Physics 111

Exam #3

March 8, 2013

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg)\times\left(2\frac{m}{s}\right) = 10\frac{kg\cdot m}{s}$
- Give standard SI units with your results.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 4 points and each free-response part is worth 6 points

Problem #1	/20
Problem #2	/16
Problem #3	/16
Problem #4	/20
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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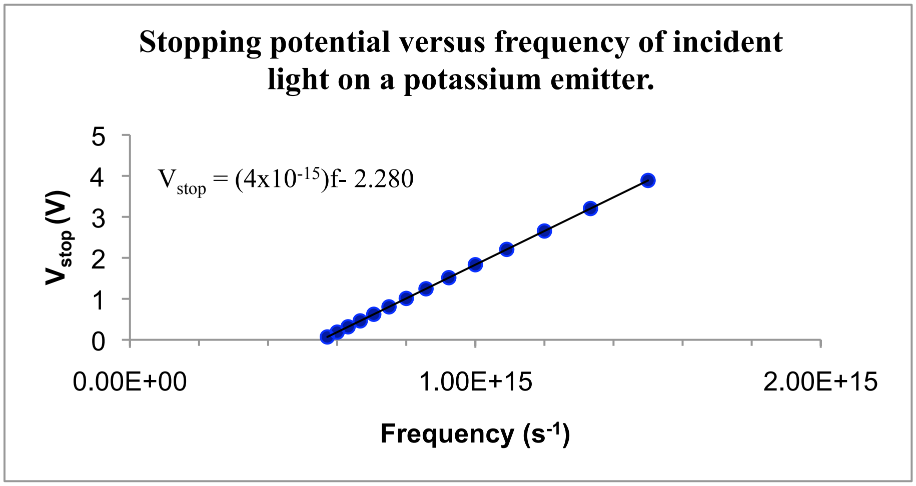
1. A potassium emitter is used in a photoelectric effect experiment. Data on the stopping potential (the potential difference needed between the emitter and collector to stop the photons from striking the collector) versus the frequency of the light incident on the emitter were taken and plotted as shown below.



a. Using information from the graph above, what is the maximum wavelength that will cause ejection of photoelectrons?

At the maximum wavelength the frequency is a minimum. This corresponds to the electron being ejected with zero kinetic energy. This corresponds to, from the graph, a zero stopping potential. Therefore,

$V_{stop} = \frac{h}{e} f - \frac{\phi}{e} = 0 \rightarrow f_{\min} = \frac{\phi/e}{h/e} = \frac{2.28V}{4 \times 10^{-15} V \cdot s} = 5.7 \times 10^{14} s^{-1}$. The maximum wavelength is given by $\lambda_{\max} = \frac{c}{f_{\min}} = \frac{3 \times 10^{8} \frac{m}{s}}{5.7 \times 10^{14} s^{-1}} = 5.26 \times 10^{-7} m = 526 nm$

b. What stopping potential would be needed to keep an electron ejected at the emitter from hitting the collector if light with a frequency $f = 1.75 \times 10^{15} s^{-1}$ was used? What was the speed of the ejected photoelectron?

$V_{stop} = \left(4 \times 10^{-15} V \cdot s\right) \times 1.75 \times 10^{15} s^{-1} - 2.28V = 4.72V$ and the speed of the ejected electron is given by

$$eV_{stop} = \tfrac{1}{2} mv^2 \rightarrow v = \sqrt{\frac{2eV_{stop}}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} C \times 4.72V}{9.11 \times 10^{-31} kg}} = 1.28 \times 10^{6} \tfrac{m}{s}$$

c. Suppose that in a different photoelectric effect experiment yellow light of intensity S_0 were incident on an ideal 100% efficient metal emitter in a phototube producing photoelectrons at a rate of N photons per second. Shining blue light of intensity ($\frac{S_0}{2}$) onto the same metal surface will

1. not produce any photoelectrons.
2. produce $2N$ photoelectrons per second.
3. produce $\frac{N}{2}$ photoelectrons per second
4. not be able to be determined.

Blue light has a shorter wavelength (and higher frequency) than yellow light. Therefore blue photons will produce photoelectrons. The intensity is proportional to the total energy and for yellow $S_{yellow} \propto N_{yellow}E_{yellow}$. For blue light, $S_{blue} = \frac{S_{yellow}}{2} \propto \frac{N_{yellow}E_{yellow}}{2} \propto N_{blue}E_{blue}$. Since the energy of the blue photons is greater than the energy of the yellow photons, the number of blue photons has to be less than the number of yellow photos, by about a factor of 2.

d. Suppose that instead of blue light with intensity S_0 incident on the emitter in part c, red light were incident on the metal emitter with intensity $2S_0$. Shining this red light onto the metal surface will

1. produce $2N$ photoelectrons per second.
2. produce $\frac{N}{2}$ photoelectrons per second.
3. produce no photoelectrons.
4. not be able to be determined.

Since yellow light works, anything with a shorter wavelength (higher frequency) will also work. However, going to a longer wavelength (and lower frequency) than yellow, I don't know if those photons have energies above the work function of the metal surface. Therefore I don't know if any photoelectrons will be produced or not.

2. An x-ray tube is shown below in Figure 1. Electrons are produced at the cathode and accelerated toward the anode. The anode is made out of silver. When the electrons interact with the target most are decelerated to rest in the anode and this deceleration produces the background radiation (called *Bremmstrahlung* or braking radiation) in Figure 2 below. However, there are times when the incident electron can eject an inner-shell electron from a silver atom. This process is called *PIXE* or *particle induced x-ray emission* and this produces (to conserve energy in the silver atom) x-rays characteristic of the target material, in this case silver and these x-rays are the two large peaks that stand up above the background. The characteristic silver x-rays are shown below where K_α x-rays are lower in energy, and the K_β x-rays are higher in energy. Filtering out the K_α x-rays, suppose that only the K_β (with an energy of $24.9keV$) x-rays are allowed to be incident on stationary electrons in a block of carbon as shown in Figure 1.

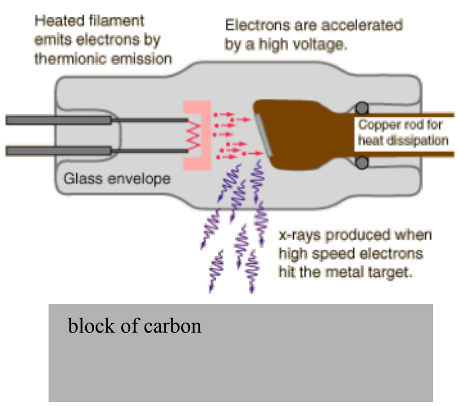


Figure 1: X-ray tube and block of carbon. Suppose that the x-rays emitted are the K_β x-rays.

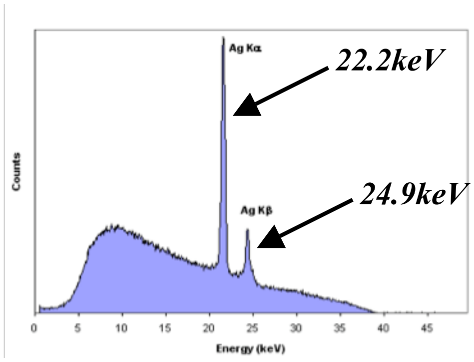


Figure 2: X-ray spectrum for silver, with the characteristic x-rays are shown as the two large peaks in the middle of the figure.

a. What are the incident and Compton scattered wavelengths for these K_β x-rays if the K_β x-rays are scattered through an angle of 75^0 with respect to their incident direction (taken as 0^0)?

The incident wavelength is determined from the energy:

$$\lambda = \frac{hc}{E} = \left[\frac{6.6\times10^{-34}Js\times3\times10^{8}\,\tfrac{m}{s}}{24.9\times10^{3}eV}\right]\times\frac{1eV}{1.6\times10^{-19}J} = 4.9925\times10^{-11}m.$$

The scattered wavelength is given from the Compton formula:

$$\lambda' = \lambda + \frac{h}{m_e c}(1-\cos\phi)$$

$$\lambda' = 4.9925\times10^{-11}m + \frac{6.6\times10^{-34}Js}{9.11\times10^{-31}kg\times3\times10^{8}\,\tfrac{m}{s}}(1-\cos 75)$$

$$\lambda' = 5.1722\times10^{-11}m$$

b. What is the recoil speed of the electron expressed as a fraction of the speed of light?

The recoil kinetic energy of the electron is the difference between the incident and scattered photon energies. The scattered photon energy is given as

$$E' = \frac{hc}{\lambda'} = \left[\frac{6.6\times10^{-34}\,Js\times 3\times10^{8}\,\frac{m}{s}}{5.1722\times10^{-11}\,m}\right]\times\frac{1eV}{1.6\times10^{-19}J} = 24.0keV$$

The kinetic energy of the electron is (assuming it to be relativistic)

$$KE_e = E - E' = 24.9keV - 24.0keV = 0.9keV = (\gamma-1)m_ec^2 = (\gamma-1)\left(511\tfrac{keV}{c^2}\right)c^2$$

$$\gamma = 1 + \frac{0.9}{511} = 1.00176 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \rightarrow v = \sqrt{1-\tfrac{1}{\gamma 2}}c = \sqrt{1-\tfrac{1}{(1.00176)^2}}c = 0.06c$$

$$v = 0.06c$$

which turns out to actually be non-relativisitic.

c. Suppose that you were to change the anode material, to say tungsten as shown in figure 3 below. The K x-rays of tungsten are very high in energy and instead the lower energy L x-rays are shown. There is no distinction made here between K and L x-rays, they are both simply x-rays. Suppose that the L_γ x-rays were incident on the same carbon block as in part a. What is the ratio of the Compton shift in the wavelength ($\Delta\lambda$) for tungsten to silver at $\phi = 75^0$?

1. 0.0
2. 0.5
3. 1.0
4. 2.2

$$\Delta\lambda = \frac{h}{m_ec}(1-\cos\phi)$$

$$\rightarrow \frac{\Delta\lambda_W}{\Delta\lambda_{Ag}} = \frac{\frac{h}{m_ec}(1-\cos\phi)}{\frac{h}{m_ec}(1-\cos\phi)} = 1$$

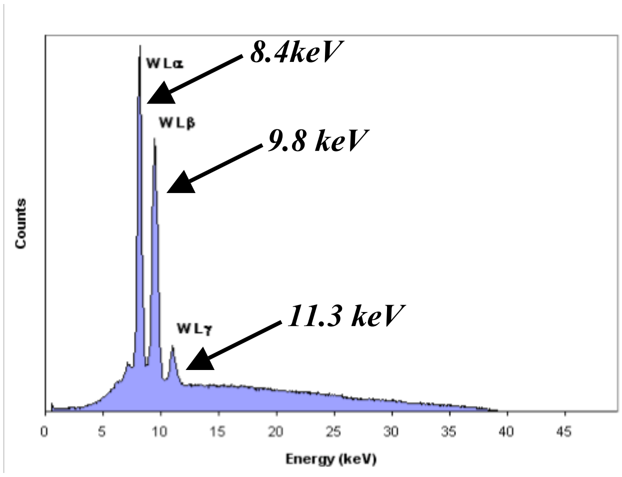


Figure 3: X-ray spectrum for tungsten, with the characteristic x-rays are shown as the three large peaks in the left of the figure.

3. Suppose that purple light ($\lambda_{purple} = 410nm$) is incident on two slits that are separated by $25\mu m$ and an interference pattern is seen on a screen $4.0m$ away.

   a. What is the spacing between adjacent interference maxima?

      The interference maxima are given by

      $$d\sin\theta_m \sim d\tan\theta_m = d\left(\frac{y_m}{D}\right) = m\lambda$$

      $$\Delta y = \frac{\Delta m\lambda D}{d} = \frac{410\times10^{-9}m\times4m}{25\times10^{-6}m} = 0.066m = 6.6cm$$

   b. Suppose that in the experiment above, two polarizers are used, where a polarizer is placed in front of each slit. The transmission axes of each polarizer are at right angles to each other and vertically polarized light is incident on the setup. If the transmission axis of the polarizer at the left slit is vertically oriented, on a viewing screen located a large distance away from the slits you will see

      1. an interference and diffraction pattern with constructive interferences whose intensities are less than they would be if the polarizers were not there.
      2. an interference and diffraction pattern with constructive interferences whose intensities are greater than they would be if the polarizers were not there.
      3. a diffraction pattern with no interference maxima or minima.
      4. no interference or diffraction pattern because the polarizers are crossed.

      Since we have vertically polarized light incident on the two polarizers in front of the slits, the left polarizer will pass all of the light while the right polarizer will pass none of the light since the incident electric field there is perpendicular to this polarizer's transmission axis. Thus light only passes through one of the slits so we lose any interference effects, but the light that passes through the left slit is diffracted and we see a diffraction pattern on the screen with no interference maxima or minima.

c. Suppose that instead of using light, you wanted to see the interference pattern produced by a beam of protons incident on a set of slits spaced $100\mu m$ apart on a screen $20m$ away. Through what potential difference would you have to accelerate the protons in order that adjacent constructive interferences are spaced $3.9cm$ apart? (Hint: Assume that the proton is non-relativistic.)

From the information given we can calculate the wavelength of the protons in the beam. Using the wavelength we can then calculate the momentum of a proton in the beam. This momentum is related to their speed and knowing their speed we can determine what potential difference is needed to accelerate the proton through to reach this speed. Therefore, the wavelength is

$$d \sin\theta_m \sim d \tan\theta_m = d\left(\frac{y_m}{D}\right) = m\lambda$$

$$\lambda = \frac{dy_1}{D} = \frac{100 \times 10^{-6} m \times 0.039m}{20m} = 1.65 \times 10^{-7} m = 165nm$$

The momentum therefore the speed of the proton is

$$p = mv = \frac{h}{\lambda} \rightarrow v = \frac{h}{m\lambda} = \frac{6.6 \times 10^{-34} Js}{1.67 \times 10^{-27} kg \times 1.65 \times 10^{-7} m} = 2.03 \tfrac{m}{s}.$$

The potential difference is determined from the work done

$$q\Delta V = \Delta KE \rightarrow V = \frac{mv^2}{2e} = \frac{1.67 \times 10^{-27} kg\left(2.03 \tfrac{m}{s}\right)^2}{2 \times 1.6 \times 10^{-19} C} = 2.1 \times 10^{-8} V$$

4. Your eye is a double convex lens and has the ability to change its focal length to accommodate objects both near and far to the lens. Over time, as you age, sometimes your eye no longer has the ability to change its focal length adequately and objects at various distances from the eye might not focus clearly on the retina. Suppose that you have the ocular condition known as *myopia*, or near-sightedness. This means objects up close are clearly focused on your retina while objects far away are not.

   a. For the person with the near-sighted eye, as the object moves farther away from the lens of your eye, the clear image of that object

      1. focuses at a point behind the retina.
      2. focuses at a point in front of the retina between your lens and retina.
      3. focuses at a point on the exterior side of your eye, that is at a point in front of your face.
      4. cannot be determined since the actual object distance and focal length of your eye is unknown.

      Since the focal length is assumed fixed for this part (no accommodation), as the object distance increases the image distances decreases. Therefore the image moves to a point between your retina and the lens of the eye. It actually doesn't move very far off of the retina, but it is enough so you cannot clearly see the far away object.

   b. If an object was placed at $25cm$ from your eye and a clear image forms on your retina located $2.5cm$ behind your lens, what is the focal length of your eye?

      The focal length is given by the thin lens equation

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{25cm}+\frac{1}{2.5cm}=\frac{1}{f_{eye}} \rightarrow f_{eye}=2.27cm$$

d. In a near-sighted eye your lens no longer has the ability to change its focal length so that objects located far away can be focused clearly on the retina. Objects can be brought into focus on your retina by using a second lens (glasses) in combination with the lens of your eye. Suppose that you want to see clearly an object located at a distance of $13m$ from your glasses. If your glasses are $1.5cm$ from your eye, what are the focal length and the type of lens that you would need to correct for *myopia*?

Here we have two lenses in combination, so we need to use the thin lens equation twice. From the sign of the focal length we're going to calculate we'll determine what type of lens is needed in the glasses. Examining the eye first we know the focal length of the eye (it's fixed – no accommodation) and the retinal distance from part b, therefore we can determine where the object should be located to form the clear image on your retina. This distance will be the image distance for the glasses that you need.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_{o,eye}} + \frac{1}{2.5cm} = \frac{1}{f_{eye}} = \frac{1}{2.27cm} \rightarrow d_{o,eye} = 25cm.$$

Then,

$$\frac{1}{d_{o,glasses}} + \frac{1}{d_{i,glasses}} = \frac{1}{13m} + \frac{1}{d_{i,glasses}} = \frac{1}{f_{glasses}},$$ where

$d_{i,glasses} = d_{o,eye} - d = 25cm - 1.5cm = 23.5cm$ from the glasses. Since this is on the same side of the lens as the object (a long ways away) that formed it, it is a virtual image and the image distance here is negative and is given by

$$\frac{1}{d_{o,glasses}} + \frac{1}{d_{i,glasses}} = \frac{1}{13m} - \frac{1}{0.235m} = \frac{1}{f_{glasses}} \rightarrow -0.239m = -23.9cm,$$ which is a diverging lens.

e. Consider the case of *presbyopia* or far-sightedness. Here objects far from the lens of the eye are focused clearly on the retina unlike the previous case of *myopia*. As the object moves closer to the eye's lens the clear image moves to a point

1. behind the retina and the correction is a diverging lens.
2. behind the retina and the correction is a converging lens.
3. between the lens of the eye and the retina and the correction is a diverging lens.
4. between the lens of the eye and the retinal and the correction is a converging lens.

   Since the focal length is assumed fixed for this part, as the object distance decreases the image distances increases. Therefore the image moves to a point behind your retina. To correct this we need to move the image distance forward and to do this we use a converging lens.

Physics 111 Equation Sheet

Electric Forces, Fields and Potentials

$$\vec{F} = k\frac{Q_1Q_2}{r^2}\hat{r}$$

$$\vec{E} = \frac{\vec{F}}{q}$$

$$\vec{E}_Q = k\frac{Q}{r^2}\hat{r}$$

$$PE = k\frac{Q_1Q_2}{r}$$

$$V(r) = k\frac{Q}{r}$$

$$E_x = -\frac{\Delta V_{B,A}}{\Delta x}$$

$$W_{A\to B} = q\Delta V_{A\to B} = -q\Delta V_{B\to A}$$

Magnetic Forces and Fields

$$F = qvB\sin\theta$$

$$F = ILB\sin\theta$$

$$\tau = NIAB\sin\theta = \mu B\sin\theta$$

$$PE = -\mu B\cos\theta$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\varepsilon_{induced} = -N\frac{\Delta\phi_B}{\Delta t} = -N\frac{\Delta(BA\cos\theta)}{\Delta t}$$

Electric Circuits

$$I = \frac{\Delta Q}{\Delta t}$$

$$V = IR = I\left(\frac{\rho L}{A}\right)$$

$$R_{series} = \sum_{i=1}^{N} R_i$$

$$\frac{1}{R_{parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$$

$$P = IV = I^2R = \frac{V^2}{R}$$

$$Q = CV = \left(\frac{\kappa\varepsilon_0 A}{d}\right)V = (\kappa C_0)V$$

$$PE = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}$$

$$Q_{\text{charge}}(t) = Q_{\max}\left(1 - e^{-\frac{t}{RC}}\right)$$

$$Q_{\text{discharge}}(t) = Q_{\max}e^{-\frac{t}{RC}}$$

$$C_{parallel} = \sum_{i=1}^{N} C_i$$

$$\frac{1}{C_{series}} = \sum_{i=1}^{N}\frac{1}{C_i}$$

Light as a Wave

$$c = f\lambda = \frac{1}{\sqrt{\varepsilon_o\mu_o}}$$

$$S(t) = \frac{energy}{time \times area} = c\varepsilon_o E^2(t) = c\frac{B^2(t)}{\mu_0}$$

$$I = S_{avg} = \tfrac{1}{2}c\varepsilon_o E_{\max}^2 = c\frac{B_{\max}^2}{2\mu_0}$$

$$P = \frac{S}{c} = \frac{Force}{Area}$$

$$S = S_o\cos^2\theta$$

$$v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{n}$$

$$\theta_{inc} = \theta_{refl}$$

$$n_1\sin\theta_1 = n_2\sin\theta_2$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$M_{total} = \prod_{i=1}^{N} M_i$$

$$d\sin\theta = m\lambda \text{ or } \left(m + \tfrac{1}{2}\right)\lambda$$

$$a\sin\phi = m'\lambda$$

Constants

$g = 9.8\,\frac{m}{s^2}$

$G = 6.67\times10^{-11}\,\frac{Nm^2}{kg^2}$

$1e = 1.6\times10^{-19}C$

$k = \frac{1}{4\pi\varepsilon_o} = 9\times10^9\,\frac{C^2}{Nm^2}$

$\varepsilon_o = 8.85\times10^{-12}\,\frac{Nm^2}{C^2}$

$1eV = 1.6\times10^{-19}J$

$\mu_o = 4\pi\times10^{-7}\,\frac{Tm}{A}$

$c = 3\times10^8\,\frac{m}{s}$

$h = 6.63\times10^{-34}Js$

$m_e = 9.11\times10^{-31}kg = \frac{0.511MeV}{c^2}$

$m_p = 1.67\times10^{-27}kg = \frac{937.1MeV}{c^2}$

$m_n = 1.69\times10^{-27}kg = \frac{948.3MeV}{c^2}$

$1amu = 1.66\times10^{-27}kg = \frac{931.5MeV}{c^2}$

$N_A = 6.02\times10^{23}$

$Ax^2 + Bx + C = 0 \to x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$

Light as a Particle & Relativity

$$E = hf = \frac{hc}{\lambda} = pc$$

$$KE_{\max} = hf - \phi = eV_{stop}$$

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\phi)$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$p = \gamma mv$$

$$E_{total} = KE + E_{rest} = \gamma mc^2$$

$$E_{total}^2 = p^2c^2 + m^2c^4$$

$$E_{rest} = mc^2$$

$$KE = (\gamma - 1)mc^2$$

Geometry

$Circles: \; C = 2\pi r = \pi D \qquad A = \pi r^2$

$Triangles: \; A = \tfrac{1}{2}bh$

$Spheres: \; A = 4\pi r^2 \qquad V = \tfrac{4}{3}\pi r^3$

Nuclear Physics

$$E_{binding} = (Zm_p + Nm_n - m_{rest})c^2$$

$$\frac{\Delta N}{\Delta t} = -\lambda N_o \to N(t) = N_o e^{-\lambda t}$$

$$A(t) = A_o e^{-\lambda t}$$

$$m(t) = m_o e^{-\lambda t}$$

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

Misc. Physics 110 Formulae

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\Delta(m\vec{v})}{\Delta t} = m\vec{a}$$

$$\vec{F} = -k\vec{y}$$

$$\vec{F}_C = m\frac{v^2}{R}\hat{r}$$

$$W = \Delta KE = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right) = -\Delta PE$$

$$PE_{gravity} = mgy$$

$$PE_{spring} = \tfrac{1}{2}ky^2$$

$$x_f = x_i + v_{ix}t + \tfrac{1}{2}a_x t^2$$

$$v_{fx} = v_{ix} + a_x t$$

$$v_{vx}^2 = v_{ix}^2 + 2a_x\Delta x$$