Physics 111

Exam #3

November 4, 2024

Name____________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. Erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}| = (5kg)\times(2\,\frac{m}{s}) = 10\,\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 6 points.

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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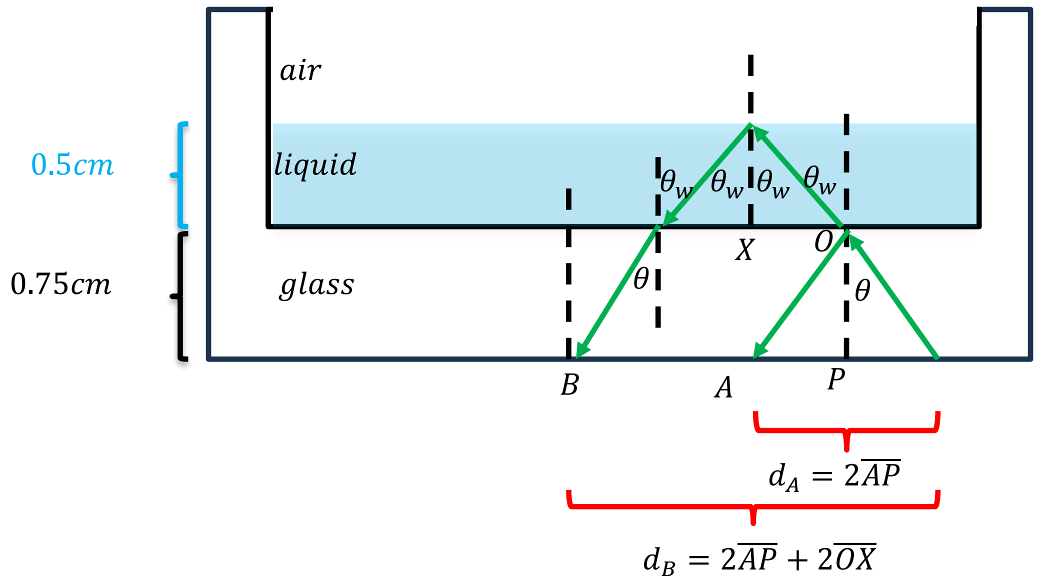
1. The figure below shows a container made from glass ($n_g = 1.5$) with a layer of water ($n_w = 1.33$) on top. Light is incident in the glass at an angle $\theta = \theta_1$ measured with respect to the normal. The bottom of the glass is frosted so that bright spots appear where light from the beam strikes this bottom surface and gets absorbed. You may assume that the reflection from this frosted surface is negligible.



a. At this angle of incidence, θ_1, two bright spots appear on the bottom surface of the glass. Let us label the spot that appears closer to point P in the diagram A, and the spot that appears farther from point P in the diagram B. The actual points A and B are not shown in the diagram. Explain the processes involved in the formation of the two spots at locations A and B and explain which spot is brighter, spot A or spot B.

Spot A, that appears closer to point P, is due to the reflection of the light at the glass/water interface.

Spot B that appears farther from point P is due to the refraction of light as it enters the water from the glass. It then reflects from the water/air surface and the ray that remains in the water again refracts at the water/glass interface producing spot B.

Since energy is conserved, every time the light reflects (from the water/air and from the water/glass interfaces after the initial reflection) lowers the energy in the subsequently transmitted beam. Thus, spot A is brighter than spot B.

b. Suppose the glass has a thickness of $0.75cm$, while the thickness of the water on top of the glass is $0.5cm$. What is the horizontal distance along the bottom surface of the glass between the spots A and B if $\theta_1 = 30^0$?

For the relevant geometry, see the original figure.

For the glass/water interface, the light enters the water at:

$$n_g \sin\theta_g = n_w \sin\theta_w \rightarrow \sin\theta_w = \frac{n_g}{n_w}\sin\theta_g = \frac{1.5}{1.33}\sin 30 \rightarrow \theta_w = 34.3^0$$

The light travels through the water and strikes the air/water interface at θ_w. From the geometry, the horizontal distance across the glass/water interface is:

$$\tan\theta_w = \frac{\overline{OX}}{t_{water}} \rightarrow \overline{OX} = t_{water}\tan\theta_w = 0.5cm\tan 34.3 = 0.34cm$$

The horizontal distance across the bottom of the glass:

$$D = d_B - d_A = (2\overline{AP} + 2\overline{OX}) - 2\overline{AP} = 2\overline{OX} = 2 \times 0.34cm = 0.68cm$$

Also, note that the beam leaves parallel to itself in the glass having been displaced horizontally across the glass surface. You could have also calculated the distance simply from $2\overline{OX} = 2 \times 0.34cm = 0.68cm$.

c. Suppose the angle of incidence in the glass is increased from θ_1 to θ_2, where $\theta_2 > \theta_1$. In this case, the spots A and B move along the bottom surface of the glass and *one of the spots becomes brighter*. Explain which way the spots move, and which spot becomes brighter and why?

Since the angle of incidence increases and the thickness of the glass and water are fixed, the horizontal distance along the bottom of the glass for both points A and B increases, and the spots move to the left.

The brightness of point A is fixed due to the reflection. So, only point B can change brightness. To increase in brightness, there needs to be more light incident on the glass from the water. To get this, the light needs to be internally reflected in the water so that no energy flows into the air and stays in the reflection from the air/water interface. This increase in energy will ultimately show up (with a little loss due to reflection at the water/glass interface) as energy transmitted back into the glass and spot B gets brighter. This does not imply (and is also not true) that spot B is brighter than spot A. Spot A is still brighter than spot B.

d. Suppose that we further increase the angle of incidence of the light from θ_2 to θ_3, where $\theta_3 > \theta_2$. At this angle θ_3, the light becomes totally internally reflected in the glass. What is the angle of incidence θ_3, in the glass so that the light will be totally internally reflected at the glass/water interface?

$$n_g \sin\theta_g = n_w \sin\theta_w \rightarrow n_g \sin\theta_3 = n_w \sin 90 \rightarrow \sin\theta_3 = \frac{n_w}{n_g} = \frac{1.33}{1.5} = 0.8867$$

$$\theta_3 = 62.5^0$$

2. Two lenses are used in an experiment. Lens #1 has a power $P_1 = +7.9D$, while the power of lens #2 is unknown. An object is placed $100mm$ to the left of lens #1 and an image is seen on a screen $258mm$ to the right of lens #2.

   a. What is the type and power of the second (lens #2) if the lenses are separated by $188mm$?

$$\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{f_1} \rightarrow \frac{1}{d_{i1}} = \frac{1}{f_1} - \frac{1}{d_{o1}} = 7.9D - \frac{1}{0.1m}$$

$$\rightarrow d_{i1} = -0.476m = -476mm$$

$$d_{o2} = d_{i1} + D = 476mm + 188mm = 664mm$$

$$\frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f_2} = P_2 \rightarrow P_1 = \frac{1}{0.664m} + \frac{1}{0.258m} = 5.4D$$

   The second lens is converging since a real image is formed and since the power is positive.

   b. The image on the screen is measured to be $3.5cm$ in length. What was the height of the original object?

$$M_T = M_1 M_2 = \frac{h_{if}}{h_o} = \left(\frac{d_{i2}}{d_{o2}}\right)\left(\frac{d_{i1}}{d_{o1}}\right) \rightarrow h_o = \frac{h_{if}}{\left(\frac{d_{i2}}{d_{o2}}\right)\left(\frac{d_{i1}}{d_{o1}}\right)}$$

$$h_o = \frac{3.5cm}{\left(\frac{258mm}{664mm}\right)\left(\frac{476mm}{100mm}\right)} = 1.9cm$$

c. Suppose that the experiment was repeated, but this time we replaced the first lens with a lens of power $P_1 = -7.9D$. With everything else the same, would we have to move the screen closer to lens #2, farther from lens #2, or make no change at all to the location of the screen, to see the clear image? Hint: This probably requires a calculation and everything else the same means that the object's location with respect to lens #1 does not change, the focal length of lens #2 does not change, and the distance between the lenses does not change.

$$\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{f_1} \rightarrow \frac{1}{d_{i1}} = \frac{1}{f_1} - \frac{1}{d_{o1}} = -7.9D - \frac{1}{0.1m}$$

$$\rightarrow d_{i1} = -0.056m = -56mm$$

$$d_{o2} = d_{i1} + D = 56mm + 188mm = 244mm$$

$$\frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f_2} \rightarrow \frac{1}{d_{i2}} = \frac{1}{f_2} - \frac{1}{d_{o2}} = 5.4D - \frac{1}{0.244m}$$

$$\rightarrow d_{i2} = 0.768m = 768mm$$

This image distance is much greater than that given in part 2a. So, we'll have to move the screen farther away from lens #2.

d. Suppose that you look at lens #1 from ***part 2a*** and notice that the shape of the lens is a circle of radius R. Further, suppose that you were to paint lens #1 in ***part 2a*** so that only a small circle of radius r, where $r < R$, near the center of the lens allowed light to pass. In this situation, what would happen to the image on the screen if everything else remained the same? Explain your answer fully. Hint: everything else the same means that the object's location with respect to lens #1 does not change, the focal length of lenses #1 and #2 do not change, and the distance between the lenses does not change. We just allow less light to pass through lens #1.

The only thing that happens to the image is it gets dimmer. Light still passes through the lens and is focused. The image location will not change since none of the other parameters has changed. Simply less light gets out of the first lens and there's less light to focus. Thus, the final image gets dimmer.

3. Light with a single wavelength $\lambda = 98nm$ is emitted from a source. The light is incident on a piece of platinum metal whose work function is ϕ. The beam makes a circular spot on the metal and electrons ejected from the metal are observed on a collector.

   a. If the electrons were ejected with a speed of $v = 0.005c$, what voltage (V_{stop}) would be needed to stop the electrons from striking the collector and what is the work function ϕ (in eV) of the metal?

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(5.11 \times 10^5 \frac{eV}{c^2}\right)(0.005c)^2 = eV_{stop} \rightarrow V_{stop} = 6.39V$$

$$K = eV_{stop} = \frac{hc}{\lambda} - \phi \rightarrow \phi = \frac{hc}{\lambda} - eV_{stop}$$

$$\phi = \left(\frac{6.63 \times 10^{-34} Js \times 3 \times 10^8 \frac{m}{s}}{98 \times 10^{-9} m}\right) \times \frac{1eV}{1.6 \times 10^{-19} J} - 6.39eV$$
$$= 12.69eV - 6.39eV$$

$$\phi = 6.3eV$$

   b. If the beam of light made a circular spot on the metal with a radius $r = 1cm$ and the rate at which electrons were incident on the collector was $8.2 \times 10^{20} s^{-1}$, what was the intensity of the incident beam of light? Hint: the efficiency of the emitter is 100%, meaning that for every incident photon, we get an ejected electron.

$$S = \frac{total\ energy}{time \times area} = \frac{N_{photons} \times E_{photon}}{time \times area}$$

$$S = 8.2 \times 10^{20} s^{-1} \times \frac{\left(\frac{6.63 \times 10^{-34} Js \times 3 \times 10^8 \frac{m}{s}}{98 \times 10^{-9} m}\right)}{\pi(0.01m)^2}$$

$$S = 5.3 \times 10^6 \frac{W}{m^2}$$

c. Suppose the intensity in part b were too large and further that we want to lower the intensity of the incident beam by using two polarizers. Initially unpolarized light from the source passes through the first polarizer with its transmission axis vertical. At what angle would you have to set the second polarizer to reduce the intensity of the emerging beam by 99%?

$$S_{out,1} = \frac{1}{2}S_0$$

$$S_{out,2} = S_{in,2}\cos^2\theta \rightarrow 0.01S_0 = 0.5S_0\cos^2\theta \rightarrow \theta = 81.9^0$$

d. The light that emerges from the polarizers is again incident on the metal surface. If the beam makes the same spot size on the platinum metal, what can you say about the kinetic energy of the ejected electrons that strike the collector, in this case? Explain your answer.

In the particle model of light, the intensity is proportional to the number of photons in the beam of light. Here we reduced the intensity meaning that we reduced the number of photons incident. The energy of each photon remains unchanged and since the energy of the photon does not change, the kinetic energy of the ejected electron does not change. The only change is that we get fewer ejected electrons, but they all have the same kinetic energy and speed.

Physics 111 Formula Sheet

Electrostatics

$F = k\frac{q_1 q_2}{r^2}$

$\vec{F} = q\vec{E}; \quad E_{pc} = k\frac{q}{r^2}; \quad E_{plate} = \frac{q}{\epsilon_0 A}$

$E = -\frac{\Delta V}{\Delta x}$

$V_{pc} = k\frac{q}{r}$

$U_e = k\frac{q_1 q_2}{r} = qV$

$W = -q\Delta V = -\Delta U_e = \Delta K$

Magnetism

$\vec{F} = q\vec{v} \times \vec{B} \rightarrow F = qvB\sin\theta$

$\vec{F} = I\vec{L} \times \vec{B} \rightarrow F = ILB\sin\theta$

$V_{Hall} = wv_d B$

$B = \frac{\mu_0 I}{2\pi r}$

$\varepsilon = \Delta V = -N\frac{\Delta\phi_B}{\Delta t}$

$\phi_B = BA\cos\theta$

Electric Circuits - Capacitors

$Q = CV; \quad C = \frac{\kappa\epsilon_0 A}{d}$

$C_{parallel} = \sum_{i=1}^{N} C_i$

$\frac{1}{C_{series}} = \sum_{i=1}^{N}\frac{1}{C_i}$

$Q_{charging}(t) = Q_{max}\left(1 - e^{-\frac{t}{\tau}}\right)$

$Q_{discharging}(t) = Q_{max}e^{-\frac{t}{\tau}}$

$I(t) = I_{max}e^{-\frac{t}{\tau}} = \frac{Q_{max}}{\tau}e^{-\frac{t}{\tau}}$

$\tau = RC$

$U_C = \frac{1}{2}qV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$

Electric Circuits - Resistors

$I = \frac{\Delta Q}{\Delta t}$

$I = neAv_d$

$n = \left(\frac{\rho_m N_A}{m}\right) \times \frac{\text{charge carriers donated}}{\text{atom}}$

$V = IR$

$R = \frac{\rho L}{A}$

$R_{series} = \sum_{i=1}^{N} R_i$

$\frac{1}{R_{parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$

$P = \frac{\Delta E}{\Delta t} = IV = I^2 R = \frac{V^2}{R}$

Light as a Wave

$c = f\lambda$

$S(t) = \frac{\text{Energy}}{\text{time}\times\text{Area}} = c\epsilon_0 E^2(t) = c\frac{B^2(t)}{\mu_0}$

$I = S_{avg} = \frac{1}{2}c\epsilon_0 E_{max}^2 = c\frac{B_{max}^2}{2\mu_0}$

$P = \begin{cases} \frac{S}{c}; & \text{absorbed} \\ \frac{2S}{c}; & \text{reflected} \end{cases}$

$S = S_0\cos^2\theta$

$v = \frac{c}{n}$

$\theta_{\text{incident}} = \theta_{\text{reflected}}$

$n_1\sin\theta_1 = n_2\sin\theta_2$

$P = \frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i}$

$M = \frac{d_i}{d_0}; \quad |M| = \frac{h_i}{h_0}$

Light as a Particle/Relativity

$E = hf = \frac{hc}{\lambda}$

$K_{max} = hf - \phi$

$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$

$\frac{1}{E'} = \frac{1}{E} + \frac{(1-\cos\phi)}{E_{rest}}; \quad E_{rest} = mc^2$

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$p = \gamma mv$

$E_{total} = E_{rest} + K = \gamma mc^2$

$K = (\gamma - 1)mc^2$

$E_{total}^2 = p^2c^2 + m^2c^4$

Nuclear Physics

$N = N_0 e^{-\lambda t}$

$m = m_0 e^{-\lambda t}$

$A = A_0 e^{-\lambda t}$

$A = \lambda N$

$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

Constants

$g = 9.8 \frac{m}{s^2}$

$1e = 1.6 \times 10^{-19} C$

$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$

$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$

$1eV = 1.6 \times 10^{-19} J$

$\mu_0 = 4\pi \times 10^{-7} \frac{Tm}{A}$

$c = 3 \times 10^8 \frac{m}{s}$

$h = 6.63 \times 10^{-34} Js = 4.14 \times 10^{-15} eVs$

$N_A = 6.02 \times 10^{23}$

$1u = 1.66 \times 10^{-27} kg = 931.5 \frac{MeV}{c^2}$

$m_p = 1.67 \times 10^{-27} kg = 937.1 \frac{MeV}{c^2}$

$m_n = 1.69 \times 10^{-27} kg = 948.3 \frac{MeV}{c^2}$

$m_e = 9.11 \times 10^{-31} kg = 0.511 \frac{MeV}{c^2}$

Physics 110 Formulas

$\vec{F} = m\vec{a}; \quad F_G = \frac{GM_1 m_2}{r^2}; \quad F_s = -ky; \quad a_c = \frac{v^2}{r}$

$W = -\Delta U_g - \Delta U_s = \Delta K$

$U_g = mgy$

$U_s = \frac{1}{2} k y^2$

$K = \frac{1}{2} m v^2$

$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2} \vec{a} t^2$

$\vec{v}_f = \vec{v}_i + \vec{a} t$

$v_f^2 = v_i^2 + 2 a_r \Delta r$

Common Metric Units

$nano\ (n) = 10^{-9}$

$micro\ (\mu) = 10^{-6}$

$milli\ (m) = 10^{-3}$

$centi\ (c) = 10^{-2}$

$kilo\ (k) = 10^{3}$

$mega\ (M) = 10^{6}$

Geometry/Algebra

Circles:	$A = \pi r^2$	$C = 2\pi r = \pi$
Spheres:	$A = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$
Triangles:	$A = \frac{1}{2} bh$	
Quadratics:	$ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$