Physics 111

Exam #3

March 3, 2023

Name____________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $\left|\vec{p}\right| \approx m\left|\vec{v}\right| = (5kg)\times\left(2\,\frac{m}{s}\right) = 10\,\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 6 points.

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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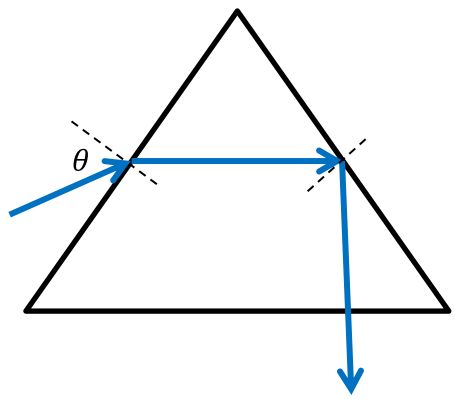
1. Suppose blue light ($\lambda_{blue} = 450nm$) was incident on the equilateral triangular piece of glass ($n_{glass} = 1.50$) as shown below. The glass is surrounded by air.

   a. Suppose that you want the blue light to be totally internally reflected at the glass/air interface on the right-hand side of the glass. To do this the light needs to strike the glass/air interface at the critical angle. What is the critical angle for the glass/air interface?



$$n_{glass} \sin\theta_c = n_{air} \sin\theta_{air}$$
$$\sin\theta_c = \frac{n_{air}}{n_{glass}} \sin\theta_{air} = \frac{1.00}{1.50} \sin 90$$
$$\theta_c = 41.8^0$$

   b. At what angle of incidence θ on the left-hand side air/glass interface would the light have to be incident to achieve total internal reflection?

$$90 = \alpha + 41.8 \rightarrow \alpha = 48.2^0$$
$$\alpha + \beta + 60 = 180 \rightarrow \beta = 180 - 60 - \alpha = 120 - 48.2 = 71.8^0$$
$$90 = \beta + \theta_g \rightarrow \theta_g = 90 - \beta = 90 - 71.8 = 18.2^0$$

$$n_{air} \sin\theta = n_{glass} \sin\theta_g \rightarrow \sin\theta = \frac{n_{glass}}{n_{air}} \sin\theta_g = \frac{1.50}{1.00} \sin 18.2$$
$$\theta = 27.9^0$$

c. The blue light that emerges through the bottom surface of the equilateral glass block is allowed to be incident on a metal emitter of unknown composition. The blue light ejects electrons from the metal surface and a voltage of $0.8123V$ is required to stop the ejected electrons from striking the collector. Using this information, what is the work function of the metal emitter?

$$K = eV_{stop} = \frac{hc}{\lambda_{blue}} - \phi \rightarrow \phi = \frac{hc}{\lambda_{blue}} - eV_{stop}$$

$$\phi = \left[\frac{6.63 \times 10^{-34}Js \times 3 \times 10^{8}\frac{m}{s}}{450 \times 10^{-9}m} \times \frac{1eV}{1.6 \times 10^{-19}J}\right] - 0.8123eV = 1.95eV$$

From the table, the element is most likely Cesium (Cs).

d. The blue light incident on the emitter has an intensity $200\frac{mW}{cm^2}$. If the rate of electron production is 95% efficient, what electron current would be measured at the collector if the emitter (and collector) has an area of $1cm^2$?

$$I = \frac{\Delta Q}{\Delta t} = \left(\frac{Ne}{t}\right) \times 95\%$$

$$S = \frac{NE_{photon}}{tA} = \frac{I}{0.95eA}E_{photon} \rightarrow I = \frac{0.95SeA}{E_{photon}}$$

$$I = \frac{0.95SeA}{E_{photon}} = \frac{0.95 \times 1.6 \times 10^{-19}C \times 200 \times 10^{-3}\frac{W}{cm^2} \times 1cm^2}{6.63 \times 10^{-34}Js \times 3 \times 10^{8}\frac{m}{s} \Big/ 450 \times 10^{-9}m} = 0.069A = 69mA$$

2. A microscopic creature is swimming in a dish and the creature is too small to see with the naked eye. To see the creature, a system of lenses is used. The first lens (closest to the creature) has a power $+4.2D$, while the second lens, located a distance $L = 1.20m$ from the first lens, has a power $+8.5D$.

   a. If the creature, in the dish, is located $30cm$ to the left of the first lens, where is the image of the creature located with respect to the second lens?

   $$\frac{1}{d_{o,1}} + \frac{1}{d_{i,1}} = P_1 \rightarrow d_{i,1} = \left[P_1 - \frac{1}{d_{o,1}}\right]^{-1} = \left[4.2D - \frac{1}{0.3m}\right]^{-1} = 1.15m$$

   The image is located $1.15m$ to the right of lens one. This is a real image.

   $$\frac{1}{d_{o,2}} + \frac{1}{d_{i,2}} = P_2 \rightarrow d_{i,2} = \left[P_2 - \frac{1}{d_{o,2}}\right]^{-1} = \left[8.5D - \frac{1}{1.20m - 1.15m}\right]^{-1} = -0.087m$$

   The image is located $8.8cm$ to the left of lens two. This is a virtual image.

   b. Looking through the two-lens system, the creature was found to be swimming with a speed of $30\frac{cm}{s}$. What is the actual speed of the swimming creature in the dish?

   $$v_{c,f} = \frac{h_{c,f}}{t} = M_1 M_2 \frac{h_{c,f}}{t} = M_1 M_2 v_{c,o} \rightarrow v_{c,o} = \frac{v_{c,f}}{M_1 M_2} = \frac{v_{c,f}}{\left(\frac{d_{i,1}}{d_{o,1}}\right)\left(\frac{d_{i,2}}{d_{o,2}}\right)}$$

   $$v_{c,o} = \frac{30\frac{cm}{s}}{\left(\frac{1.15m}{0.3m}\right)\left(\frac{0.087m}{0.05m}\right)} = 4.5\frac{cm}{s}$$

c. What type of image of the creature is produced by the second lens and what is the orientation of the creature's image in the second lens with respect to the creature's orientation in the dish? To earn full credit, be sure to fully explain your answer.

The image from the first lens is real and the image from the second lens is virtual. Thus, the final image is virtual. With respect to the first lens, the real image is always inverted with respect to the object that created the image. With respect to the second lens the virtual image is always in the same orientation as the object that created the image. Thus, the final image is inverted with respect to the orientation of the original creature.

d. Suppose the creature emits red light with an intensity $S = 10\frac{mW}{cm^2}$. What are the maximum values of the electric and magnetic fields in the red light.

$$S = \frac{1}{2}c\varepsilon_0 E_{max}^2 \rightarrow E_{max} = \sqrt{\frac{2S}{c\varepsilon_0}} = \sqrt{\frac{2 \times 10 \times 10^{-3}\frac{W}{cm^2} \times \left(\frac{100cm}{1m}\right)^2}{3 \times 10^8\frac{m}{s} \times 8.85 \times 10^{-12}\frac{C^2}{Nm^2}}} = 274.5\frac{N}{C}$$

$$E_{max} = cB_{max} \rightarrow B_{max} = \frac{E_{max}}{c} = \frac{274.5\frac{N}{C}}{3 \times 10^8\frac{m}{s}} = 9.2 \times 10^{-7}T$$

3. In a Compton effect experiment, gold x-rays of unknown energy are incident on stationary electrons in an insulating material. The x-rays are detected on a detector located at an angle of 162^0 measured with respect to the direction of the incident gold x-rays. The scattered electrons in the material are found to have a recoil speed of $v = 0.232c$.

   a. What is the kinetic energy (in keV) and momentum (in $\frac{keV}{c}$) of the scattered electrons? Express your answer to at least 4 decimal places.

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-(0.232)^2}} = 1.02805$$
$$p = \gamma m v = 1.02805 \times 511\frac{keV}{c^2} \times 0.232c = 121.8774\frac{keV}{c}$$
$$K = (\gamma - 1)mc^2 = (1.02805 - 1) \times 511\frac{keV}{c^2} \times c^2 = 14.3336keV$$

   b. What is the energy (in keV) of the incident gold x-rays? Express your answer to at least 4 decimal places.

$$\frac{1}{E'} = \frac{1}{E} + \frac{1-\cos\phi}{mc^2} \rightarrow E' = \frac{Emc^2}{mc^2 + E(1-\cos\phi)} = E - K$$

$$Emc^2 = [mc^2 + E(1-\cos\phi)] \times [E - K]$$
$$Emc^2 = Emc^2 - Kmc^2 + E^2(1-\cos\phi) - EK(1-\cos\phi)$$
$$0 = E^2(1-\cos\phi) - EK(1-\cos\phi) - Kmc^2$$

$$0 = 1.9511E^2 - 27.9657E - 7324.4696 \rightarrow E = \begin{cases} 68.8544keV \\ -54.5211keV \end{cases}$$

We choose the positive solution. The gold x-rays were incident at $68.8544keV$.

c. What is the energy (in keV) of the scattered gold x-rays? Express your answer to at least 4 decimal places.

$$E = E' + K \rightarrow E' = E - K = 68.8544keV - 14.3336keV = 54.5208keV$$

This, coincidentally, is the negative and other solution to part b.

d. At what angle were the electrons scattered through?

From the y-component of the momentum:

$$0 = \frac{E'}{c}\sin\phi - p\sin\theta \rightarrow \sin\theta = \frac{E'}{pc}\sin\phi = \frac{54.5208keV}{121.8774\frac{keV}{c} \times c}\sin 162 = 0.1382$$

$$\theta = 7.95^0$$

Physics 111 Formula Sheet

Electrostatics

$F = k\frac{q_1q_2}{r^2}$

$\vec{F} = q\vec{E}; \quad E_{pc} = k\frac{q}{r^2}; \quad E_{plate} = \frac{q}{\epsilon_0 A}$

$E = -\frac{\Delta V}{\Delta x}$

$V_{pc} = k\frac{q}{r}$

$U_e = k\frac{q_1q_2}{r} = qV$

$W = -q\Delta V = -\Delta U_e = \Delta K$

Magnetism

$\vec{F} = q\vec{v}\times\vec{B} \to F = qvB\sin\theta$

$\vec{F} = I\vec{L}\times\vec{B} \to F = ILB\sin\theta$

$V_{Hall} = wv_dB$

$B = \frac{\mu_0 I}{2\pi r}$

$\varepsilon = \Delta V = -N\frac{\Delta\phi_B}{\Delta t}$

$\phi_B = BA\cos\theta$

Electric Circuits - Capacitors

$Q = CV; \quad C = \frac{\kappa\epsilon_0 A}{d}$

$C_{parallel} = \sum_{i=1}^{N} C_i$

$\frac{1}{C_{series}} = \sum_{i=1}^{N}\frac{1}{C_i}$

$Q_{charging}(t) = Q_{max}\left(1 - e^{-\frac{t}{\tau}}\right)$

$Q_{discharging}(t) = Q_{max}e^{-\frac{t}{\tau}}$

$I(t) = I_{max}e^{-\frac{t}{\tau}} = \frac{Q_{max}}{\tau}e^{-\frac{t}{\tau}}$

$\tau = RC$

$U_C = \frac{1}{2}qV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$

Electric Circuits - Resistors

$I = \frac{\Delta Q}{\Delta t}$

$I = neAv_d; \quad n = \frac{\rho N_A}{m}$

$V = IR$

$R = \frac{\rho L}{A}$

$R_{series} = \sum_{i=1}^{N} R_i$

$\frac{1}{R_{parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$

$P = \frac{\Delta E}{\Delta t} = IV = I^2R = \frac{V^2}{R}$

Light as a Wave

$c = f\lambda$

$S(t) = \frac{\text{Energy}}{\text{time}\times\text{Area}} = c\epsilon_0 E^2(t) = c\frac{B^2(t)}{\mu_0}$

$I = S_{avg} = \frac{1}{2}c\epsilon_0 E_{max}^2 = c\frac{B_{max}^2}{2\mu_0}$

$P = \begin{cases} \frac{S}{c}; & \text{absorbed} \\ \frac{2S}{c}; & \text{reflected} \end{cases}$

$S = S_0\cos^2\theta$

$v = \frac{c}{n}$

$\theta_{\text{incident}} = \theta_{\text{reflected}}$

$n_1\sin\theta_1 = n_2\sin\theta_2$

$P = \frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i}$

$M = -\frac{d_i}{d_0}; \quad |M| = \frac{h_i}{h_0}$

Light as a Particle/Relativity

$E = hf = \frac{hc}{\lambda}$

$K_{max} = hf - \phi$

$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1-\cos\phi)$

$\frac{1}{E'} = \frac{1}{E} + \frac{(1-\cos\phi)}{E_{rest}}; \quad E_{rest} = mc^2$

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$p = \gamma mv$

$E_{total} = E_{rest} + K = \gamma mc^2$

$K = (\gamma - 1)mc^2$

$E_{total}^2 = p^2c^2 + m^2c^4$

Nuclear Physics

$N = N_0 e^{-\lambda t}$

$m = m_0 e^{-\lambda t}$

$A = A_0 e^{-\lambda t}$

$A = \lambda N$

$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

Constants

$g = 9.8\frac{m}{s^2}$

$1e = 1.6 \times 10^{-19} C$

$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$

$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$

$1eV = 1.6 \times 10^{-19} J$

$\mu_0 = 4\pi \times 10^{-7} \frac{Tm}{A}$

$c = 3 \times 10^8 \frac{m}{s}$

$h = 6.63 \times 10^{-34} Js = 4.14 \times 10^{-15} eVs$

$N_A = 6.02 \times 10^{23}$

$1u = 1.66 \times 10^{-27} kg = 931.5 \frac{MeV}{c^2}$

$m_p = 1.67 \times 10^{-27} kg = 937.1 \frac{MeV}{c^2}$

$m_n = 1.69 \times 10^{-27} kg = 948.3 \frac{MeV}{c^2}$

$m_e = 9.11 \times 10^{-31} kg = 0.511 \frac{MeV}{c^2}$

Physics 110 Formulas

$\vec{F} = m\vec{a}; \quad F_G = \frac{GM_1 m_2}{r^2}; \quad F_s = -ky; \quad a_c = \frac{v^2}{r}$

$W = -\Delta U_g - \Delta U_s = \Delta K$

$U_g = mgy$

$U_s = \frac{1}{2}ky^2$

$K = \frac{1}{2}mv^2$

$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$

$\vec{v}_f = \vec{v}_i + \vec{a}t$

$v_f^2 = v_i^2 + 2a_r \Delta r$

Common Metric Units

$nano\ (n) = 10^{-9}$

$micro\ (\mu) = 10^{-6}$

$milli\ (m) = 10^{-3}$

$centi\ (c) = 10^{-2}$

$kilo\ (k) = 10^{3}$

$mega\ (M) = 10^{6}$

Geometry/Algebra

Circles: $A = \pi r^2$ $\quad C = 2\pi r = \pi$

Spheres: $A = 4\pi r^2$ $\quad V = \frac{4}{3}\pi r^3$

Triangles: $A = \frac{1}{2}bh$

Quadratics: $ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$