$\tau = RC$

$U_C = \frac{1}{2}qV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$

Electric Circuits - Resistors

$I = \frac{\Delta Q}{\Delta t}$

$I = neAv_d$; $\quad n = \frac{\rho N_A}{m}$

$V = IR$

$R = \frac{\rho L}{A}$

$R_{series} = \sum_{i=1}^{N} R_i$

$\frac{1}{R_{parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$

$P = \frac{\Delta E}{\Delta t} = IV = I^2 R = \frac{V^2}{R}$

Light as a Wave

$c = f\lambda$

$S(t) = \frac{\text{Energy}}{\text{time}\times\text{Area}} = c\epsilon_0 E^2(t) = c\frac{B^2(t)}{\mu_0}$

$I = S_{avg} = \frac{1}{2}c\epsilon_0 E_{max}^2 = c\frac{B_{max}^2}{2\mu_0}$

$$P = \begin{cases} \frac{S}{c}; & \text{absorbed} \\ \frac{2S}{c}; & \text{reflected} \end{cases}$$

$S = S_0\cos^2\theta$

$v = \frac{c}{n}$

$\theta_{\text{incident}} = \theta_{\text{reflected}}$

$n_1\sin\theta_1 = n_2\sin\theta_2$

$P = \frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i}$

$M = \frac{d_i}{d_0}$; $\quad |M| = \frac{h_i}{h_0}$

Light as a Particle/Relativity

$E = hf = \frac{hc}{\lambda}$

$K_{max} = hf - \phi$

$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$

$\frac{1}{E'} = \frac{1}{E} + \frac{(1-\cos\phi)}{E_{rest}}$; $\quad E_{rest} = mc^2$

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$p = \gamma mv$

$E_{total} = E_{rest} + K = \gamma mc^2$

$K = (\gamma - 1)mc^2$

$E_{total}^2 = p^2c^2 + m^2c^4$

Nuclear Physics

$N = N_0 e^{-\lambda t}$

$m = m_0 e^{-\lambda t}$

$A = A_0 e^{-\lambda t}$

$A = \lambda N$

$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

Constants

$g = 9.8\frac{m}{s^2}$

$1e = 1.6 \times 10^{-19}C$

$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$

$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$

$1eV = 1.6 \times 10^{-19}J$

$\mu_0 = 4\pi \times 10^{-7}\frac{Tm}{A}$

$c = 3 \times 10^8 \frac{m}{s}$

$h = 6.63 \times 10^{-34}Js = 4.14 \times 10^{-15}eVs$

$N_A = 6.02 \times 10^{23}$

$1u = 1.66 \times 10^{-27}kg = 931.5\frac{MeV}{c^2}$

$m_p = 1.67 \times 10^{-27}kg = 937.1\frac{MeV}{c^2}$

$m_n = 1.69 \times 10^{-27}kg = 948.3\frac{MeV}{c^2}$

$m_e = 9.11 \times 10^{-31}kg = 0.511\frac{MeV}{c^2}$

Physics 110 Formulas

$\vec{F} = m\vec{a}; \quad F_G = \frac{GM_1m_2}{r^2}; \quad F_s = -ky; \quad a_c = \frac{v^2}{r}$

$W = -\Delta U_g - \Delta U_s = \Delta K$

$U_g = mgy$

$U_s = \frac{1}{2}ky^2$

$K = \frac{1}{2}mv^2$

$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$

$\vec{v}_f = \vec{v}_i + \vec{a}t$

$v_f^2 = v_i^2 + 2a_r\Delta r$

Common Metric Units

$nano\ (n) = 10^{-9}$

$micro\ (\mu) = 10^{-6}$

$milli\ (m) = 10^{-3}$

$centi\ (c) = 10^{-2}$

$kilo\ (k) = 10^{3}$

$mega\ (M) = 10^{6}$

Geometry/Algebra

Circles:	$A = \pi r^2$	$C = 2\pi r = \pi$
Spheres:	$A = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$
Triangles:	$A = \frac{1}{2}bh$	
Quadratics:	$ax^2 + bx + c = 0 \rightarrow x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$