Experiment #1: The Electrostatic Force Law

Introduction – In this experiment we will use the Coulomb balance shown in Figure 1.1 to determine the dependence of the electric force law on the separation of the charges. The apparatus is a very delicate torsion balance. A conductive sphere is mounted on an insulating rod, counterbalanced, and suspended from a thin torsion wire. A second identical sphere is mounted on a slide assembly so that it can be positioned at various distances from the sphere on the suspended wire.

To perform this experiment, you will charge both spheres and place the sphere on the slide assembly at a fixed distance from the equilibrium position of the sphere suspended on the wire. The electrostatic force between the two spheres will cause the torsion wire to twist. You will then untwist the torsion wire to bring the suspended sphere back to its equilibrium position. The torque experienced by the torsion wire is given by equation 1.1, which Hooke's law in rotational form.

$$\tau = (moment\ arm) \times F_e = \kappa\theta \rightarrow F_e \propto \theta. \quad (1.1),$$

where κ is the torsion constant of the wire. Therefore, the angle through which the torsion wire must be untwisted is proportional to the electrostatic force between the two spheres, where the magnitude of the electrostatic force is given by the electrostatic force law, or Coulomb's law,

$$F_e = k\frac{q_1 q_2}{r^2} \quad (1.2),$$

where the direction is along the line joining the two spheres in equation 1.2.

In this experiment you will attempt to determine the experimental form of the electrostatic force law, or Coulomb's law. You will use the apparatus in Figure 1.1 to determine the experimental relationship between the electrostatic force and the separation between the two charged spheres. Then you will use this relationship to determine the charge Q placed on a sphere.

Figure 1.1: The Coulomb Balance apparatus (https://www.pasco.com)

Experiment #1 The Electrostatic Force Law Pre-Lab Exercises

Read laboratory experiment #1 on the electrostatic force law, then answer the following questions in complete sentences. Be sure to print out and hand in any data and graphs you made along with the answers to these questions. The pre-laboratory exercise is due at the beginning of the laboratory period and late submissions will not be accepted.

1. A torque is created in a system by an external force applied at some distance away from the axis of rotation of the system. This distance is called the moment arm. Mass m is added (to create an external force F) at some distance from the axis of rotation shown in Figure PLE1.1 and this external force creates a torque τ which causes the system to rotate through an angle ϕ. Data are taken on the torque τ created by the weight F of the masses and the angle ϕ through which the system rotated due to those added masses and are given in Table PLE1.1. Calculate and tabulate in Table PLE1.1 the weight associated with the masses.

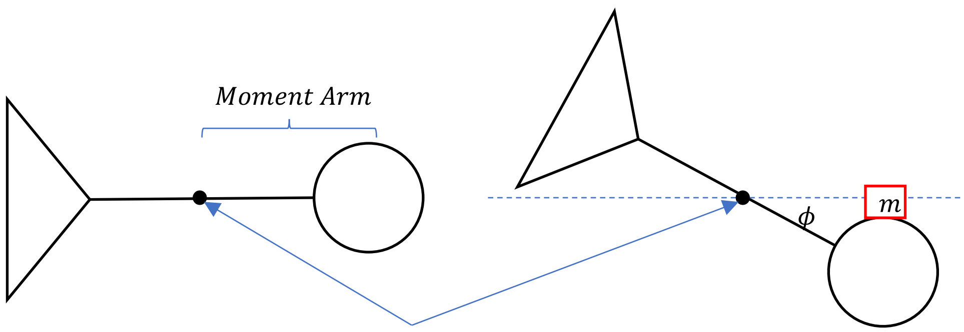

Figure PLE1.1: A system is balanced about the axis of rotation. Masses are added to the balanced system and the system rotates about the axis of rotation.

$mass\ (mg)$	$\tau\ (m \cdot N)$	$F\ (N)$	$\phi\ (degrees)$
10	0.000019404		5
20	0.000038808		11
30	0.000058212		16
40	0.000077616		20
50	0.00009702		24

Table PLE1.1: Data taken on the torque τ created and angle ϕ through which the system rotated due to masses added to the system at a fixed distance away from the axis of rotation of the system.

2. Construct a plot of the torque (on the y-axis) versus the weight (on the x-axis) according to equation 1.1 and determine the distance between the applied force and the axis of rotation, the moment arm. Explain how you determined the moment arm of the system from your plot.

   Moment arm: $M.A. =$

3. Construct a plot of the torque (on the y-axis) versus the angle through which the system rotated (on the x-axis) and determine the torsion (or rotational stiffness) constant κ of the system according to equation 1.1. Explain how you determined the torsion constant from your plot.

   Torsion constant: $\kappa =$

Experimental Procedure:

The Coulomb balance is a sensitive instrument that must be properly adjusted before you make your measurements. Impatience, air currents in the room, humidity, and static charges can affect your results. If you are careful, however, and follow the steps below, you should be able to get good results.

Activity 1: Determining the torsion constant of the wire

1. Carefully unscrew the slide assembly from the torsion balance.
2. Even more carefully turn the torsion balance on its side as is shown in Figure 1.2.
3. Use the round aluminum cylinder to help support the torsion balance.
4. You want to make sure the side arm is perpendicular the table. To do this, carefully twist the knob on the torsion balance to align the marks on the side arm and the counterweight ensuring that the rod holding the conducting sphere is parallel to the table.
5. Add a small $20mg$ mass to the conducting sphere. This will cause a torque about the torsion wire and the torsion wire will twist.
6. Record the initial angular position of the knob and slowly twist the know to realign the marks on the side arm and the counterweight.
7. Record the new angular position of the knob and determine the angle through which the wire had to be untwisted to return it to its equilibrium position. Call this value ϕ.
8. Remove the $20mg$ mass and adjust the system so that it is in equilibrium again by turning the knob back to the original setting. You may or may not get it exactly back to its original setting. If it does not go back to the original starting value, record the new starting value.

Figure 1.2: Torsion balance laid on its side with the slide assembly removed.

9. Repeat steps 5 – 7 for masses $40mg$, $50mg$, $70mg$, & $90mg$.
10. Determine an expression for the torque created by the added masses and construct a plot of the torque versus ϕ. From the plot, determine the torsion constant κ of the wire.
11. Perform a linear regression on the data to determine the uncertainty in κ.
12. Print your data, results of the linear regression analysis and the plot.

Activity 2: Determining the radial dependence of the electric force law

Below are some helpful hints before you begin the experiment with the full Coulomb balance.

Roll up your sleeves and stand a maximum comfortable distance from the Coulomb balance when performing the experiment. This will minimize the effect of static charges on your clothing interfering with the Coulomb balance.

Do not make rapid movements around the Coulomb balance because this can create air currents and the air currents will cause wild swings in the sphere suspended on the wire.

Discharge both spheres every time using the grounding probe before you charge the spheres. When charging the spheres, hold the charging probe at the end of the handle (so that your hand is as far from the sphere as possible), turn the power supply on, charge the spheres, and then immediately turn the power supply off.

Perform the measurements as quickly as possible after charging to minimize leakage effects.

Discharge and then recharge the spheres before each measurement.

1. Carefully reassemble the Coulomb balance by reattaching the slide assembly.
2. Using a Vernier caliper, make several measurements of the diameter of the sphere on the slide assembly and record these values along with their uncertainty. Determine the radius of the sphere.
3. Measure the moment arm from the wire to the center of the sphere.
4. If not already free, free the arm of the torsion pendulum and check that the etched line in the metal plate of the counterbalance, connected to the suspended sphere, matches up with the line attached to the base when the system is at rest with no charge applied. If they don't line up, turn the torsion know on the pendulum until they do. Read the dial on top and call this your "zero point." You will need to subtract this number for all your future angular displacement measures.
5. Slide the sphere on the slide assembly forward until it just touches the suspended sphere in the equilibrium position. Adjust the mark on the slide so that it shows the correct distance between the center of the spheres, which is the diameter of a sphere. This will ensure that when you set a distance on the slide scale, it will represent the center-to-center distance between the spheres, r.
6. Turn on the power supply and set the potential to $6.0kV$ and then turn off the power supply. Do not change the voltage knob on the power supply during the experiment, simply turn the supply on, charge the spheres, and then turn it back off every time, discharging both spheres between each measurement.
7. Create a data table with the column headings $r(m)$, $\theta_1(deg)$, $\theta_2(deg)$, $\theta_3(deg)$, and $\theta_{avg}(deg)$.
8. Make a note of the uncertainties in values of your measurements for of r and θ_{avg} as you do the experiment. Call these $\Delta r(m)$, the uncertainty in r, and $\Delta\theta_{avg}(deg)$, the uncertainty in θ_{avg}.
9. Slide the movable sphere as far back from the torsion apparatus as possible. Turn on the power supply and touch each sphere with the charging probe for a count of 5 seconds. Then turn the power supply back off after you've charged both spheres.

10. Position the sliding sphere at the $5.0cm$ mark on the slide assembly and then adjust the torsion knob to bring the suspended sphere back to its equilibrium position. Record the position of the sliding sphere as $r(m)$ and the angle measured on the torsion dial as θ_1 in your data table. The first time doing this it may take you a little bit of time, during which the spheres may lose some charge to the humidity in the air. If so, charge the spheres again (turn the power supply back on and touch the spheres for a few seconds) and measure the torsion angle again.
11. Repeat steps 8 – 9 two more times and record the angle measurements as θ_2 and θ_3. Consult your instructor if the angle measurements are not consistent within a few degrees of each other.
12. Repeat steps 8 – 10 to fill in your data table for distances of separation $6.0cm$, $7.0cm$, $8.0cm$, $9.0cm$, $10.0cm$, $14.0cm$ & $20.0cm$.
13. Calculate the average angle, θ_{avg}, for each value of r and record the values in your data table. Estimate the uncertainty in r and θ_{avg} and record these in the data table.
14. Create a graph of θ_{avg} versus the center-to-center separation of the spheres, r. From Hooke's law we have that $F_e \propto \theta$. Examining Coulomb's law, we see that the functional form of the relationship between θ and r should be a power law. To test this relationship with your data, you will construct two additional graphs. The first will be a graph of $\ln \theta_{avg}$ versus r and the second a graph of $\log \theta_{avg}$ versus $\log r$. One of these curves will linearize the data. If the semi-log plot linearizes the data, fit the original data of θ_{avg} and r with a trendline with an exponential of the $y = Ae^x \rightarrow \theta_{avg} = Ae^{nr}$. If, on the other hand, the log-log plot linearizes the data, fit the original data of θ_{avg} and r with a trendline with a power fit of the $y = Ax^n \rightarrow \theta_{avg} = Ar^n$. From the fit to your original data, determine the values of n and A. Make sure you display the equation and the fit on the plot that linearizes your original data of θ_{avg} and r.
15. Using the proportionality constant A, determine the charge on one of the conducting spheres.
16. Print your data and all three graphs that you constructed.

Data Analysis & Post-Laboratory Exercises

Based on your data collected and graphs generated, answer the following questions. Be sure to print out and hand in your data and graphs along with the answers to these questions.

Activity 1: Determination of the torsion constant of the system

1. Derive an expression for the torque due to the masses added to the insulating sphere and the angle ϕ through which the system rotated. Then, from the expression and your plot of the torque versus ϕ, what is the value of the torsion constant κ of the wire? Don't forget the units.

2. What is the value of κ and its uncertainty, $\Delta\kappa$, from the regression analysis? How confident are you in your result for the torsion constant? What sources of error do you think affect your value of the torsion constant?

Activity 2: Determination of the dependence of the electrostatic force law on charge separation

3. You constructed several plots in this activity. One of these plots was the angle through which the torsion arm rotated (θ_{avg}) versus the separation distance between the spheres (r). The data were clearly not linear. From this plot you then constructed two additional plots: the first, a plot of $\ln \theta_{avg}$ versus r and the second, a plot of $\log \theta_{avg}$ versus $\log r$. Which of these two plots best linearizes your data and based on this what type of relationship exists between θ_{avg} and r?

4. What is the exponent of r and the constant of proportionality A from the plot that linearized your data?

   Exponent of r:

   Proportionality constant A:

5. What is the theoretical relationship between θ and r and from question 3, what is the experimental relationship between θ and r? What is the exponent of r? How do these two expressions and exponents of r compare?

6. Based on your results for question 5, what is the relationship between the electric force (F_e) and the separation between the charged spheres (r)? What is the percent difference between your experimental value for the exponent of r and the theoretical value? If there is significant discrepancy between your theoretical and experimental values, why do you think this happened?

7. Derive an expression for the total charge on one of the isolated spheres and from this expression determine the charge on the sphere, Q_{exptl}. This is your experimental value for the charge on the sphere. How confident are you in your results for Q_{exptl}? Does your result seem reasonable? What sources of error might affect your results?

8. It can be shown that for an isolated conducting sphere of radius R the ability of the sphere to store charge, called the capacitance, is given by $C = 4\pi\varepsilon_0 R$, where $\varepsilon_0 = 8.85 \times 10^{-12}\frac{C^2}{Nm^2}$ is the permittivity of free space. Calculate the capacitance of the charged sphere, where the units of capacitance are the Farad, F.

9. It will be shown in future experiment that the amount of charge that can be stored on the system is related to the capacitance of the system and the potential difference by $Q = CV$. Calculate how much charge Q_{theo} should have been on the sphere for a potential difference of $6kV$. Calculate a percent difference between this value and the experimental one you calculated in question 7.