Physics 111

Exam #3

November 3, 2023

Name____________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $\left|\vec{p}\right| \approx m\left|\vec{v}\right| = (5kg)\times\left(2\,\frac{m}{s}\right) = 10\,\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 6 points.

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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1. Suppose that you have a $251m$ long piece of copper wire ($\rho_{Cu} = 1.68 \times 10^{-8}\Omega m$). The copper wire has a $4mm$ diameter.

   a. What is the resistance of this piece of copper wire and how many continuous circular coils of wire could you wind from this piece of wire if you wanted each coil to have a $40cm$ diameter? For the number of coils, round your answer to the nearest whole number.

$$R = \frac{\rho L}{A} = \frac{1.68 \times 10^{-8}\Omega m \times 251m}{\pi(2 \times 10^{-3}m)^2} = 0.34\Omega$$

$$N = \frac{L}{2\pi R} = \frac{251m}{2\pi \times 0.2m} = 199.7 = 200$$

   b. Suppose that the circular coils of wire were made into a $40cm$ diameter belt that could be used to monitor the breathing of a patient in the hospital. The circular set of coils of N turns (found in part a) is slid around a patient's chest. The patient is lying horizontally on a table that is oriented north-south, with the patient's head pointing north and their feet pointing south. In this location, the earth's magnetic field (of magnitude $50\mu T$) has components that point both north and vertically down into the ground at an angle of 50^0 (measured with respect to the normal of the coil of wire which points north). The patient inhales and momentarily holds their breath before exhaling. During the inhalation phase, which takes a time of $2s$, the diameter of the belt increases to $41cm$ in diameter. What is the magnitude and direction of the induced current in the coils of wire? To determine the direction assume you are standing at the patient's feet and looking north.

$$\varepsilon = \left|-N\frac{\Delta\phi_B}{\Delta t}\right| = NB\cos\theta\frac{\Delta A}{\Delta t} = NB\pi\cos\theta\frac{\left[r_f^2 - r_i^2\right]}{\Delta t}$$

$$\varepsilon = \frac{200 \times \pi \times 50 \times 10^{-6}T\cos 50}{2s}[(0.205m)^2 - (0.2m)^2]$$

$$\varepsilon = 2.05 \times 10^{-5}V$$

$$I = \frac{\varepsilon}{R} = \frac{2.05 \times 10^{-5}V}{0.34\Omega} = 6 \times 10^{-5}A = 60\mu A$$

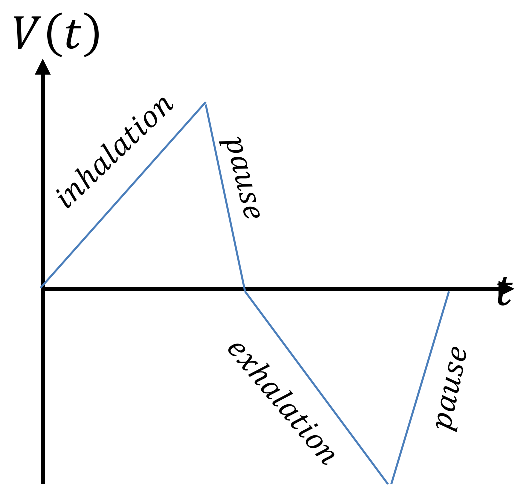
To undo the increase in magnetic flux, the current in the coils needs to flow counterclockwise.

c. Over the inhalation phase, how much energy is dissipated as heat in the coils of wire?

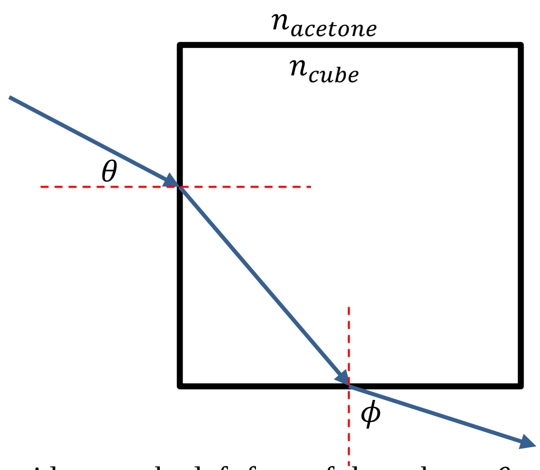
$$P = \frac{\Delta E}{\Delta t} = I^2 R \rightarrow \Delta E = P\Delta t = I^2 R \Delta t = (6 \times 10^{-5} A)^2 \times 0.34\Omega \times 2s$$

$$\Delta E = 2.5 \times 10^{-9} J = 2.5 nJ$$

d. The patient continuously inhales and exhales pausing momentarily between each inhalation and exhalation. On the axes below, what would an approximate voltage versus time graph look like for one breathing cycle (inhale – hold – exhale – hold)? To earn full credit, you need to explain the plot and why you drew it as you did. Make sure to note which parts of the plot are the inhalation, holding, exhalation and holding. This plot is what the breathing monitor would show.



2. A small cube of material has sides of length $L = 20cm$. The cube of material is submerged in acetone where the speed of light in acetone is $v_{acetone} = 0.73c$, where c is the speed of light.



a. The light is incident on the left face of the cube at $\theta = 65^0$ and exits the from the lower face at $\phi = 75^0$. What are the indices of refraction of acetone and of the cube material? Hint: $\sin(90 - \delta) = \cos\delta$.

For the acetone: $n_a = \frac{c}{v_a} = \frac{c}{0.73c} = 1.37$

Left face: $n_a \sin\theta = n_C \sin\delta$

Bottom face: $n_C \sin(90 - \delta) = n_C \cos\delta = n_a \sin\phi$

Dividing the two expressions we have:

$$\frac{n_a \sin\theta}{n_a \sin\phi} = \frac{\sin 65}{\sin 75} = \frac{n_C \sin\delta}{n_C \cos\delta} = \tan\delta \rightarrow \tan\delta = 0.9383 \rightarrow \delta = 43.2^0$$

Then from the left face:

$$n_a \sin\theta = n_C \sin\delta \rightarrow n_c = \left(\frac{\sin\theta}{\sin\delta}\right) n_a = \left(\frac{\sin 65}{\sin 43.2}\right) \times 1.37 = 1.81$$

b. Suppose that the light that was incident on the cube submerged in acetone was blue with a wavelength λ_B. Of the following scenarios below, explain fully which can or cannot happen. Simply saying it can or cannot, will earn no credit, even if the choice was correct.

1. For the light that was incident on the cube submerged in acetone, there is a reflected beam from the cube but no transmitted beam of light into the cube.

   This cannot happen. There will be a reflection, but since the index of refraction of the cube is larger than the index of refraction of the acetone, the light will enter the cube from the acetone.

2. For the light that was incident on the cube submerged in acetone, there is a transmitted beam of light into the cube but no reflected beam of light from the cube.

   This cannot happen. Whenever light strikes an interface, it will always be reflected, whether it transmits or not. So, the light has to reflect.

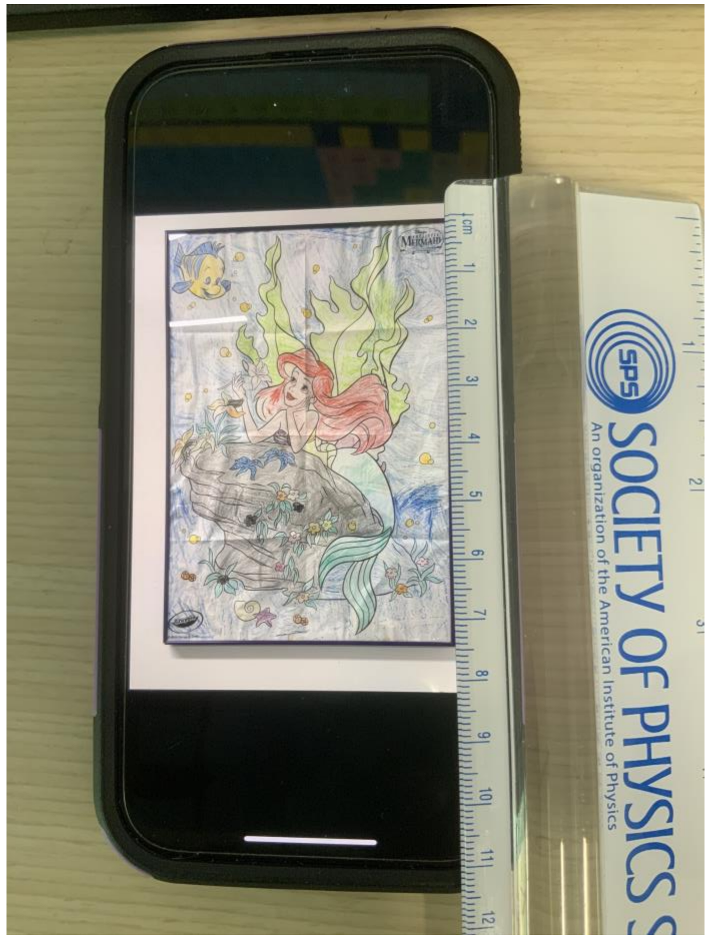
3. If the blue light were to be transmitted into the cube from the acetone its speed would increase.

   The speed of light in a material is given by $v_{material} = \frac{c}{n_{material}}$.
   For the acetone we have $v_a = 0.73c$ and for the cube $v_c = \frac{c}{n_c} = \frac{c}{1.81} = 0.55c$.
   Thus, the speed decreases as it enters the cube from the acetone.

4. If the blue light were to be transmitted into the cube from the acetone, its frequency would increase.

   This cannot happen. The frequency of the light remains constant as it crosses between the acetone and the cube. This is to make the electric and magnetic fields continuous across the interface between the two materials. If the frequency were not constant, then the could leave from a physically different location than it was incident.

c. The newest iPhone (the iPhone15) can take amazing photographs. Suppose that you are standing at a distance of $0.9m$ from a priceless piece of art that is $99cm$ tall and you take a picture of the art with your iPhone15. The photograph that you took is shown below. What is the focal length of the camera on the iPhone15 and what explain what type of lens is in the phone?



$$M = \frac{h_i}{h_o} = \frac{d_i}{d_o} \rightarrow d_i = Md_o = \left(\frac{h_i}{h_o}\right) d_o = \left(\frac{7.4cm}{99cm}\right) d_o = 0.075 d_o$$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} + \frac{1}{Md_o} = \frac{1}{f} \rightarrow \frac{1}{f} = \frac{M+1}{Md_o} \rightarrow f = \left(\frac{M}{M+1}\right) d_o = \left(\frac{0.075}{1+0.075}\right) \times 0.9m$$

$f = 0.063m = 6.3cm$ and since is positive, the lens is converging.

d. We've been using an equation in class that relates the object and image distances to the focal length of the lens. This called the thin lens equation and was first written algebraically by Sir Edmond Halley (of the comet fame) around 1693. Not to be outdone, Sir Isaac Newton also wrote a version of the thin lens equation (around 1694) that related the distance from the focal point of the lens to the object location (x), the distance from the focal point of the lens to the image location (x') to the focal length of the lens f by $xx' = f^2$. Using these definitions, show that the Newtonian form of the thin lens equation can be given by $xx' = f^2$.

From the problem statement

$$d_o = f + x$$

$$d_i = f + x'$$

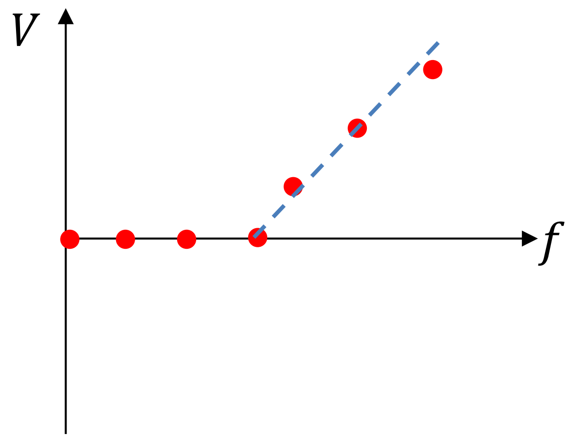
The thin lens equation of Halley:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \rightarrow \frac{1}{f + x} + \frac{1}{f + x'} = \frac{1}{f}$$

$$\frac{1}{f} = \frac{f + x' + f + x}{(f + x)(f + x')} = \frac{2f + x + x'}{f^2 + fx' + fx + xx'}$$

$$2f^2 + fx + fx' = f^2 + fx' + fx + xx' \rightarrow xx' = f^2$$

3. Monochromatic (one color) light of frequency f shines on a metal. The frequency of the light can be varied, and the maximum kinetic energy of the electrons ejected from the metal surface is measured by recording the stopping voltage of a circuit needed to stop the ejected electrons from striking the collector. The data collected were plotted below and a fit to the data gives an equation of the form $f = AV + B$, where A and B are experimental constants.



a. Explain the features plot above making sure to identify all the major parts. In addition, what is the physical interpretation of the coefficients A and B and what do they correspond to in the figure?

$K = hf - \phi \rightarrow eV_{stop} = hf - \phi \rightarrow V_{stop} = \frac{h}{e}f - \frac{\phi}{e}$

Rearranging the given equation to match the graph we have: $V = \frac{1}{A}f - \frac{B}{A}$.

Comparing the two equations we see that through the slopes of the line we can determine the coefficient A

$\rightarrow \frac{h}{e} = \frac{1}{A} \rightarrow A = \frac{e}{h}$

Comparing the two equations we see that through the intercepts of the line we can determine the coefficient B which is the minimum frequency that can eject electrons.

$\rightarrow \frac{\phi}{e} = \frac{B}{A} \rightarrow B = \frac{\phi}{e}A = \frac{\phi}{e} \times \frac{e}{h} = \frac{\phi}{h} = f_{min}$

The features of the graph show that there is a minimum frequency needed to eject electrons. Any frequency below this no electrons are ejected and thus no voltage is needed to stop them. This is the horizontal portion on the plot. The minimum frequency on the x-axis is where the plot begins to rise upward at a constant rate. Above this frequency electrons are ejected, and a voltage is needed to stop them from striking the collector. The slope of the line is Planck's constant divided by the elementary charge and there is a linear relationship between the energy of the ejected electron and the frequency of the light incident on the metal surface.

b. Suppose x-rays of frequency $f = 3 \times 10^{19} s^{-1}$ were shown onto a piece of samarium metal ($\phi = 2.73 eV$). What voltage would you need to set to stop the ejected electrons from striking the collector and what was the speed of the ejected electrons expressed as a fraction of the speed of light?

$$K = hf - \phi \rightarrow eV_{stop} = hf - \phi \rightarrow V_{stop} = \frac{h}{e} f - \frac{\phi}{e}$$

$$V_{stop} = \frac{6.63 \times 10^{-34} Js \times 3 \times 10^{19} s^{-1}}{1.6 \times 10^{-19} C} - \frac{2.73 eV}{e} = 1.24 \times 10^{5} V - 2.73 V$$

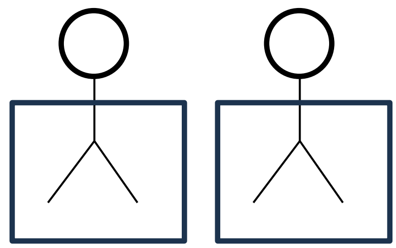
$$V_{stop} = 1.24 \times 10^{5} V$$

$$K = (\gamma - 1) mc^2 \rightarrow \gamma = \frac{K}{mc^2} + 1 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \rightarrow v = \sqrt{1 - \frac{1}{\gamma^2}}\, c$$

$$\gamma = \frac{1.24 \times 10^5 eV}{\left(5.11 \times 10^5 \frac{eV}{c^2}\right) c^2} + 1 = 1.243$$

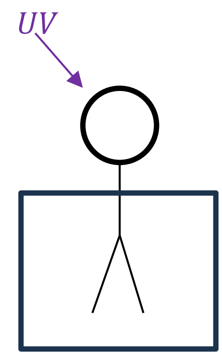
$$v = \sqrt{1 - \frac{1}{(1.243)^2}}\, c = 0.594 c$$

c. X-rays are not an ideal light source to conduct a photoelectric effect experiment since they pose an exposure and radiation hazard to the users/experimenters. Instead, suppose that you have some other light sources available and that one of those was an ultraviolet light source with frequency f_{UV} while another was a green light source with frequency f_g, where $f_g < f_{UV}$. Consider two individual identical negatively charged electroscopes shown below, where the leaves are initially separated by the same amount due to the electrostatic repulsion between the negative charges on the leaves. Now, consider the situations described below and provide an explanation for the observations seen.



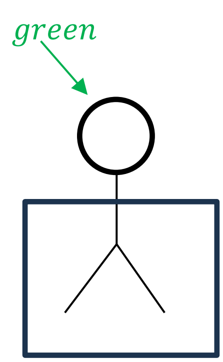
1. The ultraviolet light is shone onto one electroscope and the leaves were observed *to move closer together*. Using as many physics ideas as possible, explain how this could happen.

   The frequency of the UV light must be above the minimum frequency needed to eject electrons from the metal surface and in fact electrons are ejected. This takes negative charges out of the system and since there is less negative charge on the leaves, there is less electric force of repulsion and the leaves come together.



2. The green light is shone onto the other electroscope and the leaves are observed *not to move*. Using as many physics ideas as possible, explain how this could happen.

   The frequency of the green light (which is lower than that of the UV light) must be below the minimum frequency needed to eject electrons from the metal and thus no electrons are ejected. Since no electrons are ejected the net charge on the leaves does not change and neither does the electric force of repulsion and the leaves do not move.



d. Suppose that after the experiment, you notice that the intensity of the ultraviolet light source was greater than the intensity of the green light source. Wondering if this difference matters, you increase the intensity of the green light source until it matches that of the ultraviolet light source and redo the experiment. Fully explain what happens to the electroscope and why when the higher intensity green light is shown onto the system.

   Nothing will happen to the leaves of the electroscope. The intensity of the light is proportional to the number of photons of the light and each photon has the same energy. Changing the intensity of the light changes the number of photons incident. If the energy of the photons that are incident is below that needed to eject electrons from the metal, then no electrons will be ejected, no matter the number incident.

Physics 111 Formula Sheet

Electrostatics

$F = k\frac{q_1 q_2}{r^2}$

$\vec{F} = q\vec{E}; \quad E_{pc} = k\frac{q}{r^2}; \quad E_{plate} = \frac{q}{\epsilon_0 A}$

$E = -\frac{\Delta V}{\Delta x}$

$V_{pc} = k\frac{q}{r}$

$U_e = k\frac{q_1 q_2}{r} = qV$

$W = -q\Delta V = -\Delta U_e = \Delta K$

Electric Circuits - Capacitors

$Q = CV; \quad C = \frac{\kappa \epsilon_0 A}{d}$

$C_{parallel} = \sum_{i=1}^{N} C_i$

$\frac{1}{C_{series}} = \sum_{i=1}^{N} \frac{1}{C_i}$

$Q_{charging}(t) = Q_{max}\left(1 - e^{-\frac{t}{\tau}}\right)$

$Q_{discharging}(t) = Q_{max} e^{-\frac{t}{\tau}}$

$I(t) = I_{max} e^{-\frac{t}{\tau}} = \frac{Q_{max}}{\tau} e^{-\frac{t}{\tau}}$

$\tau = RC$

$U_C = \frac{1}{2}qV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$

Light as a Wave

$c = f\lambda$

$S(t) = \frac{\text{Energy}}{\text{time}\times\text{Area}} = c\epsilon_0 E^2(t) = c\frac{B^2(t)}{\mu_0}$

$I = S_{avg} = \frac{1}{2}c\epsilon_0 E_{max}^2 = c\frac{B_{max}^2}{2\mu_0}$

$P = \begin{cases} \frac{S}{c}; & \text{absorbed} \\ \frac{2S}{c}; & \text{reflected} \end{cases}$

$S = S_0 \cos^2 \theta$

$v = \frac{c}{n}$

$\theta_{\text{incident}} = \theta_{\text{reflected}}$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$P = \frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i}$

$M = -\frac{d_i}{d_0}; \quad |M| = \frac{h_i}{h_0}$

Magnetism

$\vec{F} = q\vec{v} \times \vec{B} \rightarrow F = qvB\sin\theta$

$\vec{F} = I\vec{L} \times \vec{B} \rightarrow F = ILB\sin\theta$

$V_{Hall} = wv_d B$

$B = \frac{\mu_0 I}{2\pi r}$

$\varepsilon = \Delta V = -N\frac{\Delta\phi_B}{\Delta t}$

$\phi_B = BA\cos\theta$

Electric Circuits - Resistors

$I = \frac{\Delta Q}{\Delta t}$

$I = neAv_d; \quad n = \frac{\rho N_A}{m}$

$V = IR$

$R = \frac{\rho L}{A}$

$R_{series} = \sum_{i=1}^{N} R_i$

$\frac{1}{R_{parallel}} = \sum_{i=1}^{N} \frac{1}{R_i}$

$P = \frac{\Delta E}{\Delta t} = IV = I^2 R = \frac{V^2}{R}$

Light as a Particle/Relativity

$E = hf = \frac{hc}{\lambda}$

$K_{max} = hf - \phi$

$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$

$\frac{1}{E'} = \frac{1}{E} + \frac{(1-\cos\phi)}{E_{rest}}; \quad E_{rest} = mc^2$

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$p = \gamma m v$

$E_{total} = E_{rest} + K = \gamma mc^2$

$K = (\gamma - 1)mc^2$

$E_{total}^2 = p^2c^2 + m^2c^4$

Nuclear Physics

$N = N_0 e^{-\lambda t}$

$m = m_0 e^{-\lambda t}$

$A = A_0 e^{-\lambda t}$

$A = \lambda N$

$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

Constants

$g = 9.8 \frac{m}{s^2}$

$1e = 1.6 \times 10^{-19} C$

$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$

$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$

$1eV = 1.6 \times 10^{-19} J$

$\mu_0 = 4\pi \times 10^{-7} \frac{Tm}{A}$

$c = 3 \times 10^8 \frac{m}{s}$

$h = 6.63 \times 10^{-34} Js = 4.14 \times 10^{-15} eVs$

$N_A = 6.02 \times 10^{23}$

$1u = 1.66 \times 10^{-27} kg = 931.5 \frac{MeV}{c^2}$

$m_p = 1.67 \times 10^{-27} kg = 937.1 \frac{MeV}{c^2}$

$m_n = 1.69 \times 10^{-27} kg = 948.3 \frac{MeV}{c^2}$

$m_e = 9.11 \times 10^{-31} kg = 0.511 \frac{MeV}{c^2}$

Physics 110 Formulas

$\vec{F} = m\vec{a}; \quad F_G = \frac{GM_1 m_2}{r^2}; \quad F_s = -ky; \quad a_c = \frac{v^2}{r}$

$W = -\Delta U_g - \Delta U_s = \Delta K$

$U_g = mgy$

$U_s = \frac{1}{2} k y^2$

$K = \frac{1}{2} m v^2$

$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2} \vec{a} t^2$

$\vec{v}_f = \vec{v}_i + \vec{a} t$

$v_f^2 = v_i^2 + 2 a_r \Delta r$

Common Metric Units

$nano\ (n) = 10^{-9}$

$micro\ (\mu) = 10^{-6}$

$milli\ (m) = 10^{-3}$

$centi\ (c) = 10^{-2}$

$kilo\ (k) = 10^{3}$

$mega\ (M) = 10^{6}$

Geometry/Algebra

Circles: $A = \pi r^2$ $\quad C = 2\pi r = \pi$

Spheres: $A = 4\pi r^2$ $\quad V = \frac{4}{3}\pi r^3$

Triangles: $A = \frac{1}{2} bh$

Quadratics: $ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$