# Physics 111

# Exam #3

# November 6, 2015

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg)\times(2\,\tfrac{m}{s}) = 10\,\tfrac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 3 points and each free-response part is worth 8 points

| | |
|---|---|
| Problem #1 | /19 |
| Problem #2 | /19 |
| Problem #3 | /19 |
| Problem #4 | /19 |
| Total | /76 |

*I affirm that I have carried out my academic endeavors with full academic honesty.*

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1. Suppose that you have two light sources that are incident on the same metal surface. One light source emits photons with wavelength $\lambda_1$ while the other light source emits photons with $\lambda_2 = 0.44\lambda_1$. The first light source produces photoelectrons with kinetic energy $0.77eV$ while the second source produces photoelectrons with energy $3.64eV$.

   a. What are $\lambda_1$, $\lambda_2$, and the work function of the metal, $\phi$ (in $eV$)?

The maximum kinetic energy for each wavelength is given by:

$$KE_1 = \frac{hc}{\lambda_1} - \phi$$

$$KE_2 = \frac{hc}{\lambda_2} - \phi = \frac{hc}{0.44\lambda_1} - \phi$$

Subtracting the two expressions gives us a way to calculate $\lambda_1$. We have

$$KE_2 - KE_1 = \left(\frac{hc}{0.44\lambda_1} - \phi\right) - \left(\frac{hc}{\lambda_1} - \phi\right) = \frac{hc}{\lambda_1}\left(\frac{1}{0.44} - 1\right) = 1.27\frac{hc}{\lambda_1}$$

$$\therefore \lambda_1 = \frac{1.27hc}{KE_2 - KE_1} = \frac{1.27 \times 6.63 \times 10^{-34} Js \times 3 \times 10^8 \frac{m}{s}}{(3.64eV - 0.77eV) \times \frac{1.6 \times 10^{-19} J}{1eV}} = 5.51 \times 10^{-7} m = 551nm$$

Then $\lambda_2 = 0.44\lambda_1 = 0.44 \times 5.51 \times 10^{-9} m = 2.43 \times 10^{-9} m = 243nm$.

Lastly, using either $\lambda_1$ or $\lambda_2$, we calculate the work function of the metal. We have:

$$KE_1 = \frac{hc}{\lambda_1} - \phi \rightarrow \phi = \frac{hc}{\lambda_1} - KE_1$$

$$\therefore \phi = \left(\frac{6.63 \times 10^{-34} Js \times 3 \times 10^8 \frac{m}{s}}{551 \times 10^{-9} m} \times \frac{1eV}{1.6 \times 10^{-19} J}\right) - 0.77eV = 1.49eV$$

b. Suppose instead of the above light sources, an infrared light source ( $\lambda_{IR} = 900nm$ ) is used on the same metal surface and further that the infrared photons have an intensity of $S_{IR}$. Compared to the number of photoelectrons produced using the infrared beam (call this $N_{IR}$ ) to the number of photoelectrons produced using a beam of photons of wavelength $\lambda_1$ (call this $N_1$ ) is

1. 0
2. identically equal to $\frac{N_{IR}}{2}$
3. identically equal to $2N_{IR}$
4. $\alpha N_{IR}$ where $\alpha$ is a constant and $0 \le \alpha \le 1$.
5. $\alpha N_{IR}$ where $\alpha$ is a constant and $\alpha > 1$.

c. We've said in class that using a model of light as a wave that, if the intensity of the beam were not large enough, then all we would have to do is wait long enough for the electron to absorb enough energy and be ejected. (Of course no measurable time lag between the incident photons arrival and the ejection of an electron has ever been measured for photons no matter the incident energy.) However, assuming there was a lag time, how long would we have to wait for an electron to be ejected? Suppose that the metal plate above were placed $1m$ away from a point light source with a power output of $1\frac{J}{s}$ and assume that the ejected photoelectron may collect its energy from a circular area of the plate whose radius is say one atomic radius $r \sim 0.1nm$ and that the light energy is spread uniformly over the metal plate.

The intensity at the electron's location due to the light source is given as:

$$S_e = \frac{P_{\text{det}}}{A_{\text{det}}} = \frac{P_{\text{det}}}{4\pi r^2} = \frac{1\frac{J}{s}}{4\pi(1m)^2} = 0.0796\frac{W}{m^2}.$$

The energy per unit time that the electron needs is

$P_{abs} = \frac{E_{\text{min}}}{t} = S_e A_e = 0.076\frac{W}{m^2} \times \pi\left(0.1\times10^{-9}m\right)^2 = 2.5\times10^{-21}W$, where the minimum energy that the electron needs to absorb is the work function of the metal $E_{\text{min}} = \phi = 1.49eV \times \frac{1.6\times10^{-19}J}{1eV} = 2.38\times10^{-19}J$.

To determine the time we'd need to wait, we divide the minimum energy by the power absorbed. We get:

$$t = \frac{E_{\text{min}}}{P_{abs}} = \frac{2.38\times10^{-19}J}{2.5\times10^{-21}\frac{J}{s}} = 95.4s = 1.6\,\text{min}$$

2. Consider a γ-ray beam produced from the radioactive decay of a $^{137}Cs$ source, with decay energy $E_\gamma = 661keV$, incident on stationary electrons in a carbon target. The radiation from this beam is scattered from free electrons in the target and is viewed at a detector oriented at an angle of $90^0$ with respect to the direction of propagation of the incident beam.

   a. What are the wavelength, momentum and energy (in $keV$) of the scattered γ-rays?

   From the incident energy we calculate the incident wavelength to be

$$E_i = \frac{hc}{\lambda_i} \rightarrow \lambda_i = \frac{hc}{E_i} = \frac{6.63\times10^{-34}\,Js\times3\times10^8\,\frac{m}{s}}{661\times10^3\,eV\times\frac{1.6\times10^{-19}\,J}{1eV}} = 1.88\times10^{-12}\,m\,.$$

   To calculate the scattered wavelength we use the Compton shift formula.

$$\lambda' = \lambda + \frac{h}{mc}(1-\cos\phi) = 1.88\times10^{-12}\,m + \frac{6.63\times10^{-34}\,Js}{9.11\times10^{-31}\,kg\times3\times10^8\,\frac{m}{s}}(1-\cos 90)$$

$$\lambda' = 4.306\times10^{-12}\,m$$

   The momentum of the scattered photon is

$$p = \frac{h}{\lambda'} = \frac{6.63\times10^{-34}\,Js}{4.306\times10^{-12}\,m} = 1.54\times10^{-22}\,\frac{kgm}{s}\,.$$

   The energy of the scattered photon is

$$E' = pc = \frac{hc}{\lambda'} = 1.54\times10^{-22}\,\frac{kgm}{s}\times3\times10^8\,\frac{m}{s}$$

$$= 4.62\times10^{-14}\,J\times\frac{1eV}{1.6\times10^{-16}\,J} = 2.89\times10^5\,eV = 289keV$$

b. What is the speed of the recoiling electron (as a fraction of the speed of light) after it interacted with the γ-rays?

Using conservation of energy we have

$$E = E' + KE_e \rightarrow KE_e = E - E' = 661keV - 289keV = 372keV$$

$$KE_e = (\gamma - 1)mc^2 = (\gamma - 1)\left(511\tfrac{keV}{c^2}\right)c^2 = 372keV$$

$$\therefore \gamma = 1 + \frac{372keV}{511keV} = 1.728 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\rightarrow v = \sqrt{1 - \frac{1}{\gamma^2}}c = \sqrt{1 - \frac{1}{(1.728)^2}}c = 0.816c$$

c. Suppose that instead of photons scattering from electrons at $90^0$ to the incident beam direction, that the photons scatter at an angle $\theta$ from a stationary *proton*. If the energy of the incident photon is $E$, the energy of the scattered photon is this case is

1. $E$
2. $\frac{E}{2}$
3. $\frac{E^2}{mc^2}$
4. $\frac{Emc^2}{E + (1 - \cos\phi)mc^2}$
5. $\frac{Emc^2(1 - \cos\phi)}{E + mc^2}$
6. $\frac{Emc^2}{E(1 - \cos\phi) + mc^2}$

3. Sea sponges in the family *Euplectella* (Venus Flower Basket) have at their base long needle-like structures called spicules, made of a combination of the mineral silica and organic compounds. Scientists have investigated whether these spicules can act as optical fibers. Each spicule consists of an inner cylindrical core made of the mineral silica, surrounded by an outer cladding-like coating made of a mixture of silica and organic compounds. All this is illustrated in Figures 1 through 3 below. In figure 3, *SS* is the outer coating while *CC* is the center core. The tough, flexible outer coating allows the spicules to bend into very small angles without cracking, unlike current optical fibers, so scientists are investigating whether they can help in designing more flexible optical fibers for medicine. Table 1 below gives some useful information for the problems below.

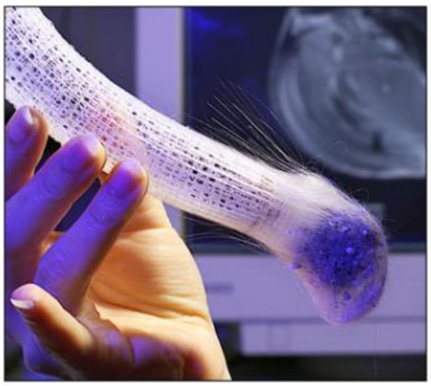

Figure 1: Photo of the sea sponge *Euplectella*. The fiber optic-like spicules are glass-like fibers projecting from the base. (Suzanne Kane/Haverford College)

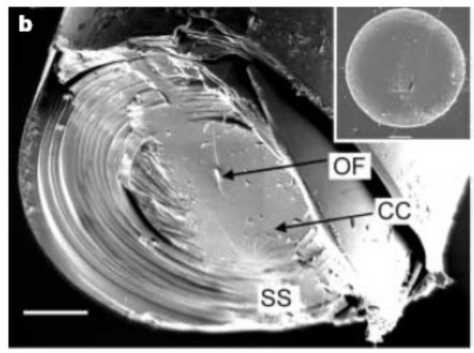


Figure 2: Cross-sectional image showing the central silica core and outer coating. (Joanna Aizenberg/Lucent Laboratories)

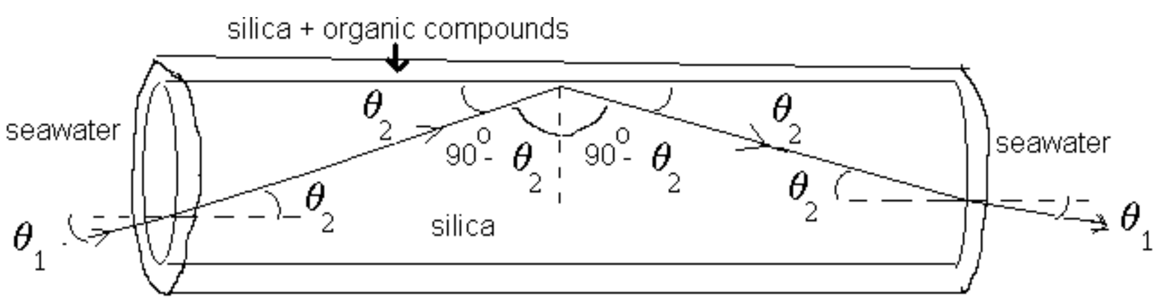


Figure 3: Representation of a ray of light entering one end of the spicule's silica center core, reflecting from the interface with the outer coating and exiting the other end. (The angles shown are not necessarily realistic for these values of index of refraction.)

| Medium | Index of refraction |
|---|---|
| Seawater | 1.34 |
| Silica (inner core of spicule) | 1.46 |
| Silica + organic compounds (outer layer of spicule) | 1.43 |

Table 1: Some useful information about the optical properties of this system.

a. Can a spicule act as an optical fiber? In other words can light entering the spicule undergo total internal reflection? If the fiber can support total internal reflection, compute the critical angle for the spicule. Explain your reasoning with words and show your calculations.

Yes the spicule can act as an optical fiber because the light is going from a higher refractive index material to a lower refractive index material. When the light enters the lower refractive index material it bends away from the normal to an angle larger than the angle of incidence in the lower refractive index material. Thus for some angle of incidence (the critical angle) the light that enters the second material will enter at an angle of $90^0$. At an angle larger than the critical angle the light will be totally internally reflected.

Applying the law of refraction at the upper surface we have:

$$n_{spiclule} \sin\theta_{spicule} = n_{spiclule+organics} \sin\theta_{spicule+organics}$$

$$\sin\theta_{spicule} = \left(\frac{n_{spiclule+organics}}{n_{spiclule}}\right)\sin\theta_{spicule+organics} = \left(\frac{1.43}{1.46}\right)\sin 90 = 0.9795$$

$$\theta_{spicule} = \theta_{critical} = \sin^{-1}(0.9795) = 78.4^0$$

b. Looking at the drawing in Figure 3 above, derive a value for the angle $\theta_2$ then compute the angle, $\theta_1$, at which light *exits* the spicule. Is the value for $\theta_1$ the largest or smallest angle that light can **enter** the spicule and still undergo total internal reflection? Justify your answer.

$$\theta_{spicule} = \theta_{critical} = 90^0 - \theta_2$$

$$\rightarrow \theta_2 = 90^0 - \theta_{critical} = 90^0 - 78.4^0 = 11.6^0$$

Applying the law of refraction on the front surface we have:

$$n_{seawater} \sin\theta_{seawater} = n_{spiclule} \sin\theta_{spicule}$$

$$\sin\theta_{seawater} = \left(\frac{n_{spiclule}}{n_{seawater}}\right)\sin\theta_{spicule} = \left(\frac{1.46}{1.34}\right)\sin 11.2 = 0.2116$$

$$\theta_{seawater} = \sin^{-1}(0.2116) = 12.2^0$$

This is the largest angle possible. If $\theta_{seawater}$ increases then $\theta_2$ increases and then the angle of incidence on the upper surface will decrease to an angle below the critical angle and thus total internal reflection will be lost.

c. When light strikes a plane boundary between two different optical media, which of the following cannot occur?
   1. There is a reflected beam but no transmitted beam.
   2. There is a transmitted beam but no reflected beam.
   3. There are both transmitted and reflected beams.
   4. The speed of light increases upon entering the second medium.

4. Suppose that you have two converging lenses that form a compound lens system. Lens #1 is on the left and has a focal length of $f_{c1} = 20cm$ while lens #2 is on the right has a focal length of $f_{c2} = 40cm$. A distance of $D = 75cm$ separates the lenses and a $1.0cm$ tall object is placed to the left of the lens #1 a distance of $d_{o1} = 15cm$.

   a. At what distance, with respect to lens #1 will the final image be located and what will be the properties of that image with respect to the original $1.0cm$ tall object?

   Lens #1: The image of the object in the first lens is given by the thin lens equation:
   $\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{f_{c1}} \rightarrow d_{i1} = \left(\frac{1}{f_{c1}} - \frac{1}{d_{o1}}\right)^{-1} = \left(\frac{1}{20cm} - \frac{1}{15cm}\right)^{-1} = -60cm$. Or $60cm$ to the left of the first lens, a virtual image.

   To find the final image location we need to determine the object distance for the first image. The object distance is $d_{o2} = d_{i1} + D = 60cm + 75cm = 135cm$.

   The final image distance is
   $\frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f_{c2}} \rightarrow d_{i2} = \left(\frac{1}{f_{c2}} - \frac{1}{d_{o2}}\right)^{-1} = \left(\frac{1}{40cm} - \frac{1}{135cm}\right)^{-1} = 56.8cm$.

   Therefore with respect to lens #1 the final image distance is
   $d_{if} = d_{i2} + D = 56.8cm + 75cm = 131.8cm$.

   The final image height is determined from the magnification:
   $$M_{total} = M_1 M_2 = \frac{h_i}{h_o} \rightarrow h_i = M_1 M_2 h_o = \left(\frac{-d_{i1}}{d_{o1}}\right)\left(\frac{-d_{i2}}{d_{o2}}\right)h_o$$
   $$\therefore h_i = \left(\frac{-d_{i1}}{d_{o1}}\right)\left(\frac{-d_{i2}}{d_{o2}}\right)h_o = \left(\frac{-(-60cm)}{15cm}\right)\left(\frac{-56.8cm}{135cm}\right)\times 1cm = -1.68cm$$

   Thus the final image is real, magnified by a factor of 1.68 times and inverted with respect to the original object.

b. In lab you developed a formula for two lenses in contact, namely $\frac{1}{f_{12}} = \frac{1}{f_1} + \frac{1}{f_2}$, where $f_{12}$ is the focal length of the two lens system and $f_1$ and $f_2$ are the focal lengths respectively of the two lenses (#1 and #2) in the system. This formula works for any two-lens system. What is the total power of the two-lens system?

1. The total power is $P_{total} = P_1 P_2$.
2. The total power is $P_{total} = \frac{P_2}{P_1}$.
3. The total power is $P_{total} = \frac{P_1}{P_2}$.
4. The total power is $P_{total} = P_1 + P_2$.
5. None of the above is the correct formula for total power.

c. Suppose that lens #1 from *part a* were replaced with a diverging lens of unknown focal length. Leaving the object in its original location, the separation between the two lenses the same, and lens#2 the same, a real image is produced at a location of $76.7cm$ to the right of lens #2. What is the focal length of the diverging lens?

Lens #2: The location of the object that produced the final real image is found using the second lens and the thin lens equation and is:

$$\frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f_c} \rightarrow d_{o2} = \left(\frac{1}{f_c} - \frac{1}{d_{i2}}\right)^{-1} = \left(\frac{1}{40cm} - \frac{1}{76.7cm}\right)^{-1} = 83.6cm .$$

We need to find the location of image with respect to the diverging lens. We know that $d_{o2} = d_{i1} + D \rightarrow d_{i1} = d_{o2} - D = 83.6cm - 75cm = 8.6cm$.

To determine the focal length of the diverging lens we use the thin lens equation again. We have:

$$\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{f_D} \rightarrow f_D = \left(\frac{1}{d_{o1}} - \frac{1}{d_{i1}}\right)^{-1} = \left(\frac{1}{15cm} - \frac{1}{8.6cm}\right)^{-1} = -20.2cm,$$ which is negative as it should be.

# Physics 111 Equation Sheet

## Electric Forces, Fields and Potentials

$$\vec{F} = k\frac{Q_1Q_2}{r^2}\hat{r}$$

$$\vec{E} = \frac{\vec{F}}{q}$$

$$\vec{E}_Q = k\frac{Q}{r^2}\hat{r}$$

$$PE = k\frac{Q_1Q_2}{r}$$

$$V(r) = k\frac{Q}{r}$$

$$E_x = -\frac{\Delta V}{\Delta x}$$

$$W = -q\Delta V$$

## Magnetic Forces and Fields

$$F = qvB\sin\theta$$

$$F = ILB\sin\theta$$

$$\tau = NIAB\sin\theta = \mu B\sin\theta$$

$$PE = -\mu B\cos\theta$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\varepsilon_{induced} = -N\frac{\Delta\phi_B}{\Delta t} = -N\frac{\Delta(BA\cos\theta)}{\Delta t}$$

## Constants

$g = 9.8\,\frac{m}{s^2}$

$1e = 1.6\times10^{-19}\,C$

$k = \frac{1}{4\pi\varepsilon_o} = 9\times10^{9}\,\frac{Nm^2}{C^2}$

$\varepsilon_o = 8.85\times10^{-12}\,\frac{C^2}{Nm^2}$

$1eV = 1.6\times10^{-19}\,J$

$\mu_o = 4\pi\times10^{-7}\,\frac{Tm}{A}$

$c = 3\times10^{8}\,\frac{m}{s}$

$h = 6.63\times10^{-34}\,Js$

$m_e = 9.11\times10^{-31}\,kg = \frac{0.511MeV}{c^2}$

$m_p = 1.67\times10^{-27}\,kg = \frac{937.1MeV}{c^2}$

$m_n = 1.69\times10^{-27}\,kg = \frac{948.3MeV}{c^2}$

$1amu = 1.66\times10^{-27}\,kg = \frac{931.5MeV}{c^2}$

## Electric Circuits

$$I = \frac{\Delta Q}{\Delta t}$$

$$V = IR = I\left(\frac{\rho L}{A}\right)$$

$$R_{series} = \sum_{i=1}^{N} R_i$$

$$\frac{1}{R_{parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$$

$$P = IV = I^2R = \frac{V^2}{R}$$

$$Q = CV = \left(\frac{\kappa\varepsilon_0 A}{d}\right)V = (\kappa C_0)V$$

$$PE = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}$$

$$Q_{charge}(t) = Q_{max}\left(1 - e^{-\frac{t}{RC}}\right)$$

$$Q_{discharge}(t) = Q_{max}e^{-\frac{t}{RC}}$$

$$C_{parallel} = \sum_{i=1}^{N} C_i$$

$$\frac{1}{C_{series}} = \sum_{i=1}^{N}\frac{1}{C_i}$$

## Light as a Particle & Relativity

$$E = hf = \frac{hc}{\lambda} = pc$$

$$KE_{max} = hf - \phi = eV_{stop}$$

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\phi)$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$p = \gamma mv$$

$$E_{total} = KE + E_{rest} = \gamma mc^2$$

$$E_{total}^2 = p^2c^2 + m^2c^4$$

$$E_{rest} = mc^2$$

$$KE = (\gamma - 1)mc^2$$

## Geometry

*Circles*: $C = 2\pi r = \pi D \qquad A = \pi r^2$

*Triangles*: $A = \tfrac{1}{2}bh$

*Spheres*: $A = 4\pi r^2 \qquad V = \tfrac{4}{3}\pi r^3$

## Light as a Wave

$$c = f\lambda = \frac{1}{\sqrt{\varepsilon_o\mu_o}}$$

$$S(t) = \frac{energy}{time \times area} = c\varepsilon_o E^2(t) = c\frac{B^2(t)}{\mu_0}$$

$$I = S_{avg} = \tfrac{1}{2}c\varepsilon_o E_{max}^2 = c\frac{B_{max}^2}{2\mu_0}$$

$$P = \frac{S}{c} = \frac{Force}{Area}$$

$$S = S_o\cos^2\theta$$

$$v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{n}$$

$$\theta_{inc} = \theta_{refl}$$

$$n_1\sin\theta_1 = n_2\sin\theta_2$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$M_{total} = \prod_{i=1}^{N} M_i$$

$$d\sin\theta = m\lambda \text{ or } \left(m + \tfrac{1}{2}\right)\lambda$$

$$a\sin\phi = m'\lambda$$

## Nuclear Physics

$$E_{binding} = \left(Zm_p + Nm_n - m_{rest}\right)c^2$$

$$\frac{\Delta N}{\Delta t} = -\lambda N_o \rightarrow N(t) = N_o e^{-\lambda t}$$

$$A(t) = A_o e^{-\lambda t}$$

$$m(t) = m_o e^{-\lambda t}$$

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

## Misc. Physics 110 Formulae

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\Delta(m\vec{v})}{\Delta t} = m\vec{a}$$

$$\vec{F} = -k\vec{y}$$

$$\vec{F}_C = m\frac{v^2}{R}\hat{r}$$

$$W = \Delta KE = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right) = -\Delta PE$$

$$PE_{gravity} = mgy$$

$$PE_{spring} = \tfrac{1}{2}ky^2$$

$$x_f = x_i + v_{ix}t + \tfrac{1}{2}a_x t^2$$

$$v_{fx} = v_{ix} + a_x t$$

$$v_{f}^2 = v_{i}^2 + 2a\Delta x$$