## Physics 111

## Exam \#2

February 25, 2022

Name $\qquad$

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}|=(5 \mathrm{~kg}) \times\left(2 \frac{\mathrm{~m}}{\mathrm{~s}}\right)=10 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 6 points.

| Problem \#1 | $/ 24$ |
| :---: | :---: |
| Problem \#2 | $/ 24$ |
| Problem \#3 | $/ 24$ |
| Total | $/ 72$ |

I affirm that I have carried out my academic endeavors with full academic honesty.

1. A circular loop of wire with radius $r=0.1 \mathrm{~m}$ and resistance $R=10 \Omega$ is held in place horizontally in a magnetic field $\vec{B}$ directed upward at an angle of $60^{\circ}$ with respect to the vertical, as shown below. The magnetic field in varies in time according to $B(t)=a(1-b t)$, where $a=4 T$ and $b=0.1 s^{-1}$ for $0 \leq t \leq 10 s$.

a. What is the magnitude and direction of the induced current in the wire loop at a time $t_{f}=10 s$ ?
b. What is the energy dissipated in the loop after a time $t_{f}=10 s$ ?
c. What is the magnitude and direction of the electric field induced in the loop at a time $t_{f}=10 s$ ?
d. At a time $t_{f}=10 s$ what are the magnitude and direction of the net force on the right side of the loop, the magnitude and direction of the force on the left side of the loop, and the net force on the loop? Consider a small length $L=0.2 \mathrm{~cm}$ on each side of the wire in your calculations.
2. A current balance is a device for investigating magnetic force on wires and is also used to measure the permeability of free space, $\mu_{0}$. In this system, the lower rail is fixed can cannot move, while the upper rail (with attached mass pan) can move. Current $I$ is flowed through the system and the upper rail moves up away from the lower rail. The experimenter must add mass to the mass pan to return the rails to their original, equilibrium, separation, $r_{e q}$.

a. What is the direction of the current flow in the upper rail so that the upper rail will move up away from the lower rail? Explain your answer fully to earn full credit.
b. Suppose that $I=12 A$ current is turned on and flows through the rails. If the centers of the rails are separated by a distance of $r=1 \mathrm{~cm}$, what is the net magnetic field at the midpoint between the rails? Take into the page as the positive direction for the magnetic field.
c. Suppose that same $I=12 A$ current is on and flows through the rails. How much mass would have to be added to the mass pan to return the system to an equilibrium separation of $r_{e q}=0.5 \mathrm{~mm}$ ? Assume that the upper/lower rails have a radius of $r_{\text {rail }}=0.25 \mathrm{~mm}$, a length $L=30 \mathrm{~cm}$, and are made of aluminum $\rho_{A l}=2700 \frac{k g}{m^{3}}$.
d. This experiment is designed to determine a value for the permeability of free space, $\mu_{0}$. To do this data are taken on the mass added to the pan as a function of the current in the rails. From these data, a plot is made of the mass added to the pan versus the square of the current flowing in the rails. What would be the slope and y-intercept of the plot and explain how you would use this plot to extract a value for the permeability of free space.
3. Topaz is a gemstone with an index of refraction $n_{T}=1.6$. Suppose a topaz gem were cut in such a way that it has the cross-section of an equilateral triangle shown below.

a. What is the critical angle for the topaz/air interface on the right and what is the speed of light in topaz?
b. If light is incident in air onto the upper surface of the topaz at an angle $\theta_{\text {air }}=$ $25^{0}$, will the light be internally reflected from in the gem? If it will not be internally reflected, at what angle will it re-enter the air?
c. Albeit expensive, suppose that topaz was used to make two thin lenses. Lens \#1 has an unknown focal length $f_{1}$ and is placed to the left of lens \#2 (with focal length $f_{2}=48 \mathrm{~mm}$ ) by an amount $D=61 \mathrm{~mm}$. A 2 cm tall object is placed 36 mm to the left of lens \#1 and a real image is seen on a screen 125 mm to the right of lens \#2. What type of lens is lens \#1 and what is its focal length, $f_{1}$ ?
d. What is the size of the image on the screen?

Electrostatics
$F=k \frac{q_{1} q_{2}}{r^{2}}$
$\vec{F}=q \vec{E} ; \quad E_{p c}=k \frac{q}{r^{2}} ; \quad E_{\text {plate }}=\frac{q}{\epsilon_{0} A}$
$E=-\frac{\Delta V}{\Delta x}$
$V=k \frac{q}{r}$
$U_{e}=k \frac{q_{1} q_{2}}{r}=q V$
$W=-q \Delta V=-\Delta U_{e}=\Delta K$
Electric Circuits - Capacitors
$Q=C V ; \quad C=\frac{\kappa \epsilon_{0} A}{d}$
$C_{\text {parallel }}=\sum_{i=1}^{N} C_{i}$
$\frac{1}{C_{\text {series }}}=\sum_{i=1}^{N} \frac{1}{C_{i}}$
$Q_{\text {charging }}(t)=Q_{\max }\left(1-e^{-\frac{t}{\tau}}\right)$
$Q_{\text {discharging }}(t)=Q_{\text {max }} e^{-\frac{t}{\tau}}$
$I(t)=I_{\text {max }} e^{-\frac{t}{\tau}}=\frac{Q_{\text {max }}}{\tau} e^{-\frac{t}{\tau}}$
$\tau=R C$
$U_{C}=\frac{1}{2} q V=\frac{1}{2} C V^{2}=\frac{Q^{2}}{2 C}$
Light as a Wave
$c=f \lambda$
$S(t)=\frac{\text { Energy }}{\text { time } \times \text { Area }}=c \epsilon_{0} E^{2}(t)=c \frac{B^{2}(t)}{\mu_{0}}$
$I=S_{\text {avg }}=\frac{1}{2} c \epsilon_{0} E_{\text {max }}^{2}=c \frac{B_{\max }^{2}}{2 \mu_{0}}$
$P= \begin{cases}\frac{s}{c} ; & \text { absorbed } \\ \frac{2 s}{c} ; & \text { reflected }\end{cases}$
$S=S_{0} \cos ^{2} \theta$
$v=\frac{c}{n}$
$\theta_{\text {incident }}=\theta_{\text {reflected }}$
$n_{1} \sin \theta_{1}=n_{2} \sin \theta_{2}$
$P=\frac{1}{f}=\frac{1}{d_{0}}+\frac{1}{d_{i}}$
$M=\frac{d_{i}}{d_{0}} ; \quad|M|=\frac{h_{i}}{h_{0}}$
$M_{\text {total }}=\prod_{i=1}^{N} M_{i}$

Magnetism
$\vec{F}=q \vec{v} \times \vec{B} \rightarrow F=q v B \sin \theta$
$\vec{F}=I \vec{L} \times \vec{B} \rightarrow F=I L B \sin \theta$
$B=\frac{\mu_{0} I}{2 \pi r}$
$\varepsilon=\Delta V=-N \frac{\Delta \phi_{B}}{\Delta t}$
$\phi_{B}=B A \cos \theta$

Electric Circuits - Resistors
$I=\frac{\Delta Q}{\Delta t}$
$I=n e A v_{d} ; \quad n=\frac{\rho N_{A}}{m}$
$V=I R$
$R=\frac{\rho L}{A}$
$R_{\text {series }}=\sum_{i=1}^{N} R_{i}$
$\frac{1}{R_{\text {parallel }}}=\sum_{i=1}^{N} \frac{1}{R_{i}}$
$P=\frac{\Delta E}{\Delta t}=I V=I^{2} R=\frac{V^{2}}{R}$

Light as a Particle/Relativity
$E=h f=\frac{h c}{\lambda}$
$K_{\text {max }}=h f-\phi$
$\Delta \lambda=\lambda^{\prime}-\lambda=\frac{h}{m c}(1-\cos \phi)$
$\frac{1}{E^{\prime}}=\frac{1}{E}+\frac{(1-\cos \phi)}{E_{\text {rest }}} ; \quad E_{\text {rest }}=m c^{2}$
$\gamma=\frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}}$
$p=\gamma m v$
$E_{\text {total }}=E_{\text {rest }}+K=\gamma m c^{2}$
$K=(\gamma-1) m c^{2}$
$E_{\text {total }}^{2}=p^{2} c^{2}+m^{2} c^{4}$

Nuclear Physics
$N=N_{0} e^{-\lambda t}$
$m=m_{0} e^{-\lambda t}$
$A=A_{0} e^{-\lambda t}$
$A=\lambda N$
$t_{\frac{1}{2}}=\frac{\ln 2}{\lambda}$

## Constants

$g=9.8 \frac{m}{s^{2}}$
$1 e=1.6 \times 10^{-19} \mathrm{C}$
$k=\frac{1}{4 \pi \epsilon_{0}}=9 \times 10^{9 \mathrm{Nm}^{2}} \frac{\mathrm{C}^{2}}{}$
$\epsilon_{0}=8.85 \times 10^{-12} \frac{\mathrm{C}^{2}}{N m^{2}}$
$1 \mathrm{eV}=1.6 \times 10^{-19} \mathrm{~J}$
$\mu_{0}=4 \pi \times 10^{-7 \frac{7 m}{A}}$
$c=3 \times 10^{8} \frac{\mathrm{~m}}{\mathrm{~s}}$
$h=6.63 \times 10^{-34} \mathrm{Js}=4.14 \times 10^{-15} \mathrm{eVs}$
$N_{A}=6.02 \times 10^{23}$
$1 u=1.66 \times 10^{-27} \mathrm{~kg}=931.5 \frac{\mathrm{MeV}}{\mathrm{c}^{2}}$
$m_{p}=1.67 \times 10^{-27} \mathrm{~kg}=937.1 \frac{\mathrm{MeV}}{\mathrm{c}^{2}}$
$m_{n}=1.69 \times 10^{-27} \mathrm{~kg}=948.3 \frac{\mathrm{MeV}}{\mathrm{c}^{2}}$
$m_{e}=9.11 \times 10^{-31} \mathrm{~kg}=0.511 \frac{\mathrm{MeV}}{c^{2}}$

Physics 110 Formulas

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\begin{aligned}
& \vec{F}=m \vec{a} ; \quad F_{G}=\frac{G M_{1} m_{2}}{r^{2}} ; \quad F_{S}=-k y ; \quad a_{c}=\frac{v^{2}}{r} \\
& W=-\Delta U_{g}-\Delta U_{s}=\Delta K \\
& U_{g}=m g y \\
& U_{s}=\frac{1}{2} k y^{2} \\
& K=\frac{1}{2} m v^{2} \\
& \vec{r}_{f}=\vec{r}_{i}+\vec{v}_{i} t+\frac{1}{2} \vec{a} t^{2} \\
& \vec{v}_{f}=\vec{v}_{i}+\vec{a} t \\
& v_{f}^{2}=v_{i}^{2}+2 a_{r} \Delta r
\end{aligned}
$$

## Common Metric Units

nano $(n)=10^{-9}$
micro $(\mu)=10^{-6}$
$\operatorname{milli}(m)=10^{-3}$
centi $(c)=10^{-2}$
kilo $(k)=10^{3}$
mega $(M)=10^{6}$

Geometry/Algebra
$\begin{array}{lll}\text { Circles: } & A=\pi r^{2} & C=2 \pi r=\pi \\ \text { Spheres: } & A=4 \pi r^{2} & V=\frac{4}{3} \pi r^{3}\end{array}$
Triangles: $\quad A=\frac{1}{2} b h$
Quadratics: $\quad a x^{2}+b x+c=0 \rightarrow x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$

## PERIODIC TABLE OF ELEMENTS



