Experiment #4: The Horizontal Component of the Earth's Magnetic Field and the Magnetic Force Law

Introduction: When performing experiments involving magnetic fields one needs to be aware of other external magnetic fields in the vicinity of the experiment. These other external magnetic fields could be very large and their effects on your experiment should be minimized if possible. One external magnetic field is due to the earth. The earth has a magnetic field with components that point parallel to the ground in a northernly direction as well as a component that points vertically into the ground. As a first experiment we will try to determine how large of a magnetic field is produced by the earth parallel to the ground. We call this the horizontal component of the earth's magnetic field, $\vec{B}_{H,earth}$. To determine the horizontal component of the Earth's magnetic field $\vec{B}_{H,earth}$, we will compare the unknown $\vec{B}_{H,earth}$with another magnetic field $\vec{B}$ whose strength is known.

When placed in the earth's magnetic field a compass should point north as shown below in Figure 4.1. If we place the north pointing compass at the center of a $N = 15$ turn loop of wire of radius R_{coil}, as shown in Figure 4.2, we can generate another magnetic field $\vec{B}$ perpendicular to the earth's magnetic field. The magnetic field generated from this loop of wire is calculated by the Biot-Savart Law and the magnitude is given by

$$\left|\vec{B}\right| = \frac{\mu_0 N I}{2R_{coil}} \qquad (4.1),$$

where $\mu_0 = 4\pi \times 10^{-7}\frac{Tm}{A}$ is the permeability of free space. By the right-hand rule if the current I flows through the coils in Figure 4.2 clockwise, the magnetic field produced by the current would point parallel to the ground through the loop and point into the plane of the page. Alternatively, if the current I flows counterclockwise though the coils, the magnetic field produced by the current would point parallel to the ground through the loop and point out of the plane of the page.

Figure 4.1: Compass points north in the earth's magnetic field.

Figure 4.2: A current carrying coil of wire will produce a magnetic field perpendicular to the earth's magnetic field.

Suppose that the current in the coils of the loop of wire was flowing counterclockwise. By the right-hand rule this would produce a magnetic field perpendicular to the horizontal component of the earth's magnetic field pointing east. We can control the deflection angle θ, by varying the current I flowing in the coils of wire. Using the geometry shown in Figure 4.3 below we see that the horizontal component of the earth's magnetic field can be calculated according to

$$\tan\theta = \frac{B}{B_{H,earth}} \qquad (4.2).$$

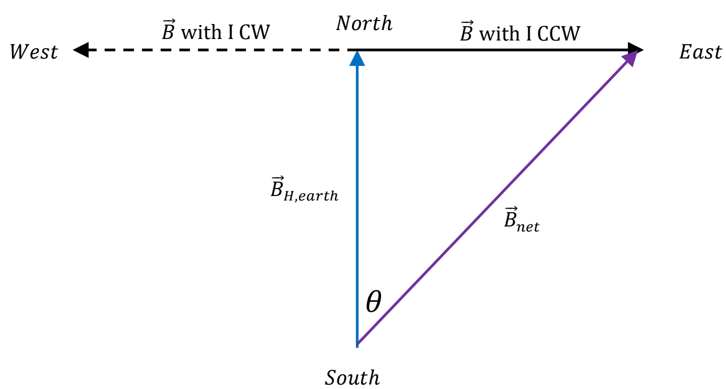

Figure 4.3: A schematic of the geometry involved between the earth's magnetic field and the magnetic field produced by the current in the coils of wire.

Assuming that the earth's magnetic field is actually small compared to the magnetic fields generated in the next experiment, we will determine the experimental form of the magnetic force law for a charge q moving with a speed v through a magnetic field B when the speed is perpendicular to the magnetic field.

To determine the experimental form of the magnetic force law, we will take a beam of charged particles (electrons with charge $q = -e$) and pass the beam through a magnetic field $\vec{B}$ perpendicular to the velocity $\vec{v}$ of the beam. By changing the accelerating potential difference that the electrons are accelerated through we can control the velocity of the charges and, for a fixed value of the magnetic field, determine the relationship between the magnetic force and the velocity of the electrons. Then by changing the current in a set of Helmholtz coils we can determine the relationship between the magnetic force and the magnetic field through which the electrons pass, for a constant value of velocity of the electron. In addition, we will determine two values for the charge-to-mass ratio $\frac{e}{m}$ for the electron.

The apparatus we will be using consists of a special vacuum tube designed for this experiment and a set of Helmholtz coils to produce the magnetic field. Three power supplies are used to produce the magnetic field, the filament current that heats the wire, the source of electrons, and the accelerating voltage that defines the energy of the electrons. The beam of electrons is produced by an electron gun composed of a heater that heats a cathode electrode, which emits electrons. Figure 4.4 is a photograph of the apparatus without the power supplies attached.

Figure 4.4: Photograph of the apparatus. The Helmholtz coils generate a uniform magnetic field that the electron beam passes through.

The kinetic energy gained by an electron is equal to the electric potential energy lost intraveling through a potential difference V. We can write this as

$$W = \Delta K \rightarrow -q\Delta V = eV = \frac{1}{2}mv^2 \qquad (4.3)$$

where we assume that the electron starts approximately from rest. From equation 4.3, the speed of the electrons as they emerge from the electron gun. The beam is made visible by the addition of a little inert neon gas in the tube; some of it evaporates in the tube and glows when electrons strike it. The path of the electron beam becomes circular when a magnetic field is applied, and the components of the tube are shown in Figure 4.5.

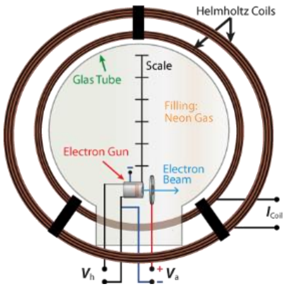

Figure 4.5: Schematic of the assembly showing the electron guns and the pins used to measure the electron beam radius. Photo: https://virtuelle-experimente.de/en/b-feld/b-feld/versuchsaufbau.php

A pair of circular coils of wire each of radius R_{coil} separated by a distance R_{coil} is used to generate a uniform magnetic field at the center of the coils. This setup for the coils of wire is called a Helmholtz Coil. The magnitude of the B-field is expressed in terms of the current I through the

Helmholtz coils and certain constants associated with the coil. By the Biot-Savart law for two coils of wire, each with N turns and radius R_{coil}, separated by a distance R_{coil}, the magnetic field at the center of Helmholtz coil is given by

$$B = \frac{8\mu_0 NI}{\sqrt{125}R_{coil}} \qquad (4.4),$$

where, N $(= 130)$ is the number of turns of the wire in each coil, I is the current through the coils, in amps, R_{coil} $(= 0.15m)$ is the mean radius of the coils (you should check this), and $\mu_0$is the permeability of free space, $\mu_0 = 4\pi \times 10^{-7}\frac{Tm}{A}$. The radius of the orbit of the electrons is a circle is such that the required centripetal force is furnished by the magnetic force. Therefore, we have

$$F = qvB = ma_c = m\frac{v^2}{R} \qquad (4.5)$$

Substitution of equations 4.3 and 4.4 into equation 4.5 yields

$$R^2 = \left(\frac{250mR_{coil}^2}{64\mu_0^2 N^2 e}\right)\frac{V}{I^2} \qquad (4.6)$$

Equation 4.6 will be used to determine the expression for the magnetic force as a function of the velocity of the charge for a constant magnetic field as well as the expression for the magnetic force as a function of the magnetic field for a constant speed of the charge.

Experiment #4 The Magnetic Field of the Earth and the Magnetic Force Law Pre-Laboratory Exercises

Read laboratory experiment #4 on the magnetic field of the earth and the magnetic force law, then answer the following questions in complete sentences. Be sure to print out and hand in any data and graphs you made along with the answers to these questions. The pre-laboratory exercise is due at the beginning of the laboratory period and late submissions will not be accepted.

1. The magnetic field along the axis of a ring (taken to be the z-axis) with N turns of wire, of radius R_{coil} with a current I flowing can be calculated from the Biot-Savart law. Without worrying about the details of the Biot-Savart law the magnitude of the magnetic field along the axis of the ring is given by

$$B_{coil} = \frac{\mu_o N I R_{coil}^2}{2(z^2 + R_{coil}^2)^{\frac{3}{2}}} \quad (PLE4.1),$$

   where the direction along the z-axis of the coil is given by the right-hand-rule. Show that the magnitude of the magnetic field, given by equation 4.1 at the center of the coil, can be determined from equation PLE4.1.

2. Look up and cite the horizontal component of the Earth's magnetic field.

3. Suppose that you have two coils of wire, each with N turns of wire, radius R_{coil}, separated by a distance R_{coil} as shown in the in Figure PLE4.1 below. The coil of wire on the left is located at a point $z = 0$, while the other coil of wire on the right is located at a distance R_{coil} along the z-axis from the first coil. Using the fact that magnetic fields are vector quantities determine the magnitude of the net magnetic field at the midpoint between the two coils of wire at $z = \frac{R_{coil}}{2}$ if the currents flow in the same direction in the coils and have equal magnitudes I. In addition, show that the net magnetic field at the midpoint between the two coils of wire is given by equation 4.4. In the side view of Figure PLE4.1 the current is flowing out of the page on the top of the coils and into the page on the bottom of the coils.

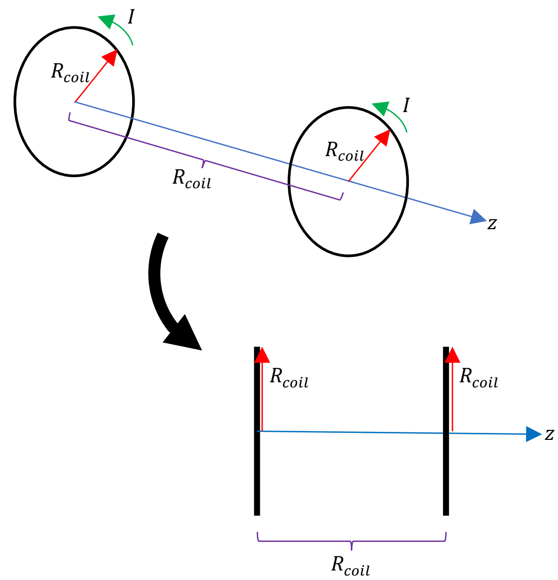

Figure PLE4.1: Two rings, each of radius R_{coil}, N turns, with current I flowing, lying along a common axis separated by a distance R_{coil}.

Experimental Procedure:

Activity 1: Determining the horizontal component of the Earth's magnetic field

1. Your circuit is mostly prewired, and you should neither take the wires out of their connections nor unplug any wires while the apparatus powered up.
2. Because the magnetic field at your lab station is mainly from the earth, you should expect that a compass needle points roughly north. Align the compass so that the red end of the compass needle and its outlined image are both pointing north as seen in Figure 1.
3. Place the compass carefully at the center of the wire loop and align the coils so that the coils point north and that the compass needle also points north.
4. Slowly turn up the current in the coils until the compass needle points 10^0 to the east. Using the digital multimeter record the current in mA. In the circuit there is a 220Ω resistor to prevent the current from getting too large and burning out the fuse of the multimeter.
5. Next, slowly increase the current until the compass needle points to the east at 20^0, 30^0, 40^0, 50^0, and 60^0. Record the current on the digital multimeter at each of these angles.
6. Turn off the power supply and reverse the leads going to the coils from the battery. Now repeat steps 4 and 5 for the compass needle deflecting to the west.
7. Repeat steps 4, 5, and 6 one more time. This will give you two values for the east and west deflection angles and two values for the currents. Average each of the east deflection angles and their corresponding currents. Then average each of the west deflection angles and their corresponding currents. You should have 12 data points, 6 angles and 6 currents east and 6 angles and 6 currents west.
8. For each current, determine the magnitude of the magnetic field, B, from $B = \frac{\mu_0 NI}{2R}$ and construct a plot of B versus $\tan\theta$ for each of your east and west deflection angles. Determine the value for the horizontal component of the Earth's magnetic field, $B_{H,earth}$, from the plot.
9. Perform a regression analysis on the data of B versus $\tan\theta$ to determine the slope and the uncertainty in the slope of the line, $B_{H,earth}$.

Activity 2: The magnetic force as a function of speed for a fixed magnetic field.

1. Your circuit should be prewired, and you should neither take the wires out of their connections nor unplug any wires while the apparatus is powered up. If your circuit is not wired, please consult with your instructor.
2. To keep the magnetic field a constant, select a value for the current in the Helmholtz coils. Record the value of the current, in amps.

Constant current: $I =$

3. Next vary the value of the accelerating potential difference (do not exceed 300V for long periods of time on the tube) until your electron beam passes between each set of pins. You may not be able to get the smallest sets of pins. Don't worry if you cannot. The pins are spaced every $2cm$. Record the value of the electron beam radius (for beam diameters $5cm$, $6cm$, $7cm$, $8cm$, $9cm$, and $10cm$ along with the value of the accelerating potential difference you needed to steer the beam through these diameters in the table below.

Beam Diameter (cm)	Beam Radius (cm)	Accelerating Voltage (V)
5		
6		
7		
8		
9		
10		

4. Using equation 4.6, plot R versus V and determine the experimental relationship between R and V. Your expression should have the form of $R = AV^n$, where A is the proportionality constant and n is the exponent of V.

   $R = AV^n =$

Activity 3: The magnetic force as a function of magnetic field for a fixed speed.

1. Choose a constant value for the accelerating potential difference (preferably one that you've already done) and record this value below.

   Constant accelerating voltage: $V =$

2. Record the value of the electron beam radius (for beam diameters $5cm$, $6cm$, $7cm$, $8cm$, $9cm$, and $10cm$ along with the value of the current through the Helmholtz coils that you needed to steer the beam through these diameters in the table below.

Beam Diameter (cm)	Beam Radius (cm)	Coil Current (A)
5		
6		
7		
8		
9		
10		

3. Using equation 4.6, plot R versus I and determine the experimental relationship between R and I. Your expression should have the form of $R = BI^m$, where B is the proportionality constant and m is the exponent of I.

   $R = BI^m =$

Data Analysis & Post-Laboratory Exercises

Based on your data collected, graphs generated, and equations of fits to the data, answer the following questions. Be sure to print out and hand in your data and graphs along with the answers to these questions.

Activity 1:

1. From your plot of the magnetic field of the small coil of wire B_{coil} versus $\tan\theta$, what is the value of the horizontal component of the Earth's magnetic field and its uncertainty.

2. What percent error exists between your experimental value for the horizontal component of the Earth's magnetic field and the theoretical value?

3. What are some sources of uncertainty in your experimental value for the horizontal component of the Earth's magnetic field?

4. Using equation 4.4 and a current of $I = 1.0A$, calculate the magnitude of the magnetic field between the two circular coils of wire? How does this compare to the magnetic field of the Earth? Do you need to consider the horizontal component of the Earth's magnetic field when doing *Activity 2* and *Activity 3*? Explain.

Activity 2:

1. What assumptions do you need to make to perform the magnetic force law experiment?

2. To determine the how the magnetic force law depends on force F and velocity v, you made a plot of the square of the electron beam radius R versus the accelerating potential difference V according to equation 4.6.
   a. How does the speed of the electron relate to the accelerating potential difference?
   b. How does the magnetic force relate to the radius of the beam of electrons?
   c. What do these two expressions imply theoretically about your graph of the electron beam radius R versus the accelerating potential difference V and the magnetic force F versus the velocity v of the charge? Does your data agree with theory?
   d. What is an approximate experimental expression for how the magnetic force F depends on the velocity v of the charge?

3. What is the constant of proportionality from your graph of R and V? What the value of this constant from equation 4.6? This is the theoretical value.

   From the plot: $C_{exptl} =$

   From equation 4.6: $C_{theo} =$

4. What is your experimental value for the charge-to-mass ratio of the electron using the proportionality constant from your plot of R and V? How does it compare to the accepted value? Explain any discrepancies between your experimental and theoretical values.

   $\left.\frac{e}{m}\right)_{theo} =$

   Using the proportionality constant: $\left.\frac{e}{m}\right)_{exptl} =$

Activity 3:

1. To determine the how the magnetic force law depends on force F and magnetic field B, you made a plot of the electron beam radius R and the current through the Helmholtz coils I according to equation 4.6.
   a. How does the magnetic field produced by the Helmholtz coils relate to the current through the Helmholtz coils?
   b. How does the magnetic force relate to the electron beam radius?
   c. What do these two expressions imply theoretically about your graph of electron beam radius R and the current through the coils I and the magnetic force F versus the magnetic field B? Does your data agree with theory?
   d. What is an approximate experimental expression for how the magnetic force F depends on the magnetic field B the charge experiences?

2. What is the constant of proportionality from your graph of R and I? What the value of this constant from equation 4.6? This is the theoretical value.

   From the plot: $C_{exptl} =$

   From equation 4.6: $C_{theo} =$

3. What is your experimental value for the charge-to-mass ratio of the electron using the proportionality constant from your plot of R and I? How does it compare to the accepted value? Explain any discrepancies between your experimental and theoretical values.

$$\left.\frac{e}{m}\right)_{theo} =$$

Using the proportionality constant: $\left.\frac{e}{m}\right)_{exptl} =$

4. What are your *expressions for the experimental magnetic force law* using your results from *Activity 2* question 2 and *Activity 3* question 1? What are the theoretical relationships? Comment on your result being sure to address any discrepancies between your theoretical and experimental results.

   Magnetic Force versus velocity:

   Magnetic force versus magnetic field:

5. What is the average value for your charge-to-mass ratio for the electron? How does this compare to the accepted value? Calculate a percent difference.

6. What are your sources of uncertainty? Are these sources, as you would expect, given the assumptions you made at the beginning of the experiment? Are there things you would modify or change after having performed the experiment?