## Physics 111

Exam \#2

February 13, 2013

Name $\qquad$

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}|=(5 \mathrm{~kg}) \times\left(2 \frac{\mathrm{~m}}{\mathrm{~s}}\right)=10 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}}$
- Give standard SI units with your results.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 4 points and each free-response part is worth 6 points

| Problem \#1 | $/ 28$ |
| :---: | :---: |
| Problem \#2 | $/ 24$ |
| Problem \#3 | $/ 34$ |
| Total | $/ 86$ |

I affirm that I have carried out my academic endeavors with full academic honesty.

1. Consider the arrangement of resistors each $100 \Omega$ connected to a 15 V battery.

a. What are $R_{e q}$ and $I_{\text {total }}$ ?
$\mathrm{R}_{1} \& \mathrm{R}_{2}$ in series $\rightarrow R_{12}=200 \Omega$
$\mathrm{R}_{3} \& \mathrm{R}_{4}$ in series $\rightarrow R_{34}=200 \Omega$
$\mathrm{R}_{12}, \mathrm{R}_{34}, \& \mathrm{R}_{5}$ in parallel, $\frac{1}{R_{12345}}=\frac{1}{R_{12}}+\frac{1}{R_{34}}+\frac{1}{R_{5}}=\frac{5}{200 \Omega} \rightarrow R_{12345}=50 \Omega$
$\mathrm{R}_{12345}, \mathrm{R}_{6}, \mathrm{R}_{7}, \& \mathrm{R}_{8}$ in series $\rightarrow R_{12345678}=R_{\text {eq }}=350 \Omega$
Therefore, by Ohm's law, $I_{\text {total }}=\frac{V}{R_{e q}}=\frac{15 \mathrm{~V}}{350 \Omega}=0.043 \mathrm{~A}=43 \mathrm{~mA}$
b. What are $V_{R_{5}}$ and $V_{R_{8}}$ ?

Since the total current flows through $\mathrm{R}_{8}$, the potential difference across $\mathrm{R}_{8}$ is $V_{R_{8}}=I_{\text {total }} R_{8}=0.043 \mathrm{~A} \times 100 \Omega=4.3 \mathrm{~V}$.

Energy is conserved in the circuit, so we have that $V_{12345}=V_{B}-3 V_{R_{8}}=15 \mathrm{~V}-(3 \times 4.3 \mathrm{~V})=2.1 \mathrm{~V}$.

Elements in parallel have the same potential differences across them, then $V_{R_{5}}=2.1 \mathrm{~V}$.
c. What are $I_{U}, I_{M}$, and $I_{L}$ ? Is charge conserved? Explain.

The currents in each branch are found using Ohm's law. Thus,

$$
\begin{aligned}
& V_{R_{12}}=I_{U} R_{12} \rightarrow I_{U}=\frac{V_{R_{12}}}{R_{12}}=\frac{2.1 \mathrm{~V}}{200 \Omega}=0.0105 \mathrm{~A}=10.5 \mathrm{~mA}, \\
& V_{R_{5}}=I_{M} R_{5} \rightarrow I_{M}=\frac{V_{R_{5}}}{R_{5}}=\frac{2.1 \mathrm{~V}}{100 \Omega}=0.0210 \mathrm{~A}=21 \mathrm{~mA}, \text { and } \\
& V_{R_{34}}=I_{L} R_{34} \rightarrow I_{L}=\frac{V_{R_{34}}}{R_{34}}=\frac{2.1 \mathrm{~V}}{200 \Omega}=0.0105 \mathrm{~A}=10.5 \mathrm{~mA}
\end{aligned}
$$

Adding these currents gives the total current, so charge is conserved.
d. What is the total energy dissipated by the circuit one minute after the battery is connected to the circuit?

$$
P=\frac{E}{t} \rightarrow E=P t=\left(I_{\text {total }}^{2} R_{\text {eq }}\right) t=(0.043 A)^{2} \times 350 \Omega \times 60 s=38.8 \mathrm{~J}
$$

e. What would happen to $I_{\text {total }}$ and the power produced by the battery if a $100 \Omega$ resistor were placed in series between $R_{1}$ and $R_{2}$ as well as placing $100 \Omega$ resistor in series between $R_{3}$ and $R_{4}$ ?

1. $I \quad \downarrow$ and $P \quad \uparrow$.
(2.) $I \downarrow$ and $P \downarrow$.
2. $I \uparrow$ and $P \uparrow$.
3. $I \uparrow$ and $P \downarrow$

Since the effective resistance increases then the total current decreases by Ohm's law. The power produced by the battery decreases using any one of the equations for the power.
2. A current balance is a device with two sets of bars through which a current I can pass. The lower bar is fixed, while the upper bar is able to pivot about the back edge (with the mirror) as shown in the figure. The separation distance $r_{e q}$ between the bars is fixed and this is called the equilibrium separation. When the upper bar is perturbed from equilibrium the experimenter needs to bring the system of bars back into equilibrium by adding masses to the pan attached to the upsper bar.

a. The diagram on the right is a schematic of the upper and lower bars of the current balance. Given the directions of the currents in the upper and lower bars, the direction of the magnetic force on the upper bar, due to the current flowing in the lower bar is
1.) pointing up the plane of the paper.
2. pointing down the plane of the paper.
3. pointing into the paper.
4. pointing out of the paper.
b. The current in the circuit is produced from a 120 V battery (not shown, but the leads are on the left side of the picture) connected to a variable resistor (also not shown). A variable resistor is a resistor whose resistance can be changed. Suppose that the resistance in this circuit can be changed from $1 \Omega$ to $40 \Omega$. What range of masses (in milligrams) would be required to be added to the pan on the upper bar to return the system to an equilibrium separation of $r_{e q}=\frac{1}{2} \mathrm{~cm}$ if the bars have a length $L=30 \mathrm{~cm}$ ?

To balance the system, the upward magnetic force is countered by adding weight to the mass pan. Therefore,
$F_{B}-F_{W}=F_{n e t}=0 \rightarrow I L B=I L\left(\frac{\mu_{0} I}{2 \pi r_{e q}}\right)=\frac{\mu_{0} I^{2} L}{2 \pi r_{e q}}=m g$. And, the current in the wires is given by Ohm's law, so we have $\frac{\mu_{0} I^{2} L}{2 \pi r_{e q}}=\frac{\mu_{0} L}{2 \pi r_{e q}}\left(\frac{V}{R}\right)^{2}=\frac{\mu_{0} L V^{2}}{2 \pi r_{e q} R^{2}}=m g$.
For each resistor we can calculate the mass that needs to be added. For

$$
R=1 \Omega, m=\frac{\mu_{0} L V^{2}}{2 \pi r_{e q} g R^{2}}=\frac{2 \times 10^{-7} \frac{T \cdot m}{A} \times 0.3 \mathrm{~m} \times(120 \mathrm{~V})^{2}}{0.005 \mathrm{~m} \times 9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}} \times(1 \Omega)^{2}}=0.01782 \mathrm{~kg}=17800 \mathrm{mg}
$$

and for $R=40 \Omega$,

$$
m=\frac{\mu_{0} L V^{2}}{2 \pi r_{e q} g R^{2}}=\frac{2 \times 10^{-7} \frac{\mathrm{~T} \cdot \mathrm{~m}}{A} \times 0.3 \mathrm{~m} \times(120 \mathrm{~V})^{2}}{0.005 \mathrm{~m} \times 9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}} \times(40 \Omega)^{2}}=0.0111 \mathrm{~kg}=11.1 \mathrm{mg}
$$

c. Suppose that you go into the lab and use a current balance and take data on the current through the bars versus the mass added to the pan, where the current range is determined from the 120 V power supply and the variable resistance in part b . You then construct the following graph. What is the experimental value of the permeability of free space?


Since

$$
I^{2}=\left(\frac{2 \pi r_{e q} g}{\mu_{0} L}\right) m=\left(850694 \frac{A^{2}}{k g}\right) m \rightarrow \frac{2 \pi r_{e q} g}{\mu_{0} L}=850694 \frac{\mathrm{~A}^{2}}{k g}
$$

$$
\therefore \mu_{0}=\frac{2 \pi r_{e q} g}{850694 \frac{\mathrm{~A}^{2}}{\mathrm{~kg}} L}=\frac{2 \pi \times 0.005 \mathrm{~m} \times 9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}}{850694 \frac{\mathrm{~A}^{2}}{\mathrm{~kg}} \times 0.3 \mathrm{~m}}=3.84 \pi \times 10^{-7} \frac{\mathrm{~T} \cdot \mathrm{~m}}{\mathrm{~A}}
$$

d. Suppose that the 120 V battery that supplies the current to the circuit were turned around. Which of the following quantities would change direction, if any?
1.) $B$ and $I$.
2. $F_{\text {weight }}$ and $B$.
3. $I$ and $F_{B}$.
4. $B$ and $F_{B}$.
5. None of these quantities change direction.
e. Suppose that you replaced the 120 V battery in the current balance experiment with one that is 10 times larger (everything else being the same, like the resistance range and equilibrium separation value) and again you took data on the current and the mass added to return the system to equilibrium. The slope of your curve fit would

1. increase because you are drawing more current in the circuit.
2. decrease as more mass is added to return the system to equilibrium.
3.) remain the same since as the current is increased more mass has to be added to return the system to equilibrium.
3. change, but in a way that cannot be known without performing the experiment and taking data.
4. Consider the circuit shown below where a blue bar (of length 0.25 m ) rotates in the plane of the paper on a low friction wire. The bar passes through the earth's magnetic field (pointing down into the page and perpendicular to the plane of the circuit. The earth's magnetic field is taken as being a constant and has strength of

a. At time $t=0$, the blue bar is positioned as shown at angle $\theta_{i}=0$. The bar is rotating counterclockwise (when viewed from above) at a constant rate of $\omega=12.6 \frac{\mathrm{rad}}{\mathrm{s}}$ (called the angular velocity). At a later time $t>0$, the bar has swept out some angle $\theta_{f}=\Delta \theta>0$. What fraction of the area of a circle $(\Delta A)$ has been swept out at this time $t>0$ ? (Hint: there are $2 \pi$ radians in a circle.)

The fraction of the area of the circle swept out after a time $t>0$ is given by $\Delta A=$ fraction $\times A=\left(\frac{\Delta \theta}{2 \pi}\right) \pi r^{2}=\frac{(\Delta \theta) L^{2}}{2}$.
b. Starting from the definition of Faraday's law, what is the induced potential difference developed across the bar? (Hint: the angular velocity is defined as the change of angle $\theta$ with respect to time, $\frac{\Delta \theta}{\Delta t}$.)

$$
\varepsilon=\left|\frac{\Delta \phi_{B}}{\Delta t}\right|=\left|\frac{\Delta(B A \cos \theta)}{\Delta t}\right|=\left|\frac{B \Delta A}{\Delta t}\right|=\left|\frac{B(\Delta \theta) L^{2}}{2 \Delta t}\right|=\left|\frac{B \omega L^{2}}{2}\right| \text {. Inserting the known }
$$ values, we have that the induced potential difference across the bar is

$$
\varepsilon=\left|\frac{B \omega L^{2}}{2}\right|=\left|\frac{50 \times 10^{-6} T \times 12.6 \frac{\mathrm{rad}}{\mathrm{~s}} \times(0.25 \mathrm{~m})^{2}}{2}\right|=1.97 \times 10^{-5} \mathrm{~V}=19.7 \mu \mathrm{~V} .
$$

c. If the circuit has a resistance of $100 \Omega$, how much current is developed? What is the direction of the current flow in the bar?

The magnitude of the current is given by Ohm's law
$I=\frac{\varepsilon}{R}=\frac{1.97 \times 10^{-5} \mathrm{~V}}{100 \Omega}=1.97 \times 10^{-7} \mathrm{~A}=197 \mathrm{nA}$. The direction of the current flow is to oppose the change in magnetic flux. Since the magnetic flux through the loop of wire is decreasing the induced magnetic field produced by the wire has to point into the page and therefore the current will flow clockwise in the circuit. And, the current will flow from the top of the bar down to the pivot.
d. What is the electric field along the bar?
$E=\frac{\Delta V}{\Delta x}=\frac{1.97 \times 10^{-5} \mathrm{~V}}{0.25 \mathrm{~m}}=7.9 \times 10^{-5} \frac{\mathrm{~V}}{\mathrm{~m}}$ and is points along decreasing electric potentials. Since the current is flowing from the top of the bar toward the pivot, the top of the bar is at the higher electric potential and the pivot is at the lower electric potential. Therefore the electric field points from the top of the bar towards the pivot.
e. What external force would you need to apply the bar so that it rotates at a constant angular velocity?

By the right-hand-rule, the magnetic force opposes the velocity, so the external force would need to point in the direction of the bar's rotation. And if the bar is to rotate at a constant angular velocity then the magnetic force and the external force have to have the same magnitude given by

$$
F_{\text {external }}=F_{B}=I L B=1.97 \times 10^{-7} \mathrm{~A} \times 0.25 \mathrm{~m} \times 50 \times 10^{-6} \mathrm{~T}=2.46 \times 10^{-12} \mathrm{~N}=2.46 \mathrm{pN}
$$

f. Suppose that you have a new bar that is half as long as the original bar. If the angular speed of the bar were twice as great as that of the part a, the induced potential difference would

1. increase.
2. decrease.
3. remain the same.
4. change in a way that cannot be determined since the magnetic force that acts on the bar is not known.

$$
\varepsilon_{\text {new }}=\frac{B \omega_{\text {new }} L_{\text {new }}^{2}}{2}=\frac{B\left(2 \omega_{\text {old }}\right)\left(\frac{L_{\text {old }}^{2}}{4}\right)}{2}=\frac{1}{2}\left(\frac{B \omega_{\text {old }} L_{\text {old }}^{2}}{2}\right)=\frac{\varepsilon_{\text {old }}}{2}
$$

## Physics 111 Equation Sheet

Electric Forces, Fields and Potentials

$$
\begin{aligned}
& \vec{F}=k \frac{Q_{1} Q_{2}}{r^{2}} \hat{r} \\
& \vec{E}=\frac{\vec{F}}{q} \\
& \vec{E}_{Q}=k \frac{Q}{r^{2}} \hat{r} \\
& P E=k \frac{Q_{Q} Q_{2}}{r} \\
& V(r)=k \frac{Q}{r} \\
& E_{x}=-\frac{\Delta V_{B, A}}{\Delta x} \\
& W_{A \rightarrow B}=q \Delta V_{A \rightarrow B}=-q \Delta V_{B \rightarrow A}
\end{aligned}
$$

Magnetic Forces and Fields
$F=q v B \sin \theta$
$F=I l B \sin \theta$
$\tau=N I A B \sin \theta=\mu B \sin \theta$
$P E=-\mu B \cos \theta$
$B=\frac{\mu_{0} I}{2 \pi r}$
$\varepsilon_{\text {induced }}=-N \frac{\Delta \phi_{B}}{\Delta t}=-N \frac{\Delta(B A \cos \theta)}{\Delta t}$
Constants
$g=9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$
$G=6.67 \times 10^{-11} \frac{1 \mathrm{Nm}^{2}}{\mathrm{~kg}^{2}}$
$1 e=1.6 \times 10^{-19} \mathrm{C}$
$k=\frac{1}{4 \pi \varepsilon_{o}}=9 \times 10^{9} \frac{\frac{c}{}^{2}}{N m^{2}}$
$\varepsilon_{o}=8.85 \times 10^{-12} \frac{\mathrm{Nm}{ }^{2}}{\mathrm{c}^{2}}$
$1 \mathrm{eV}=1.6 \times 10^{-19} \mathrm{~J}$
$\mu_{o}=4 \pi \times 10^{-7} \frac{\mathrm{~T}}{\mathrm{I}} \mathrm{A}$
$c=3 \times 10^{8} \frac{\mathrm{~m}}{s}$
$h=6.63 \times 10^{-34} \mathrm{Js}$
$m_{e}=9.11 \times 10^{-31} \mathrm{~kg}=\frac{0.511 \mathrm{MeV}}{c^{2}}$
$m_{p}=1.67 \times 10^{-27} \mathrm{~kg}=\frac{937.1 \mathrm{MeV}}{c^{2}}$
$m_{n}=1.69 \times 10^{-27} \mathrm{~kg}=\frac{948.3 \mathrm{MeV}}{c^{2}}$
$1 \mathrm{amu}=1.66 \times 10^{-27} \mathrm{~kg}=\frac{931.5 \mathrm{MeV}}{c^{2}}$
$N_{A}=6.02 \times 10^{23}$
$A x^{2}+B x+C=0 \rightarrow x=\frac{-B \pm \sqrt{B^{2}-4 A C}}{2 A}$

Electric Circuits
$I=\frac{\Delta Q}{\Delta t}$
$V=I R=I\left(\frac{\rho L}{A}\right)$
$R_{\text {series }}=\sum_{i=1}^{N} R_{i}$
$\frac{1}{R_{\text {parallel }}}=\sum_{i=1}^{N} \frac{1}{R_{i}}$
$P=I V=I^{2} R=\frac{V^{2}}{R}$
$Q=C V=\left(\frac{\kappa \varepsilon_{0} A}{d}\right) V=\left(\kappa C_{0}\right) V \quad v=\frac{1}{\sqrt{\varepsilon \mu}}=\frac{c}{n}$
$P E=\frac{1}{2} Q V=\frac{1}{2} C V^{2}=\frac{Q^{2}}{2 C}$
$Q_{\text {charge }}(t)=Q_{\max }\left(1-e^{-\frac{t}{R C}}\right) \quad \frac{1}{f}=\frac{1}{d_{o}}+\frac{1}{d_{i}}$
$Q_{\text {discharge }}(t)=Q_{\text {max }} e^{-\frac{t}{R C}}$
$C_{p \text { arallel }}=\sum_{i=1}^{N} C_{i}$
$\frac{1}{C_{\text {series }}}=\sum_{i=1}^{N} \frac{1}{C_{i}}$

Light as a Wave
$c=f \lambda=\frac{1}{\sqrt{\varepsilon_{o} \mu_{o}}}$
$S(t)=\frac{\text { energy }}{\text { time } \times \text { area }}=c \varepsilon_{o} E^{2}(t)=c \frac{B^{2}(t)}{\mu_{0}}$
$I=S_{\text {avg }}=\frac{1}{2} c \varepsilon_{o} E_{\max }^{2}=c \frac{B_{\max }^{2}}{2 \mu_{0}}$
$P=\frac{S}{c}=\frac{\text { Force }}{\text { Area }}$
$S=S_{o} \cos ^{2} \theta$
$\theta_{\text {inc }}=\theta_{\text {refl }}$
$n_{1} \sin \theta_{1}=n_{2} \sin \theta_{2}$
$M=\frac{h_{i}}{h_{o}}=-\frac{d_{i}}{d_{o}}$
$M_{\text {total }}=\prod_{i=1}^{N} M_{i}$
$d \sin \theta=m \lambda$ or $\left(m+\frac{1}{2}\right) \lambda$
$a \sin \phi=m^{\prime} \lambda$

Light as a Particle \& Relativity Nuclear Physics

$$
\begin{array}{ll}
E=h f=\frac{h c}{\lambda}=p c & \left.E_{\text {binding }}=\left(Z m_{p}+N m_{n}-m_{\text {rest }}\right)\right)^{2} \\
K E_{\max }=h f-\phi=e V_{\text {stop }} & \frac{\Delta N}{\Delta t}=-\lambda N_{o} \rightarrow N(t)=N_{o} e^{-\lambda t} \\
\Delta \lambda=\frac{h}{m_{e} c}(1-\cos \phi) & A(t)=A_{o} e^{-\lambda t} \\
\gamma=\frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}} & m(t)=m_{o} e^{-\lambda t} \\
\frac{t_{1}}{2}=\frac{\ln 2}{\lambda}
\end{array}
$$

$$
p=\gamma m v
$$

$$
E_{\text {total }}=K E+E_{\text {rest }}=\gamma m c^{2}
$$

$$
E_{\text {total }}^{2}=p^{2} c^{2}+m^{2} c^{4}
$$

Geometry
Circles: $C=2 \pi r=\pi D \quad A=\pi r^{2}$
Triangles: $A=\frac{1}{2} b h$
Misc. Physics 110
Formulae
$\vec{F}=\frac{\Delta \vec{p}}{\Delta t}=\frac{\Delta(m v)}{\Delta t}=m \vec{a}$
$\vec{F}=-k \vec{y}$

$$
E_{r e s t}=m c^{2}
$$

$$
K E=(\gamma-1) m c^{2}
$$

$\vec{F}_{C}=m \frac{v^{2}}{R} \hat{r}$
$W=\Delta K E=\frac{1}{2} m\left(v_{f}^{2}-v_{i}^{2}\right)=-\Delta P E$
$P E_{\text {gravity }}=m g y$
$P E_{\text {spring }}=\frac{1}{2} k y^{2}$

Spheres: $A=4 \pi r^{2} \quad V=\frac{4}{3} \pi r^{3}$
$x_{f}=x_{i}+v_{i x} t+\frac{1}{2} a_{x} t^{2}$
$v_{f x}=v_{i x}+a_{x} t$
$v_{v x}^{2}=v_{i x}^{2}+2 a_{x} \Delta x$

