Physics 111

Exam #3

March 4, 2011

Name____________________________________

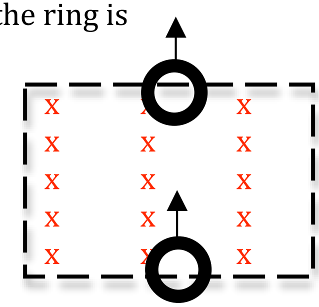
Multiple Choice	/16
Problem #1	/21
Problem #2	/21
Problem #3	/21
Problem #4	/21
Total	/100

Part I: Multiple-Choice: *Circle the best answer to each question. Any other marks will not be given credit. Each multiple-choice question is worth 4 points for a total of 16 points.*

1. Coherent light passes through a rectangular aperture (a single slit) of height h and width w. If h of the slit were halved and the w was doubled, the corresponding height and width of the diffraction pattern on a screen far away will change according to

   a. its height will double and width will halve
   b. its height and width will double
   c. its height and width will halve
   d. its height will halve and width will double

2. A plastic ring is thrown vertically upward (indicated by the arrows) through a region of space containing a magnetic field, shown below right. Due to electromagnetic induction only the direction of the current in the ring is



   a. clockwise.
   b. counterclockwise.
   c. alternating both clockwise and counterclockwise.
   d. zero.

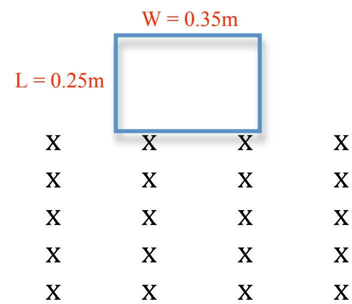
3. Light in a transparent medium with index of refraction n_1 is incident upon a plane interface with a second medium with index of refraction n_2 ($< n_1$). The incident light makes a small angle θ with respect to the normal to the plane interface. Some of the light is reflected at the interface and some of it is refracted. Compared to the speed of the refracted light, the speed of the reflected light is

   a. greater.
   b. the same.
   c. less.
   d. unable to be determined from the given information.

4. A beam of light is linearly polarized with it's electric field pointing vertically. You wish to rotate its direction of polarization by 90° (so that the electric field is pointing horizontally) using one or more *ideal* polarizing sheets. To get maximum transmitted intensity, you should use how many Polaroid sheets?
   a. One.
   b. Two.
   c. Three.
   d. As many as possible.
   e. It is not possible to rotate the polarization by 90°.

Part II: Free Response Problems: *The four problems below are worth 84 points total and each subpart is worth 7 points each. Please show all work in order to receive partial credit. If your solutions are illegible or illogical no credit will be given. A number with no work shown (even if correct) will be given no credit. Please use the back of the page if necessary, but number the problem you are working on.*

1. A single loop of wire (with dimensions shown) is held at the top edge of a region of space where there is a *2T* magnetic field that point into the page everywhere and is perpendicular to the plane of the wire loop. The wire loop has a mass of *100g* and a resistance of *0.25Ω*.



a. If the bar is released from rest with its bottom edge at the very top edge of the region of magnetic field, what is its speed just as the loop's top edge completely enters the region of magnetic field? (Hints: Assume over this interval that the net force (and hence the acceleration) across the conductor's width is zero. When the loop is completely inside of the field, the net force is not zero.) If you cannot calculate a speed use $v = 1.0m/s$.

$$\sum F_y: \quad F_B - F_w = IwB - mg = 0 \rightarrow IwB = \frac{\varepsilon}{R}wB = \frac{Bwv}{R}wB = mg$$

$$\therefore v = \frac{mgR}{w^2B^2} = \frac{0.1kg \times 9.8\frac{m}{s^2} \times 0.25\Omega}{(0.35m \times 2T)^2} = 0.5\frac{m}{s}$$

b. What are the potential difference and the magnitude of the electric field that are induced over the width of the wire?

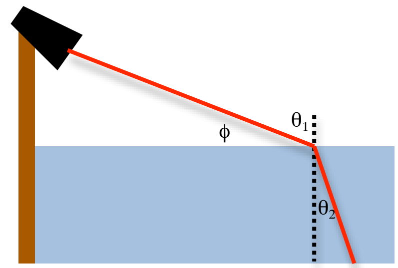
$$\varepsilon = Bwv = 2T \times 0.35m \times 0.5\frac{m}{s} = 0.35V$$

$$E = \frac{\Delta V}{\Delta x} = \frac{0.35V}{0.35m} = 1\frac{V}{m}$$

c. What are the magnitude and direction of the induced current in the loop of wire?

$I = \dfrac{\varepsilon}{R} = \dfrac{0.35V}{0.25\Omega} = 1.4A$ and the current flow is *CCW* to oppose the change in flux.

2. A spotlight is mounted on the left side and above a swimming pool and this light source sends out a beam of light into the pool as shown by the red line. The pool is *2.0m* deep with the light is mounted on the left side of the pool *1.2m* above the surface of the water and the light strikes the water's surface *2.4m* from the left edge of the pool.



a. What is the angle of refraction that the light makes upon entering the water? (The indices of refraction of air and water are $n_{air} = 1.00$ and $n_{water} = 1.33$ respectively.)

The angle of incidence is given from the geometry

$$\tan\phi = \frac{1.2m}{2.4m} = 0.5 \rightarrow \phi = \tan^{-1}(0.5) = 26.6^o$$

$$\theta_1 + \phi = 90 \rightarrow \theta_1 = 63.4^o$$

The angle of refraction is given from the law of refraction

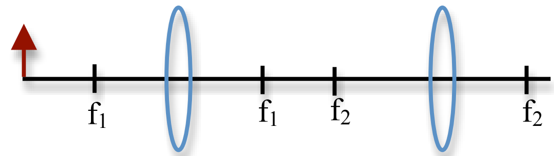
$$n_{air}\sin\theta_1 = n_{water}\sin\theta_2 \rightarrow 1.00\sin(63.4) = 1.33\sin\theta_2 \rightarrow \theta_2 = 42.2^o$$

b. How far from the left edge of the pool light strike the bottom of the pool?

From the projection of the normal to where the light strikes the bottom of the pool we have from the geometry of the system:

$\tan\theta_2 = \dfrac{x}{2.0m} \rightarrow x = 2.0m\tan(42.2) = 1.8m$. Therefore from the left edge we have the *total distance = 2.4m + 1.8m = 4.2m*.

c. Given the system of converging lenses below, what is the final image height if the object is 2.4cm tall and the focal lengths of lens #1 and lens #2 are *127mm* and *48mm* respectively? Suppose that the object is placed *450mm* to the left of lens #1 and that the lenses are separated by *200mm*.



For lens 1:

$$\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_1}\rightarrow d_{i1}=\left(\frac{1}{f_1}-\frac{1}{d_{o1}}\right)^{-1}=\left(\frac{1}{127mm}-\frac{1}{450mm}\right)^{-1}=176.9mm$$

$$M_1=-\frac{d_{i1}}{d_{o1}}=-\frac{176.9mm}{450mm}=-0.39$$

and $D=d_{i1}+d_{o2}\rightarrow d_{o2}=200mm-176.9mm=23.1mm$

For Lens 2:

$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2}\rightarrow d_{i2}=\left(\frac{1}{f_2}-\frac{1}{d_{o2}}\right)^{-1}=\left(\frac{1}{48mm}-\frac{1}{23.1mm}\right)^{-1}=-44.5mm$$

$$M_2=-\frac{d_{i2}}{d_{o2}}=-\frac{-44.5mm}{23.1mm}=1.9$$

Therefore the final image height is

$M_T=M_1M_2=\frac{h_{i2}}{h_o}\rightarrow h_{i2}=M_1M_2h_o=(0.39)(1.9)\times 2.4cm=1.8cm$ and if you want, the final image properties are virtual, inverted with respect to the object and reduced in size.

3. Suppose that you conduct an experiment to study the interference and diffraction of a beam of particles. Suppose that you have a beam of *10.0MeV* protons (this is their kinetic energy) and these protons are incident on a pair of double slits of width *a* and separation *d*. On a screen *18.0m* from the slits, there is a pattern of constructive interferences produced. You measure the distance between the centers of each adjacent constructive interference and you find they are all evenly spaced by *7.2mm*, and addition you count 11 visible constructive interference spots within the central diffraction envelope.

a. What is the wavelength of the protons used for the experiment?

$$KE = 10MeV = (\gamma - 1)m_p c^2 = (\gamma - 1) \times 937.1MeV \rightarrow \gamma = 1.011$$

$$\gamma = 1.011 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \rightarrow v = \sqrt{1 - \frac{1}{\gamma^2}} c = 0.15c$$

$$p = \gamma m v = 1.011 \times 1.67 \times 10^{-27} kg \times 0.15 \times 3 \times 10^8 \tfrac{m}{s} = 7.60 \times 10^{-20} \tfrac{kgm}{s}$$

$$\therefore \lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34} Js}{7.60 \times 10^{-20} \tfrac{kgm}{s}} = 8.7 \times 10^{-15} m$$

b. What is the separation *d* between the two slits?

$$d \frac{y_1}{D} = \lambda \rightarrow d = \frac{\lambda D}{y_1} = \frac{8.7 \times 10^{-15} m \times 18m}{7.2 \times 10^{-3} m} = 2.2 \times 10^{-11} m$$

c. What is the width *a* of one of the slits?

$$\#_{total} = 13 = 2\left(\frac{d}{a}\right) + 1 \rightarrow \frac{d}{a} = 6 \rightarrow a = \frac{d}{6} = \frac{2.2 \times 10^{-11} m}{6} = 3.6 \times 10^{-12} m$$

4. Suppose that you are conducting an experiment to investigate the photoelectric effect using a cesium target ($\phi_{Cs} = 2.9eV$.)

   a. If a *1.0W* beam of *400nm* photons are used to illuminate the *Cs* target, what will be the maximum *KE* of the electrons produced and assuming that the process of photoelectron production is 100% efficient what will be the photocurrent of electrons produced?

   $E = \frac{hc}{\lambda} = \left[\frac{6.63 \times 10^{-34} Js \times 3 \times 10^{8} \frac{m}{s}}{400 \times 10^{-9} m}\right] \times \frac{1eV}{1.6 \times 10^{-19} J} = 3.11eV$. Thus the KE of the ejected electron is $KE = E - \phi = 3.11eV - 2.9eV = 0.21eV$.

   The photocurrent is calculated from the number of photons per second, since the process is 100% efficient. Here we have

   $$1W = 1\frac{J}{s} = \frac{NE}{s} \rightarrow \frac{N}{s} = \frac{1W}{E} = \frac{1W}{4.97 \times 10^{-19} J} = 2.0 \times 10^{18} \frac{photons}{s}$$

   $$I = Ne = 2.0 \times 10^{18} \frac{photons}{s} \times \frac{1.6 \times 10^{-19} C}{1e} = 0.32A$$

   b. What potential difference, V_{stop}, would you need to stop the electrons from reaching the opposite plate from where they were ejected? (Hint: It takes work to stop the ejected electron from reaching the opposite plate.)

   $$KE = eV_{stop} \rightarrow V_{stop} = \frac{KE}{e} = \frac{0.21eV}{e} = 0.21V$$

   c. Suppose instead of the *400nm* photons used in *part a*, you decide to use *500nm* photons instead. If again you have a *1.0W* beam of *500nm* photons incident on the *Cs* target, what will be the maximum *KE* of the electrons produced and assuming that the process of photoelectron production is 100% efficient what will be the photocurrent produced?

   $$\phi = hf_{\min} = \frac{hc}{\lambda_{\max}} \rightarrow \lambda_{\max} = \frac{hc}{\phi} = \frac{6.63 \times 10^{-34} Js \times 3 \times 10^{8} \frac{m}{s}}{2.9eV \times \frac{1.6 \times 10^{-19} J}{1eV}} = 4.29 \times 10^{-7} m = 424nm.$$

   Since this is the maximum wavelength, then 500nm light will produce no ejected electrons and thus no photocurrent will be produced.

Physics 111 Equation Sheet

Electric Forces, Fields and Potentials

$$\vec{F} = k\frac{Q_1Q_2}{r^2}\hat{r}$$
$$\vec{E} = \frac{\vec{F}}{q}$$
$$\vec{E}_Q = k\frac{Q}{r^2}\hat{r}$$
$$PE = k\frac{Q_1Q_2}{r}$$
$$V(r) = k\frac{Q}{r}$$
$$E_x = -\frac{\Delta V_{B,A}}{\Delta x}$$
$$W_{A,B} = q\Delta V_{A,B}$$

Magnetic Forces and Fields

$$F = qvB\sin\theta$$
$$F = IlB\sin\theta$$
$$\tau = NIAB\sin\theta = \mu B\sin\theta$$
$$PE = -\mu B\cos\theta$$
$$B = \frac{\mu_0 I}{2\pi r}$$
$$\varepsilon_{induced} = -N\frac{\Delta\phi_B}{\Delta t} = -N\frac{\Delta(BA\cos\theta)}{\Delta t}$$

Constants

$g = 9.8\,\frac{m}{s^2}$

$G = 6.67\times10^{-11}\,\frac{Nm^2}{kg^2}$

$1e = 1.6\times10^{-19}C$

$k = \frac{1}{4\pi\varepsilon_o} = 9\times10^9\,\frac{C^2}{Nm^2}$

$\varepsilon_o = 8.85\times10^{-12}\,\frac{Nm^2}{C^2}$

$1eV = 1.6\times10^{-19}J$

$\mu_o = 4\pi\times10^{-7}\,\frac{Tm}{A}$

$c = 3\times10^8\,\frac{m}{s}$

$h = 6.63\times10^{-34}Js$

$m_e = 9.11\times10^{-31}kg = \frac{0.511MeV}{c^2}$

$m_p = 1.67\times10^{-27}kg = \frac{937.1MeV}{c^2}$

$m_n = 1.69\times10^{-27}kg = \frac{948.3MeV}{c^2}$

$1amu = 1.66\times10^{-27}kg = \frac{931.5MeV}{c^2}$

$N_A = 6.02\times10^{23}$

$Ax^2 + Bx + C = 0 \rightarrow x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$

Electric Circuits

$$I = \frac{\Delta Q}{\Delta t}$$
$$V = IR = I\left(\frac{\rho L}{A}\right)$$
$$R_{series} = \sum_{i=1}^{N} R_i$$
$$\frac{1}{R_{parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$$
$$P = IV = I^2R = \frac{V^2}{R}$$
$$Q = CV = \left(\frac{\kappa\varepsilon_0 A}{d}\right)V = (\kappa C_0)V$$
$$PE = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}$$
$$Q_{charge}(t) = Q_{max}\left(1-e^{-\frac{t}{RC}}\right)$$
$$Q_{discharge}(t) = Q_{max}e^{-\frac{t}{RC}}$$
$$C_{parallel} = \sum_{i=1}^{N} C_i$$
$$\frac{1}{C_{series}} = \sum_{i=1}^{N}\frac{1}{C_i}$$

Light as a Particle & Relativity

$$E = hf = \frac{hc}{\lambda} = pc$$
$$KE_{max} = hf - \phi = eV_{stop}$$
$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\phi)$$
$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$p = \gamma mv$$
$$E_{total} = KE + E_{rest} = \gamma mc^2$$
$$E_{total}^2 = p^2c^2 + m^2c^4$$
$$E_{rest} = mc^2$$
$$KE = (\gamma-1)mc^2$$

Geometry

Circles: $C = 2\pi r = \pi D \qquad A = \pi r^2$

Triangles: $A = \tfrac{1}{2}bh$

Spheres: $A = 4\pi r^2 \qquad V = \tfrac{4}{3}\pi r^3$

Light as a Wave

$$c = f\lambda = \frac{1}{\sqrt{\varepsilon_o\mu_o}}$$
$$S(t) = \frac{energy}{time\times area} = c\varepsilon_o E^2(t) = c\frac{B^2(t)}{\mu_0}$$
$$I = S_{avg} = \tfrac{1}{2}c\varepsilon_o E_{max}^2 = c\frac{B_{max}^2}{2\mu_0}$$
$$P = \frac{S}{c} = \frac{Force}{Area}$$
$$S = S_o\cos^2\theta$$
$$v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{n}$$
$$\theta_{inc} = \theta_{refl}$$
$$n_1\sin\theta_1 = n_2\sin\theta_2$$
$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$
$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$
$$M_{total} = \prod_{i=1}^{N} M_i$$
$$d\sin\theta_m = d\tan\theta_m = d\frac{y_m}{D} = m\lambda \text{ or } (m+\tfrac{1}{2})\lambda$$
$$a\sin\phi_{m'} = a\tan\phi_{m'} = a\frac{y_{m'}}{D} = m'\lambda$$

Nuclear Physics

$$E_{binding} = (Zm_p + Nm_n - m_{rest})c^2$$
$$\frac{\Delta N}{\Delta t} = -\lambda N_o \rightarrow N(t) = N_o e^{-\lambda t}$$
$$A(t) = A_o e^{-\lambda t}$$
$$m(t) = m_o e^{-\lambda t}$$
$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

Misc. Physics 110 Formulae

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\Delta(m\vec{v})}{\Delta t} = m\vec{a}$$
$$\vec{F} = -k\vec{y}$$
$$\vec{F}_C = m\frac{v^2}{R}\hat{r}$$
$$W = \Delta KE = \tfrac{1}{2}m(v_f^2 - v_i^2) = -\Delta PE$$
$$PE_{gravity} = mgy$$
$$PE_{spring} = \tfrac{1}{2}ky^2$$
$$x_f = x_i + v_{ix}t + \tfrac{1}{2}a_x t^2$$
$$v_{fx} = v_{ix} + a_x t$$
$$v_{vx}^2 = v_{ix}^2 + 2a_x\Delta x$$