## Physics 111

## Exam \#3

## March 7, 2014

Name $\qquad$

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}|=(5 \mathrm{~kg}) \times\left(2 \frac{\mathrm{~m}}{\mathrm{~s}}\right)=10 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 3 points, short answer 6 points, and each free-response part is worth 9 points

| Problem \#1 | $/ 24$ |
| :---: | :---: |
| Problem \#2 | $/ 21$ |
| Problem \#3 | $/ 27$ |
| Total | $/ 72$ |

I affirm that I have carried out my academic endeavors with full academic honesty.

1. Suppose that red light from a helium-neon laser $\left(\lambda_{H e-N e}=632.8 \mathrm{~nm}\right)$ is incident on a sample of plastic (surrounded by air) as shown below, where $n_{p}=1.27$.
a. What angle of incidence $(\theta)$ of the laser beam on the left-most surface would be needed so that the laser light is totally internally reflected in the plastic material? Is this a maximum or minimum angle of incidence? Explain your answer.


At the upper surface, we apply the law of refraction to determine the critical angle. We have

$$
n_{1} \sin \theta_{1}=n_{2} \sin \theta_{2} \rightarrow n_{p} \sin \theta_{c}=n_{\text {air }} \sin 90
$$

$$
\theta_{c}=\sin ^{-1}\left(\frac{1}{n_{p}}\right)=\sin ^{-1}\left(\frac{1}{1.27}\right)=51.9
$$

Then using the geometry, the angle of refraction with respect to the normal on the front surface is $\theta_{2}=90-\theta_{c}=31.8^{\circ}$.

Therefore the angle of incidence can be determined by applying the law of refraction on the front surface.

$$
\begin{aligned}
& n_{1} \sin \theta_{1}=n_{2} \sin \theta_{2} \rightarrow n_{\text {air }} \sin \theta=n_{p} \sin \theta_{2} \\
& \theta=\sin ^{-1}\left(\frac{n_{p}}{n_{\text {air }}} \sin \theta_{2}\right)=\sin ^{-1}\left(\frac{1.27}{1.00} \sin (38.1)\right)=51.6
\end{aligned}
$$

This is a maximum angle. If $\theta$ increases then $\theta_{2}$ increases and then the angle of incidence on the upper surface falls below the critical angle and the ray will no longer be TIR.
b. Suppose that instead of red light you decided to use blue light and that you make the blue light incident along the normal to the surface on the block of plastic above. You determine that it takes the blue light a time $t_{1}$ to travel through the block of plastic. Now suppose that you have a second material of unknown refractive index but that this second material is the same length as the block of plastic from part a. If the same experiment is performed, it is found that the blue light takes a time $t_{2}<t_{1}$ to travel through this unknown material. The index of refraction of the second material is given as
(1.) $n_{2}=\left(\frac{t_{2}}{t_{1}}\right) n_{1}$.
2. $n_{2}=\left(\frac{t_{1}}{t_{2}}\right) n_{1}$.
3. $n_{2}=\left(\frac{t_{2}}{t_{1}}\right) \frac{n_{1}}{c}$
4. $n_{2}=\left(\frac{v_{2} t_{2}}{v_{1} t_{1}}\right) n_{1}$
c. Suppose that the light that exits the slab of plastic from the right-most surface is allowed to be incident on a cadmium surface (called the emitter) with work function $\phi_{C d}=4.07 \mathrm{eV}$. What potential difference between the emitter and collector would be needed to stop the photoelectrons from reaching the collector? If no potential difference is possible for the red $\mathrm{He}-\mathrm{Ne}$ laser light, what is the first "color" of light (from the table below) that will produce photoelectrons?

| "Color" | $\lambda(\mathrm{nm})$ | "Color" | $\lambda(\mathrm{nm})$ |
| :--- | :--- | :--- | :--- |
| $\mu$-waves | $1 \times 10^{8}$ | IR | $3 \times 10^{4}$ |
| Red | 650 | Orange | 625 |
| Yellow | 590 | Green | 565 |
| Blue | 520 | Violet | 435 |
| UV | 300 | X-rays | 10 |

The stopping potential is given by
$K E_{\max }=e V_{\text {stop }}=h f-\phi=\frac{h c}{\lambda}-\phi$
$V_{\text {stop }}=\frac{h c}{e \lambda}-\frac{\phi}{e}=\frac{6.6 \times 10^{-34} \mathrm{~J} \times 3 \times 10^{8} \frac{\mathrm{~m}}{s}}{1.6 \times 10^{-19} \mathrm{C} \times 632.8 \times 10^{-9} \mathrm{~m}}-\frac{4.07 \mathrm{eV}}{e}=1.96 \mathrm{~V}-4.07 \mathrm{~V}=-2.1 \mathrm{~V}$
This is negative and therefore no electrons are ejected using this wavelength. To determine the wavelength of light that is needed, we set the KE of the ejected electron equal to zero and determine the maximum wavelength needed to just unbind the electron. Thus we have

$$
\begin{aligned}
& K E_{\max }=0=h f_{\min }-\phi=\frac{h c}{\lambda_{\max }}-\phi \\
& \lambda_{\max }=\frac{h c}{\phi}=\frac{6.6 \times 10^{-34} \mathrm{Js} \times 3 \times 10^{8} \frac{\mathrm{~m}}{\mathrm{~s}}}{\left(4.07 \mathrm{eV} \times \frac{1.6 \times 10^{-19} \mathrm{~J}}{1 \mathrm{eV}}\right)}=3.04 \times 10^{-7} \mathrm{~m}=304 \mathrm{~nm}
\end{aligned}
$$

Thus the largest wavelength that can be used is 304 nm , so anything smaller than this will produce photoelectrons. So we select the "color" of ultraviolet .
d. Suppose that instead of red light from the $\mathrm{He}-\mathrm{Ne}$ laser you used a light source with a higher frequency for the incident photon, but with the same intensity. Increasing the frequency of the light that is incident on the emitter will

1. increase the number of photons ejected.
2. increase the number of electrons ejected.
3. decrease the speed of the ejected electrons.
4. increase the speed of the ejected electrons.
5. Suppose that a 1 mm diameter beam of molybdenum x-rays $\left(E_{M o}=17.15 \mathrm{keV}\right)$ are incident on a sample composed of tissue and bone (the primarily composition of which are carbon atoms). An x-ray detector, placed at a $160^{\circ}$ angle with respect to the incident beam, detects x-rays scattered off of stationary electrons in the carbon atoms.
a. What are the energy of the scattered x-ray and the speed of the recoiling electron (as a fraction of the speed of light)?

The energy is given from the Compton wavelength formula. We have
$\lambda^{\prime}=\lambda+\frac{h}{m_{e} c}(1-\cos \phi) \rightarrow \frac{\lambda^{\prime}}{h c}=\frac{\lambda}{h c}+\frac{1}{m_{e} c^{2}}(1-\cos \phi)$
$\frac{1}{E^{\prime}}=\frac{1}{E}+\frac{1}{m_{e} c^{2}}(1-\cos \phi)=\frac{1}{17.15 \mathrm{keV}}+\frac{1}{\left(511 \frac{\mathrm{keV}}{c^{2}}\right) c^{2}}(1-\cos 160)=0.062 \mathrm{keV}^{-1}$
$E^{\prime}=16.1 \mathrm{keV}$

The kinetic energy of the electron is given as the difference between the incident and scattered photon energies and setting this energy equal to the relativistic kinetic energy formula we determine the speed of the recoiling electron as a fraction of the speed of light. We have

$$
\begin{aligned}
& K E_{e}=1.05 \mathrm{keV}=(\gamma-1) m_{e} c^{2}=(\gamma-1) m_{e} c^{2}=(\gamma-1)\left(511 \frac{\mathrm{keV}}{\mathrm{c}^{2}}\right) c^{2} \\
& \gamma=1.00206=\frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}} \rightarrow v=0.064 c
\end{aligned}
$$

b. Suppose the sample is composed of 1 cm of tissue $\left(\mu_{T}=0.2045 \mathrm{~cm}^{-1}\right)$ and 0.5 cm of bone ( $\left.\mu_{B}=0.4660 \mathrm{~cm}^{-1}\right)$. If the beam, passing through both the bone and the tissue, exits the sample with a diameter of 1.14 mm , what is the power of the exiting beam?

The incident intensity is given as
$S_{0}=\frac{\text { Energy }}{\text { time } \cdot \text { area }}=\frac{17.15 \mathrm{keV} / \mathrm{s}}{\pi\left(0.5 \times 10^{-3} \mathrm{~m}\right)^{2}}=2.18 \times 10^{7} \frac{\mathrm{keV}}{\mathrm{m}^{2} \mathrm{~s}}=3.5 \times 10^{-9} \frac{\mathrm{~W}}{\mathrm{~m}^{2}}$.
The intensity of the transmitted beam through the bone and tissue sample is given by $S=S_{0} e^{-\left(\mu_{t} x_{1}+\mu_{b} x_{b}\right)}=\left(2.18 \times 10^{7} \frac{\mathrm{keV}}{\mathrm{m}^{2} \mathrm{~s}}\right) e^{-\left[\left(0.2045 \mathrm{~cm}^{-1} \times 1 \mathrm{~cm}\right)+\left(0.4660 \mathrm{~cm}^{-1} \times 0.5 \mathrm{~cm}\right)\right]}=1.41 \times 10^{7} \frac{\mathrm{keV}}{\mathrm{m}^{2} \mathrm{~s}}$.

Therefore the power of the exiting beam is the product of the intensity and the area of the exit spot. We have
$P=S \times A=1.14 \times 10^{7} \frac{\mathrm{keV}}{\mathrm{m}^{2} \mathrm{~s}} \times\left(\pi\left(0.57 \times 10^{-3} \mathrm{~m}\right)^{2}\right)=11.6 \frac{\mathrm{keV}}{\mathrm{s}}=1.86 \times 10^{-15} \mathrm{~W}$.
c. Suppose that you perform two separate Compton effect experiments one using visible light and the other using x-rays. The scattered radiation is observed at the same scattering angle. For these two experiments

1. $x$-rays have the greater shift in wavelength and the greater change in photon energy.
2. the two radiation types have the same shift in wavelength and the x rays have the greater change in photon energy.
3. the two radiation types have the same shift in wavelength and the visible light has the greater change in photon energy.
4. the two radiation types have the same shift in wavelength and the same change in photon energy.
5. visible light has the greater shift in wavelength and the greater shift in photon energy.
6. Your eye is a double convex lens and has the ability to change its focal length to accommodate objects both near and far to the lens. Over time, as you age, sometimes your eye no longer has the ability to change its focal length adequately and objects at various distances from the eye might not focus clearly on the retina as they once used to. Suppose that you have the ocular condition known as presbyopia, or farsightedness. This means objects far away from your eye are clearly focused on your retina while objects up close are not.
a. For the person with the far-sighted eye, as the object moves towards the lens of your eye, the clear image of that object
7. focuses at a point behind the retina.
8. focuses at a point in front of the retina between your lens and retina.
9. focuses at a point on the exterior side of your eye, that is at a point in front of your face.
10. cannot be determined since the actual object distance and focal length of your eye is unknown.
b. If an object was placed at $3 m$ from your eye and a clear image forms on your retina located 2.5 cm behind your lens, what is the focal length of your eye?

$$
\frac{1}{d_{o}}+\frac{1}{d_{i}}=\frac{1}{f_{\text {eye }}} \rightarrow \frac{1}{3 m}+\frac{1}{0.025 m}=\frac{1}{f_{\text {eye }}} \rightarrow f_{\text {eye }}=0.0248 \mathrm{~m}=2.48 \mathrm{~cm}
$$

c. In a far-sighted eye your lens no longer has the ability to change its focal length so that objects located far away can be focused clearly on the retina. Objects can be brought into focus on your retina by using a second lens (glasses) in combination with the lens of your eye. Suppose that you want to see clearly an object located at a distance of 26 cm from your glasses. If your glasses are 1.5 cm from your eye, what are the power and the type of lens that you would need to correct for presbyopia?

For the eye, I want to image to be on my retina and I can't change the focal length of my eye. So, I will use these two numbers to see where the image should be so that I can see it. This object distance will be the image distance for the lenses that I'm going to use in my glasses and from here I can determine the power and type of the glasses I need.

For the eye:
$\frac{1}{d_{o e}}+\frac{1}{d_{i e}}=\frac{1}{f_{e y e}} \rightarrow \frac{1}{d_{o e}}+\frac{1}{0.025 m}=\frac{1}{0.0248 m} \rightarrow d_{o e}=3.1 \mathrm{~m}$. This is the location of the image in the glasses I need. It will be on the same side of the lens as the object, so it is a virtual image.

For the glasses:
I need the image distance of the object from the lens of the glasses. This is determined from $d_{o e}=d_{i e}+D_{e \rightarrow g} \rightarrow d_{i e}=d_{o e}-D_{e \rightarrow g}=3.1 \mathrm{~m}-0.015 \mathrm{~m}=3.085 \mathrm{~m}$

To determine the focal length of the lens (and type)
$\frac{1}{d_{o g}}+\frac{1}{d_{i g}}=\frac{1}{f_{g}} \rightarrow \frac{1}{0.26 m}-\frac{1}{3.085 m}=\frac{1}{f_{g}} \rightarrow f_{g}=0.284 \mathrm{~m}=28.4 \mathrm{~cm}$. Since the
focal length is positive, a converging lens will be used. The power of the lens is given by: $P=\frac{1}{f}=\frac{1}{0.284 m}=+3.5 \mathrm{D}$
d. On a cold day in February, a stranger arrives in the village of Iping (a small village located on the southern coast of England). He wears gloves and dark glasses, even inside, and his face is covered in bandages. Soon crimes occur that cannot be explained, and the townspeople realize the unthinkable truth: the strange man is invisible - and he is slowly going mad. The stranger is a dangerous enemy who must be stopped. But if no one can see him, how can he be caught? This is the plot of H.G. Wells classic novel The Invisible Man. Using the theory that if a person's refractive image can be adjusted in such a way as to mimic the refractive quality of air, an obscure scientist invents a way to render skin, bones, and blood invisible, and tries the formula on himself. Now he can go anywhere, menace anyone - sight unseen. He has only two problems: he cannot become visible again - and he has gone murderously insane. Using what you know about the reflection of light, the refraction of light, and lenses, would the invisible man be blind and therefore be unable to carry out a reign of terror in the city of Iping? Or would he be able to see and therefore capable of terrorizing a city? To earn full credit you must support your answer. Simply saying yes or no will not earn you full credit for the question.

Having a refractive index of the body equal to that of air means that any light rays incident on you will not notice any difference in your refractive index compared to that of the air surrounding you. Thus the light that enters the invisible lens of your eye will not be refracted and therefore no image will be formed on your invisible retina. So you would be blind. So, unfortunately for the story line, the invisible man was blind.

Physics 111 Equation Sheet
Electric Forces, Fields and Potentials

$$
\begin{aligned}
& \vec{F}=k \frac{Q_{1} Q_{2}}{r^{2}} \hat{r} \\
& \vec{E}=\frac{\vec{F}}{q} \\
& \vec{E}_{Q}=k \frac{Q}{r^{2}} \hat{r} \\
& P E=k \frac{Q_{1} Q_{2}}{r} \\
& V(r)=k \frac{Q}{r} \\
& E_{x}=-\frac{\Delta V}{\Delta x} \\
& W=-q \Delta V
\end{aligned}
$$

Magnetic Forces and Fields
$F=q v B \sin \theta$
$F=I l B \sin \theta$
$\tau=N I A B \sin \theta=\mu B \sin \theta$
$P E=-\mu B \cos \theta$
$B=\frac{\mu_{0} I}{2 \pi r}$
$\varepsilon_{\text {induced }}=-N \frac{\Delta \phi_{B}}{\Delta t}=-N \frac{\Delta(B A \cos \theta)}{\Delta t}$

## Constants

$g=9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$
$1 e=1.6 \times 10^{-19} \mathrm{C}$
$k=\frac{1}{4 \pi \varepsilon_{o}}=9 \times 10^{9} \frac{\mathrm{~N} \mathrm{~N}^{2}}{\mathrm{c}^{2}}$
$\varepsilon_{0}=8.85 \times 10^{-12} \frac{\mathrm{C}^{2}}{\mathrm{Nm}^{2}}$
$1 \mathrm{eV}=1.6 \times 10^{-19} \mathrm{~J}$
$\mu_{o}=4 \pi \times 10^{-7} \frac{7 m}{A}$
$c=3 \times 10^{8} \frac{\mathrm{~m}}{\mathrm{~s}}$
$h=6.63 \times 10^{-34} \mathrm{Js}$
$m_{e}=9.11 \times 10^{-31} \mathrm{~kg}=\frac{0.511 \mathrm{MeV}}{c^{2}}$
$m_{p}=1.67 \times 10^{-27} \mathrm{~kg}=\frac{937.1 \mathrm{MeV}}{c^{2}}$
$m_{n}=1.69 \times 10^{-27} \mathrm{~kg}=\frac{948.3 \mathrm{MeV}}{c^{2}}$
$1 \mathrm{amu}=1.66 \times 10^{-27} \mathrm{~kg}=\frac{931.5 \mathrm{MeV}}{\mathrm{c}^{2}}$
$N_{A}=6.02 \times 10^{23}$
$A v^{2}+R v+C-n \rightarrow v-B \pm \sqrt{B^{2}-4 A C}$

Electric Circuits

$$
\begin{aligned}
& I=\frac{\Delta Q}{\Delta t} \\
& V=I R=I\left(\frac{\rho L}{A}\right) \\
& R_{\text {series }}=\sum_{i=1}^{N} R_{i} \\
& \frac{1}{R_{\text {parallel }}}=\sum_{i=1}^{N} \frac{1}{R_{i}} \\
& P=I V=I^{2} R=\frac{V^{2}}{R} \\
& Q=C V=\left(\frac{\kappa \varepsilon_{0} A}{d}\right) V=\left(\kappa C_{0}\right) V \\
& P E=\frac{1}{2} Q V=\frac{1}{2} C V^{2}=\frac{Q^{2}}{2 C} \\
& Q_{\text {charge }}(t)=Q_{\max }\left(1-e^{-\frac{t}{R C}}\right) \\
& Q_{\text {discharge }}(t)=Q_{\max } e^{-\frac{t}{R C}} \\
& C_{\text {parallel }}=\sum_{i=1}^{N} C_{i} \\
& \frac{1}{C_{\text {series }}}=\sum_{i=1}^{N} \frac{1}{C_{i}}
\end{aligned}
$$

Light as a Particle \& Relativity

$$
\begin{aligned}
& E=h f=\frac{h c}{\lambda}=p c \\
& K E_{\max }=h f-\phi=e V_{\text {stop }} \\
& \Delta \lambda=\frac{h}{m_{e} c}(1-\cos \phi) \\
& \gamma=\frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}} \\
& p=\gamma m v \\
& E_{\text {total }}=K E+E_{\text {rest }}=\gamma m c^{2} \\
& E_{\text {total }}^{2}=p^{2} c^{2}+m^{2} c^{4} \\
& E_{\text {rest }}=m c^{2}
\end{aligned}
$$

$$
K E=(\gamma-1) m c^{2}
$$

Geometry
Circles: $C=2 \pi r=\pi D \quad A=\pi r^{2}$
Triangles: $A=\frac{1}{2} b h$
Spheres: $A=4 \pi r^{2} \quad V=\frac{4}{3} \pi r^{3}$

Light as a Wave

$$
\begin{aligned}
& c=f \lambda=\frac{1}{\sqrt{\varepsilon_{o} \mu_{o}}} \\
& S(t)=\frac{\text { energy }}{\text { time } \times \text { area }}=c \varepsilon_{o} E^{2}(t)=c \frac{B^{2}(t)}{\mu_{0}} \\
& I=S_{\text {avg }}=\frac{1}{2} c \varepsilon_{o} E_{\max }^{2}=c \frac{B_{\max }^{2}}{2 \mu_{0}} \\
& P=\frac{S}{c}=\frac{\text { Force }}{\text { Area }} \\
& S=S_{o} \cos ^{2} \theta \\
& v=\frac{1}{\sqrt{\varepsilon \mu}}=\frac{c}{n} \\
& \theta_{\text {inc }}=\theta_{\text {refl }} \\
& n_{1} \sin \theta_{1}=n_{2} \sin \theta_{2} \\
& \frac{1}{f}=\frac{1}{d_{o}}+\frac{1}{d_{i}} \\
& P=\frac{1}{f} \\
& M=\frac{h_{i}}{h_{o}}=-\frac{d_{i}}{d_{o}} \\
& M \text { total }=\prod_{i=1}^{N} M_{i} \\
& S=S_{o} e^{-\mu x} \\
& C T \#=H U=\left(\frac{\mu-\mu_{w}}{\mu_{w}}\right) \times 1000
\end{aligned}
$$

Nuclear Physics
$E_{b \dot{n d i n g}}=\left(Z m_{p}+N m_{n}-m_{r \varepsilon t}\right) c$
$\frac{\Delta N}{\Delta t}=-\lambda N_{o} \rightarrow N(t)=N_{o} e^{-\lambda t}$
$A(t)=A_{o} e^{-\lambda t}$
$m(t)=m_{o} e^{-\lambda t}$
$t_{\frac{1}{2}}=\frac{\ln 2}{\lambda}$
Misc. Physics 110
Formulae
$\vec{F}=\frac{\Delta \vec{p}}{\Delta t}=\frac{\Delta(m v)}{\Delta t}=m \vec{a}$
$\vec{F}=-k \vec{y}$
$\vec{F}_{C}=m \frac{v^{2}}{R} \hat{r}$
$W=\Delta K E=\frac{1}{2} m\left(v_{f}^{2}-v_{i}^{2}\right)=-\Delta P E$
$P E_{g r a v i t y}=m g y$
$P E_{\text {spring }}=\frac{1}{2} k y^{2}$
$x_{f}=x_{i}+v_{i x} t+\frac{1}{2} a_{x} t^{2}$
$v_{f x}=v_{i x}+a_{x} t$
$v_{f x}^{2}=v_{i x}^{2}+2 a_{x} \Delta x$

