Physics 111

Exam #3

November 3, 2025

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So, erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}| = (5kg)\times(2\frac{m}{s}) = 10\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 6 points.

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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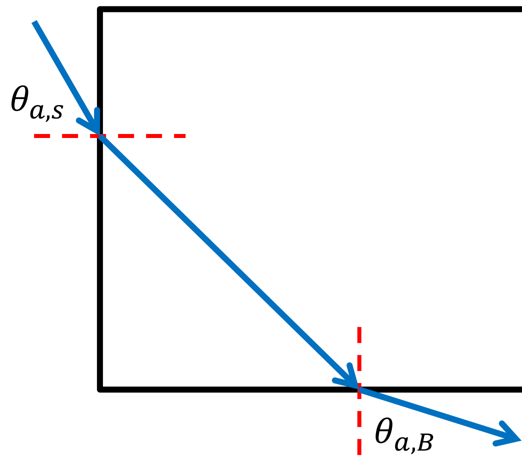
1. Light from a blue laser pointer, rated at $2mW$, is aimed at a block of transparent material of unknown index of refraction n_m . The block of material is surrounded on all sides by air and the light strikes the left-hand side at $\theta_{a,S} = 45^0$ and exits the bottom at $\theta_{a,B} = 76^0$, as shown in the diagram below.



a. What is the index of refraction of the material? You may need the fact that $\sin(90 - \alpha) = \cos\alpha$.

$$n_a \sin\theta_{a,S} = n_m \sin\theta_{m,S}$$

$$n_m \sin(90 - \theta_{m,S}) = n_m \cos\theta_{m,S} = n_a \sin\theta_{a,B}$$

$$\frac{n_a \sin\theta_{a,S}}{n_a \sin\theta_{a,B}} = \frac{n_m \sin\theta_{m,S}}{n_m \cos\theta_{m,S}} \rightarrow \tan\theta_{m,S} = \frac{\sin\theta_{a,S}}{\sin\theta_{a,B}} = \frac{\sin 45}{\sin 76} \rightarrow \theta_{m,S} = 36.1^0$$

$$n_a \sin\theta_{a,S} = n_m \sin\theta_{m,S} \rightarrow n_m = \left(\frac{\sin\theta_{a,S}}{\sin\theta_{m,S}}\right) n_{air} = \left(\frac{\sin 45}{\sin 36.1}\right) \times 1.0 = 1.2$$

b. What is the critical angle of incidence on the lower surface and what is the maximum angle of incidence of the light on the left-hand surface that will lead to total internal reflection of the light in the material?

$$n_m \sin\theta_c = n_a \sin 90 \rightarrow \theta_c = \sin^{-1}\left(\frac{n_a}{n_m}\right) = \sin^{-1}\left(\frac{1.0}{1.2}\right) = 56.1^0$$

To see if the actual angle of incidence produces total internal reflection or not, we need to see what angle we actually need on the left-hand side.

$$\theta_{m,S} + \theta_c = 90^0 \rightarrow \theta_{m,S} = 90^0 - \theta_c = 90^0 - 56.1^0 = 33.6^0$$

$$n_a \sin\theta_{a,S} = n_m \sin\theta_{m,S} \rightarrow \sin\theta_{a,S} = \left(\frac{n_m}{n_1}\right)\sin\theta_{m,S} = \left(\frac{1.2}{1.0}\right)\sin 33.6$$

$$\rightarrow \theta_{a,S} = 41.6^0$$

This is the maximum angle of incidence and any angle smaller than this will produce internal reflection. As $\theta_{a,S}$ gets smaller, $\theta_{m,S}$ gets smaller and the angle of the light incident on the lower surface would be greater than the critical angle and we'd get TIR. At the angle we have set (which is larger than this maximum angle), we will not get TIR as can be seen by the exiting light at the bottom surface.

c. Suppose the blue light that exits the transparent material is incident on a calcium surface ($\phi = 2.9eV$). Electrons are ejected with a maximum kinetic energy $0.12eV$. What is the wavelength of the blue light that was used?

$$K = \frac{hc}{\lambda} - \phi = \rightarrow \lambda = \frac{hc}{K+\phi} = \frac{6.63\times10^{-34}Js \times 3\times10^8\frac{m}{s}}{(0.12eV + 2.9eV)\times\frac{1.6\times10^{-19}J}{1eV}}$$

$$\lambda = 4.12\times10^{-7}m = 412nm$$

d. How many photons are incident on the calcium surface per second and what is the power carried away by those ejected electrons? Assume the efficiency of photoelectron production is 80%.

$$P = \frac{energy}{time} = \frac{number\ of\ photos}{time} \times \frac{energy}{photon} \rightarrow \frac{number\ of\ photons}{time} = \frac{P}{\frac{energy}{photon}}$$

$$\frac{number\ of\ photons}{time} = 2 \times 10^{-3}\frac{J}{s} \times \frac{1photon}{(0.12eV + 2.9eV) \times \frac{1.6 \times 10^{-19}J}{1eV}}$$

$$\frac{number\ of\ photons}{time} = 4.1 \times 10^{15}s^{-1}$$

For every say, 10 photons in, we get 8 electrons out since the system is 80% efficient.

$$P_e = \frac{number\ of\ photons}{time} \times \frac{0.8e}{1photon} \times \frac{energy}{1e}$$

$$P_e = 4.1 \times 10^{15}s^{-1} \times 0.8 \times 0.12eV \times \frac{1.6 \times 10^{-19}J}{1eV}$$

$$P_e = 6.36 \times 10^{-5}W = 0.0636mW$$

2. Students were conducting a laboratory experiment involving a thin converging lens. Objects are placed at various distances from the lens and data are taken on the corresponding real image distances, where the object and image distances were all measured in millimeters. A plot of $\frac{1}{d_o}$ versus $\frac{1}{d_i}$ was generated and the equation of the fit to the data was found to be $y = -0.98x + 0.021$.

   a. Suppose that the converging lens is placed in a holder and an object is placed to the left of the lens and a screen to the right of the lens. The object to screen distance is fixed at $D = 240mm$. At what position(s) can the lens be placed, with respect to the object, so that a sharp image is formed on the screen?

$$b = \frac{1}{f} \rightarrow f = \frac{1}{b} = \frac{1}{0.021mm^{-1}} = 47.6mm$$

$$\frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{d_o} + \frac{1}{D - d_o} = \frac{D}{d_o D - d_o^2} \rightarrow d_o^2 - d_o D + fD = 0$$

$$d_{o1} = \frac{D + \sqrt{D^2 - 4fD}}{2} = \frac{240mm + \sqrt{(240mm)^2 - 4 \times 47.6mm \times 240mm}}{2}$$
$$d_{o1} = 174.5mm$$

$$d_{o2} = \frac{D - \sqrt{D^2 - 4fD}}{2} = \frac{240mm - \sqrt{(240mm)^2 - 4 \times 47.6mm \times 240mm}}{2}$$
$$d_{o1} = 65.5mm$$

   b. For the position(s) in part a, what will be the image size of a $1cm$ tall object on the screen?

$$M_1 = \frac{d_{i1}}{d_{o1}} = \frac{D - d_{o1}}{d_{01}} = \frac{240mm - 174.5mm}{174.5mm} = 0.38 \rightarrow h_{i1} = M_1 h_o = 0.38cm$$

$$M_2 = \frac{d_{i2}}{d_{o2}} = \frac{D - d_{o2}}{d_{o2}} = \frac{240mm - 65.5mm}{65.5mm} = 2.7 \rightarrow h_{i1} = M_1 h_o = 2.7cm$$

c. Suppose that the lens from part a (call it lens A) is used in combination with a second lens (call this lens B) of unknown focal length f_B. A $1cm$ tall object is placed $75mm$ to the left of lens B and Lens A is placed $50mm$ to the right of lens B. In this configuration a real image is seen $67mm$ to the right of lens A. What type of lens is lens B and what is its focal length?

$$\frac{1}{f_A} = \frac{1}{d_{oA}} + \frac{1}{d_{iA}} \rightarrow d_{oA} = \left(\frac{1}{f_A} - \frac{1}{d_{oA}}\right)^{-1} = \left(\frac{1}{47.6mm} - \frac{1}{67mm}\right)^{-1} = 164.4mm$$

$$d_{oA} = D + d_{iB} \rightarrow d_{iB} = d_{oA} - D = 164.4mm - 50mm = 114.4mm$$

$$\frac{1}{f_B} = \frac{1}{d_{oB}} + \frac{1}{d_{iB}} \rightarrow f_B = \left(\frac{1}{d_{oB}} - \frac{1}{d_{iB}}\right)^{-1} = \left(\frac{1}{75mm} - \frac{1}{114.4mm}\right)^{-1} = 217.8mm$$

Since the focal length is positive, lens B is a converging lens.

d. Lenses of type B are used as corrective lenses in a pair of glasses. What refractive error of the eye will these lenses correct. Be sure explain what the eye condition is and why theses lenses are the appropriate corrective type. What prescription would the eye doctor need to write for these set of glasses?

The prescription is the power of the lens:

$$P = \frac{1}{f} = \frac{1}{0.2178m} = +4.6D$$

Converging lenses are used to correct for hyperopia or farsightedness. In a farsighted eye, the eye can focus the image of a distant object onto the retina. When the object comes in closer to the eye, the resulting image moves behind the retina and is thus out of focus. To correct for this, a converging lens is used. The object located near the eye (the near point) is placed inside the focal length of the corrective lenses to produce a virtual image at a distant point (the far point) in space where the eye can focus it onto the retina.

3. X-rays from a mercury light source with wavelength $\lambda = 0.0176nm$ are incident on a block of carbon.

   a. What is the energy (to three decimal places and in keV), momentum (in $\frac{keV}{c}$) and frequency (in Hz or s^{-1}) of the incident mercury x-rays?

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8 \frac{m}{s}}{0.0176 \times 10^{-9} m} = 1.7 \times 10^{19} s^{-1}$$

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} Js \times 3 \times 10^8 \frac{m}{s}}{0.0176 \times 10^{-9} m} \times \frac{1eV}{1.6 \times 10^{-19} J} \times \frac{1keV}{1000eV} = 70.632keV$$

$$E = pc \rightarrow p = \frac{E}{c} = \frac{70.632keV}{c} = 70.632 \frac{keV}{c}$$

   b. If the x-rays are observed to scatter through an angle $\phi = 160^0$, what is the energy (in keV and to three decimal places) of the scattered x-rays?

$$\frac{1}{E'} = \frac{1}{E} + \frac{1 - \cos\phi}{mc^2} = \frac{1}{70.632keV} + \frac{1 - \cos 160}{\left(511 \frac{keV}{c^2}\right) c^2} \rightarrow E' = 55.699keV$$

c. Expressed as a fraction of the speed of light, what is the speed of the scattered electron?

$$E = E' + K \rightarrow K = E - E' = 70.632keV - 55.699keV = 14.933keV$$

$$K = (\gamma - 1)mc^2 \rightarrow \gamma = \frac{K}{mc^2} + 1 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \rightarrow v = \sqrt{1 - \frac{1}{\gamma^2}}c$$

$$\gamma = \frac{14.933keV}{\left(511\frac{keV}{c^2}\right)c^2} + 1 = 1.02922$$

$$v = \sqrt{1 - \frac{1}{\gamma^2}}c = \sqrt{1 - \frac{1}{(1.02922)^2}}c = 0.237c$$

d. At what angle θ was the electron scattered through? Measure your angle with respect to the direction of the incident x-rays, taken to be 0^0.

From conservation of momentum in the direction perpendicular to the incident x-rays we have:

$$0 = p_{photon}\sin\phi - p_e\sin\theta \rightarrow \sin\theta = \left(\frac{p_{photon}}{p_e}\right)\sin\phi = \left(\frac{p_{photon} \times c}{p_e \times c}\right)\sin\phi$$

$$\sin\theta = \left(\frac{E_{photon}}{\gamma mvc}\right)\sin\phi = \left(\frac{55.699keV}{1.02922 \times 511\frac{keV}{c^2} \times 0.237c \times c}\right)\sin 160$$

$$\theta = \sin^{-1}(0.1528) = 8.8^0$$

Physics 111 Formula Sheet

Electrostatics

$F = k\frac{q_1 q_2}{r^2}$

$\vec{F} = q\vec{E}; \quad E_{pc} = k\frac{q}{r^2}; \quad E_{plate} = \frac{q}{\epsilon_0 A}$

$E = -\frac{\Delta V}{\Delta x}$

$V_{pc} = k\frac{q}{r}$

$U_e = k\frac{q_1 q_2}{r} = qV$

$W = -q\Delta V = -\Delta U_e = \Delta K$

Magnetism

$\vec{F} = q\vec{v} \times \vec{B} \to F = qvB\sin\theta$

$\vec{F} = I\vec{L} \times \vec{B} \to F = ILB\sin\theta$

$V_{Hall} = wv_d B$

$B = \frac{\mu_0 I}{2\pi r}$

$\varepsilon = \Delta V = -N\frac{\Delta\phi_B}{\Delta t}$

$\phi_B = BA\cos\theta$

Electric Circuits - Capacitors

$Q = CV; \quad C = \frac{\kappa\epsilon_0 A}{d}$

$C_{parallel} = \sum_{i=1}^{N} C_i$

$\frac{1}{C_{series}} = \sum_{i=1}^{N} \frac{1}{C_i}$

$Q_{charging}(t) = Q_{max}\left(1 - e^{-\frac{t}{\tau}}\right)$

$Q_{discharging}(t) = Q_{max}e^{-\frac{t}{\tau}}$

$I(t) = I_{max}e^{-\frac{t}{\tau}} = \frac{Q_{max}}{\tau}e^{-\frac{t}{\tau}}$

$\tau = RC$

$U_C = \frac{1}{2}qV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$

Electric Circuits - Resistors

$I = \frac{\Delta Q}{\Delta t}$

$I = neAv_d; \quad n = \frac{\rho N_A}{m}$

$V = IR$

$R = \frac{\rho L}{A}$

$R_{series} = \sum_{i=1}^{N} R_i$

$\frac{1}{R_{parallel}} = \sum_{i=1}^{N} \frac{1}{R_i}$

$P = \frac{\Delta E}{\Delta t} = IV = I^2 R = \frac{V^2}{R}$

Light as a Wave

$c = f\lambda$

$S(t) = \frac{\text{Energy}}{\text{time}\times\text{Area}} = c\epsilon_0 E^2(t) = c\frac{B^2(t)}{\mu_0}$

$I = S_{avg} = \frac{1}{2}c\epsilon_0 E_{max}^2 = c\frac{B_{max}^2}{2\mu_0}$

$P = \begin{cases} \frac{S}{c}; & \text{absorbed} \\ \frac{2S}{c}; & \text{reflected} \end{cases}$

$S = S_0\cos^2\theta$

$v = \frac{c}{n}$

$\theta_{\text{incident}} = \theta_{\text{reflected}}$

$n_1\sin\theta_1 = n_2\sin\theta_2$

$P = \frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i}$

$M = \frac{d_i}{d_0}; \quad |M| = \frac{h_i}{h_0}$

Light as a Particle/Relativity

$E = hf = \frac{hc}{\lambda}$

$K_{max} = hf - \phi$

$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$

$\frac{1}{E'} = \frac{1}{E} + \frac{(1-\cos\phi)}{E_{rest}}; \quad E_{rest} = mc^2$

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$p = \gamma mv$

$E_{total} = E_{rest} + K = \gamma mc^2$

$K = (\gamma - 1)mc^2$

$E_{total}^2 = p^2c^2 + m^2c^4$

Nuclear Physics

$N = N_0 e^{-\lambda t}$

$m = m_0 e^{-\lambda t}$

$A = A_0 e^{-\lambda t}$

$A = \lambda N$

$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

Constants

$g = 9.8 \frac{m}{s^2}$

$1e = 1.6 \times 10^{-19} C$

$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$

$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$

$1eV = 1.6 \times 10^{-19} J$

$\mu_0 = 4\pi \times 10^{-7} \frac{Tm}{A}$

$c = 3 \times 10^8 \frac{m}{s}$

$h = 6.63 \times 10^{-34} Js = 4.14 \times 10^{-15} eVs$

$N_A = 6.02 \times 10^{23}$

$1u = 1.66 \times 10^{-27} kg = 931.5 \frac{MeV}{c^2}$

$m_p = 1.67 \times 10^{-27} kg = 937.1 \frac{MeV}{c^2}$

$m_n = 1.69 \times 10^{-27} kg = 948.3 \frac{MeV}{c^2}$

$m_e = 9.11 \times 10^{-31} kg = 0.511 \frac{MeV}{c^2}$

Physics 110 Formulas

$\vec{F} = m\vec{a}; \quad F_G = \frac{GM_1 m_2}{r^2}; \quad F_s = -ky; \quad a_c = \frac{v^2}{r}$

$W = -\Delta U_g - \Delta U_s = \Delta K$

$U_g = mgy$

$U_s = \frac{1}{2} k y^2$

$K = \frac{1}{2} m v^2$

$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2} \vec{a} t^2$

$\vec{v}_f = \vec{v}_i + \vec{a} t$

$v_f^2 = v_i^2 + 2 a_r \Delta r$

Common Metric Units

$nano\ (n) = 10^{-9}$

$micro\ (\mu) = 10^{-6}$

$milli\ (m) = 10^{-3}$

$centi\ (c) = 10^{-2}$

$kilo\ (k) = 10^{3}$

$mega\ (M) = 10^{6}$

Geometry/Algebra

Circles: $A = \pi r^2$ $\quad C = 2\pi r = \pi$

Spheres: $A = 4\pi r^2$ $\quad V = \frac{4}{3}\pi r^3$

Triangles: $A = \frac{1}{2} bh$

Quadratics: $ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$