Physics 111

Exam #2

February 9, 2011

Name_________________________________

Multiple Choice	/16
Problem #1	/28
Problem #2	/28
Problem #3	/28
Total	/100

Part I: Multiple-Choice: *Circle the best answer to each question. Any other marks will not be given credit. Each multiple-choice question is worth 4 points for a total of 16 points.*

1. Two loops of wire of the same physical size are oriented parallel to one another and carry the same magnitudes of current but the current flow is in opposite directions in each of the wires. If the current flows counterclockwise in the bottom loop while the same magnitude of current flows clockwise in the upper loop, the force felt by the upper loop of wire due to the current flowing in the lower loop of wire would tend to

    a. make the upper loop grow and be attracted to the lower loop of wire.
    b. make the upper loop grow and be repelled from the lower loop of wire.
    c. make the upper loop shrink and be attracted to the lower loop of wire.
    d. make the upper loop shrink and be repelled from the lower loop of wire.

2. A plane flies at constant speed due south through the Earth's magnetic field that has components that point both north and vertically down. In this situation,

    a. the left side of the plane becomes positively charged.
    b. the right side of the plane becomes positively charged.
    c. the top of the plane becomes positively charged.
    d. the bottom of the plane becomes positively charged.

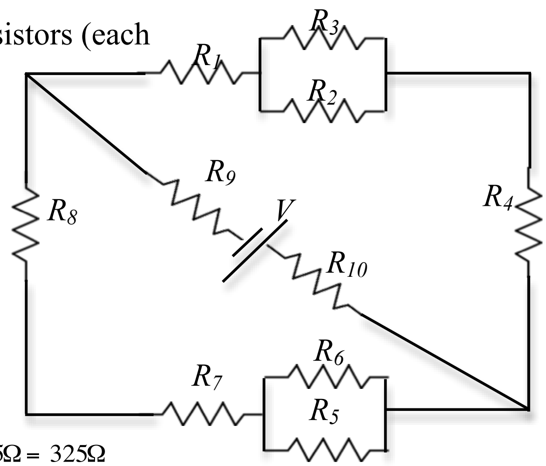
3. A circuit has a single battery (with constant potential) and some resistors in it. If the equivalent resistance of the circuit is increases by a factor of 4, what happens to the total current and power dissipated by the entire circuit?

    a. Both decrease by a factor of 4.
    b. Both increase by a factor of 4.
    c. Both remain constant.
    d. Both decrease by a factor of 16.
    e. Both increase by a factor of 16.

4. A long straight wire is connected to a battery of constant potential and some resistors. This wire produces a magnetic field of strength B at a perpendicular distance r away from the wire. What happens to the equivalent resistance of the circuit and the magnetic field strength at the same perpendicular distance r if the current in the circuit were doubled?

    a. The equivalent resistance halves and the magnetic field strength halves.
    b. The equivalent resistance halves and the magnetic field strength doubles.
    c. The equivalent resistance doubles and the magnetic field strength halves.
    d. The equivalent resistance doubles and the magnetic field strength doubles.

Part II: Free Response Problems: *The three problems below are worth 84 points total and each subpart is worth 7 points each. Please show all work in order to receive partial credit. If your solutions are illegible or illogical no credit will be given. A number with no work shown (even if correct) will be given no credit. Please use the back of the page if necessary, but number the problem you are working on.*



1. You are given the circuit on the right that has 10 resistors (each with a resistance of 100Ω) that are wired to a 12.0V battery.

R_2 and R_3 in parallel $\rightarrow R_{23} = \left(\frac{2}{100\Omega}\right)^{-1} = 50\Omega$

R_1, R_4 and R_{23} in series $\rightarrow R_{1234} = 100\Omega + 100\Omega + 50\Omega = 250\Omega$

R_5 and R_6 in parallel $\rightarrow R_{56} = \left(\frac{2}{100\Omega}\right)^{-1} = 50\Omega$

R_7, R_8 and R_{56} in series $\rightarrow R_{5678} = 100\Omega + 100\Omega + 50\Omega = 250\Omega$

R_{1234} in parallel with $R_{5678} \rightarrow R_{12345678} = \left(\frac{2}{250\Omega}\right)^{-1} = 125\Omega$

R_9, R_{10} and $R_{12345678}$ in series $\rightarrow R_{12345678910} = 100\Omega + 100\Omega + 125\Omega = 325\Omega$

a. What are the equivalent resistance R_{eq} of the circuit and total current I_{total} produced by the battery?

$R_{eq} = R_{12345678910} = 325\Omega$ and $I_{total} = \frac{V}{R_{eq}} = \frac{12V}{325\Omega} = 0.0369A = 36.9mA$

b. What is the potential drop across resistors R_8 and R_{10}?

$V_{10} = I_{total}R_{10} = 0.0369A \times 100\Omega = 3.69V$. The potential drop across R_9 is the same as V_{10}. Thus the potential drop across the remainder of the circuit is $12V - 2(3.69V) = 4.62V$. This is the potential drop across the equivalent resistors R_{1234} and R_{5678}. Therefore, the current in the branch with equivalent resistance R_{1234} and R_{5678} will be the same (since both branches have the same equivalent resistance) given by

$I_{1234} = I_{5678} = \frac{V_{5678}}{R_{5678}} = \frac{4.62V}{250\Omega} = 0.0185A = 18.5mA$. Taking the total current and dividing by 2 since the current has to split evenly between the upper and lower branch could also obtain this current. Therefore the potential drop across resistor R_8 is given by $V_8 = I_{5678}R_8 = 0.0185A \times 100\Omega = 1.85V$

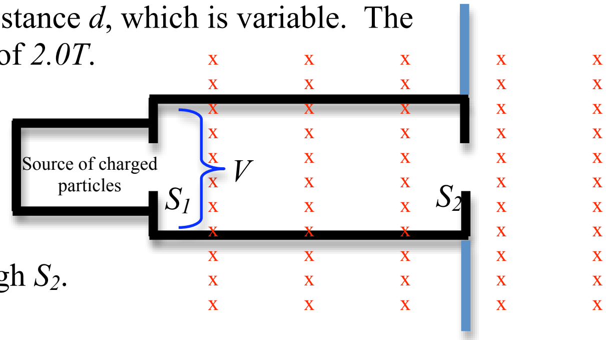
c. What currents flow through resistors R_1 and R_7?

From part b, $I_7 = I_1 = I_{5678} = I_{1234} = 0.0185A = 18.5mA$.

d. Suppose that the circuit is powered up and allowed to run for one hour. How much energy has been dissipated across resistor R_2?

$$E = Pt = I_2^2R_2t = \left(\frac{0.0185A}{2}\right)^2 \times 100\Omega \times 3600s = 30.8J$$

2. Consider the mass spectrometer shown below where there is a source of charged particles and these particles are incident at S_1, travel through the velocity selector and exit at S_2. All of the particles that are incident at S_1 have the same charge ($+q$) but may differ in mass. There is a *50kV* potential difference across the capacitor (located between S_1 and S_2 and with the upper plate at charge $+Q$ and the lower plate at charge $-Q$) and the plates are separated by a distance d, which is variable. The magnetic field everywhere has a value of *2.0T*.



a. Derive an expression for the orbital trajectory r of a particle as a function of the stated parameters and the mass of the particle, m, after it passes out through S_2.

$$F_B = qvB = \frac{mv^2}{r} \rightarrow r = \frac{mv}{qB} = \frac{mV}{qdB^2}$$

$$\text{where, } \sum F_y : -qE + qvB = ma_y = 0 \rightarrow v = \frac{E}{B} = \frac{\Delta V / \Delta x}{B} = \frac{V}{dB}$$

b. Suppose that you have two singly ionized isotopes of sodium, ${}^{22}_{11}Na$ and ${}^{24}_{11}Na$, that are the source of charged particles, by what amount would the two isotopes be separated when they strike the detector (either one of the blue plates) if the separation between the capacitor plates is *5mm*? Will the isotopes strike the detector above or below the exit at S_2?

The $+q$ charges will feel a magnetic force and both will strike above S_2. The distance each charge strikes the upper plate is two times the orbital radius. Using the result in part a, we have

$$r_{22} = \frac{mV}{qdB^2} = \frac{\left(22amu \times \frac{1.66 \times 10^{-27} kg}{1amu}\right) \times 50 \times 10^3 V}{1.6 \times 10^{-19} C \times 0.005m \times (2T)^2} = 0.57m \rightarrow d_{22} = 2r_{22} = 1.14m$$

and

$$r_{24} = \frac{mV}{qdB^2} = \frac{\left(24amu \times \frac{1.66 \times 10^{-27} kg}{1amu}\right) \times 50 \times 10^3 V}{1.6 \times 10^{-19} C \times 0.005m \times (2T)^2} = 0.60m \rightarrow d_{22} = 2r_{22} = 1.2m.$$

Thus the two charges will be separated by *1.20m – 1.14m = 0.06m = 6cm*.

c. What is the time of flight for each of the two isotopes of in the magnetic field?

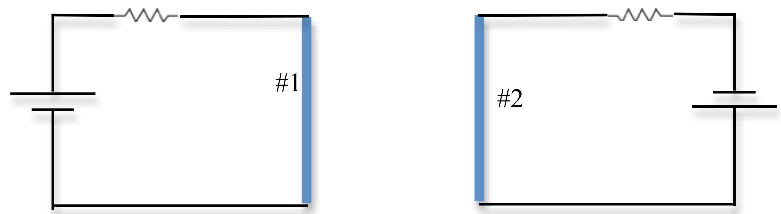
The time of flight is given by $t = \frac{T}{2} = \frac{2\pi r}{2v} = \frac{\pi m V}{\left(\frac{V}{Bd}\right) \times qdB^2} = \frac{\pi m}{qB}$. Therefore we have $t_{22} = \frac{\pi m}{qB} = \frac{\pi \times \left(22 amu \times \frac{1.66 \times 10^{-27} kg}{1 amu}\right)}{1.6 \times 10^{-19} C \times 2T} = 3.6 \times 10^{-7} s$ and

$$t_{24} = \frac{\pi m}{qB} = \frac{\pi \times \left(24 amu \times \frac{1.66 \times 10^{-27} kg}{1 amu}\right)}{1.6 \times 10^{-19} C \times 2T} = 3.9 \times 10^{-7} s$$

d. Suppose instead you have the nuclear decay of barium (${}^{137}_{56}Ba^* \rightarrow {}^{137}_{56}Ba + {}^{0}_{0}\gamma$) in which a gamma ray (${}^{0}_{0}\gamma$) is produced at the source and these are incident at S_1 and emerge at S_2. Where would the gamma ray, a form of electromagnetic radiation, strike the detector?

Since a gamma ray has no charge it is unaffected by the magnetic field. Therefore it will not strike any of the detectors in the diagram above.

3. Consider the circuit shown below in which a long straight wire #1 is connected to a battery rated at *12.0V* and a *125Ω* resistor, while another long straight wire #2 is connected to a *10.0V* battery and a *275Ω* resistor and the two wires (#1 & #2) are separated by *0.5m* and that each blue segment of wire has a length of 0.25m.



a. What magnetic force would wire #1's feel due to the current flowing in wire #2? Suppose that this force were strong enough to accelerate and actually move the entire circuit #1 assumed to have total mass *m*, would the circuit experience a constant acceleration and would its velocity after it has been displaced by a distance *Δx* be given by $v_f^2 = 2a\Delta x$? Justify your answer.

The velocity will not be given by the above equation, since the acceleration is not constant. This is because the magnetic field is not constant (it varies inversely with separation) and thus the magnetic force is not constant. The force on wire #1 is given as:

$$F_{1,2} = I_1 l_1 B_{1,2} = \left(\frac{V_1}{R_1}\right) l_1 \left(\frac{\mu_0 I_2}{2\pi d}\right) = \frac{\mu_0 l_1 V_1 V_2}{2\pi R_1 R_2 d} = \frac{2\times10^{-7}\,\frac{Tm}{A}\times 0.25m \times 12V \times 10V}{125\Omega \times 275\Omega \times 0.5m} = 3.5\times10^{-10} N$$

directed away from wire #2.

b. Assuming that the wires are in their original configuration and remain stationary, what is the net magnetic field at the midpoint between the two wires?

Assuming out of the page is the positive direction, we have

$$B_{net} = B_1 + B_2 = \frac{\mu_0}{2\pi}\left(\frac{V_1}{R_1 r_1} + \frac{V_1}{R_2 r_2}\right) = \frac{2\times10^{-7}\,\frac{T}{Am}}{0.25m}\left(\frac{12V}{125\Omega} + \frac{10V}{275\Omega}\right) = 1.1\times10^{-7} T \text{ out of}$$

the page.

c. Suppose a square loop of wire with sides of length *10cm* were oriented with its face perpendicular to the net magnetic field and the center of the loop at the midpoint between the two wires. What would the net force be on the loop if there was a current $I_{loop} = 25mA$ flowing clockwise? (Hint: Assume that the net magnetic field is taken as being constant across the face of the loop of wire and the value of that field is your calculated value in part b.)

By the right hand rule, with the magnetic field taken as constant across the loop of wire, and directed out of the page, we have the force directed up the plane of the page on the bottom of the loop of wire and the force on the top of the wire directed down the plane of the page. These forces are equal in magnitude and oppositely directed so they cancel. Analogously, on the left side of the loop the force is directed to the right and on the right side of the loop the force is directed to the left and thus these forces are equal and opposite so the net force here is zero too. Thus the net force on the loop of wire is zero.

d. Suppose that you made the same *25*-turn loop of wire exactly as in part c, what would the net torque be on the loop if there was a current $I_{loop} = 25mA$ flowing clockwise? In what direction would the loop of wire rotate?

Here the forces on the top and bottom of the wire act opposite each other as well as the forces on the right and left sides, the net torque is zero. The forces would tend to squeeze the wire but not rotate it. Also, the magnetic moment is parallel to the external field, so the net torque is $\tau = \mu B \sin 0 = 0$.

Physics 111 Equation Sheet

Electric Forces, Fields and Potentials

$$\vec{F} = k\frac{Q_1Q_2}{r^2}\hat{r}$$

$$\vec{E} = \frac{\vec{F}}{q}$$

$$\vec{E}_Q = k\frac{Q}{r^2}\hat{r}$$

$$PE = k\frac{Q_1Q_2}{r}$$

$$V(r) = k\frac{Q}{r}$$

$$E_x = -\frac{\Delta V_{B,A}}{\Delta x}$$

$$W_{A,B} = q\Delta V_{A,B}$$

Magnetic Forces and Fields

$$F = qvB\sin\theta$$

$$F = IlB\sin\theta$$

$$\tau = NIAB\sin\theta = \mu B\sin\theta$$

$$PE = -\mu B\cos\theta$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\varepsilon_{induced} = -N\frac{\Delta\phi_B}{\Delta t} = -N\frac{\Delta(BA\cos\theta)}{\Delta t}$$

Electric Circuits

$$I = \frac{\Delta Q}{\Delta t}$$

$$V = IR = I\left(\frac{\rho L}{A}\right)$$

$$R_{series} = \sum_{i=1}^{N} R_i$$

$$\frac{1}{R_{parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$$

$$P = IV = I^2R = \frac{V^2}{R}$$

$$Q = CV = \left(\frac{\kappa\varepsilon_0 A}{d}\right)V = (\kappa C_0)V$$

$$PE = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}$$

$$Q_{\text{charge}}(t) = Q_{\max}\left(1 - e^{-\frac{t}{RC}}\right)$$

$$Q_{\text{discharge}}(t) = Q_{\max}e^{-\frac{t}{RC}}$$

$$C_{parallel} = \sum_{i=1}^{N} C_i$$

$$\frac{1}{C_{series}} = \sum_{i=1}^{N}\frac{1}{C_i}$$

Light as a Wave

Constants

$g = 9.8\,\frac{m}{s^2}$

$G = 6.67\times10^{-11}\,\frac{Nm^2}{kg^2}$

$1e = 1.6\times10^{-19}C$

$k = \frac{1}{4\pi\varepsilon_o} = 9\times10^9\,\frac{C^2}{Nm^2}$

$\varepsilon_o = 8.85\times10^{-12}\,\frac{Nm^2}{C^2}$

$1eV = 1.6\times10^{-19}J$

$\mu_o = 4\pi\times10^{-7}\,\frac{Tm}{A}$

$c = 3\times10^8\,\frac{m}{s}$

$h = 6.63\times10^{-34}Js$

$m_e = 9.11\times10^{-31}kg = \frac{0.511MeV}{c^2}$

$m_p = 1.67\times10^{-27}kg = \frac{937.1MeV}{c^2}$

$m_n = 1.69\times10^{-27}kg = \frac{948.3MeV}{c^2}$

$1amu = 1.66\times10^{-27}kg = \frac{931.5MeV}{c^2}$

$N_A = 6.02\times10^{23}$

$Ax^2 + Bx + C = 0 \rightarrow x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$

Light as a Particle & Relativity

$$E = hf = \frac{hc}{\lambda} = pc$$

$$KE_{\max} = hf - \phi = eV_{stop}$$

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\phi)$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$p = \gamma mv$$

$$E_{total} = KE + E_{rest} = \gamma mc^2$$

$$E_{total}^2 = p^2c^2 + m^2c^4$$

$$E_{rest} = mc^2$$

$$KE = (\gamma - 1)mc^2$$

Geometry

Circles: $C = 2\pi r = \pi D \qquad A = \pi r^2$

Triangles: $A = \tfrac{1}{2}bh$

Spheres: $A = 4\pi r^2 \qquad V = \tfrac{4}{3}\pi r^3$

Nuclear Physics

$$E_{binding} = (Zm_p + Nm_n - m_{rest})c^2$$

$$\frac{\Delta N}{\Delta t} = -\lambda N_o \rightarrow N(t) = N_o e^{-\lambda t}$$

$$A(t) = A_o e^{-\lambda t}$$

$$m(t) = m_o e^{-\lambda t}$$

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

Misc. Physics 110 Formulae

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\Delta(mv)}{\Delta t} = m\vec{a}$$

$$\vec{F} = -k\vec{y}$$

$$\vec{F}_C = m\frac{v^2}{R}\hat{r}$$

$$W = \Delta KE = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right) = -\Delta PE$$

$$PE_{gravity} = mgy$$

$$PE_{spring} = \tfrac{1}{2}ky^2$$

$$x_f = x_i + v_{ix}t + \tfrac{1}{2}a_x t^2$$

$$v_{fx} = v_{ix} + a_x t$$

$$v_{vx}^2 = v_{ix}^2 + 2a_x\Delta x$$