Experiment #5: Light as a Wave - Geometric Optics and Lenses

Introduction: Geometric optics treats light as rays that travel in straight lines and refract at the boundaries between different optical media. In a simple thin lens, the law of refraction combined with some simple geometry yields the thin lens and magnification equations.

Consider the thin double-convex (converging) lens below of focal length f_c. An object with height h_o is placed a distance d_o (the object distance) to the left of the center of the converging lens as shown in Figure 5.1. Rays of light are drawn to locate the image of the object using the refraction of light at the front and back surfaces of the lens. If the ray diagram in Figure 5.1 is drawn to scale, the ray diagram can be used to measure, with respect to the lens, the location of the image d_i (the image distance) and the height of the image h_i. Using the geometry from the ray diagram in Figure 5.1, we see that the magnification M of the image (the ratio of the image height to the object height) can be determined from

$$\tan\alpha = \frac{h_o}{d_o} = \frac{h_i}{d_i} \rightarrow M = \frac{h_i}{h_o} = \frac{d_i}{d_o} \quad (5.1).$$

If the ray-diagram were drawn to scale, we could measure the image height h_i directly, or from equation 5.1, we could calculate the image height h_i if we knew where the image was located, d_i. From the scale drawing of the ray diagram, we could measure the image distance d_i. However, we'd like to be able to predict where the image should be for a given focal length lens and object distance d_o from the lens. Again, using the geometry from the ray diagram in Figure 5.1, we have

$$\tan\beta = \frac{h_o}{d_o - f_c} = \frac{h_i}{f_c} \quad (5.2).$$

Using equation 5.2, we can determine an expression for the image distance d_i for the image of the object produced by the lens. This is shown in equation 5.3.

$$\frac{h_i}{h_o} = \frac{d_i}{d_o} = \frac{f_c}{d_o - f_c} \rightarrow d_i = \frac{f_c d_o}{d_o - f_c} \quad (5.3)$$

Equation 5.3 is the thin lens equation for a converging lens. It is, however, not the most useful form. Cross-multiplying and dividing equation 5.3 by the product $d_o d_i f_c$ will yield the thin lens equation, equation 5.4.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f_c} \quad (5.4)$$

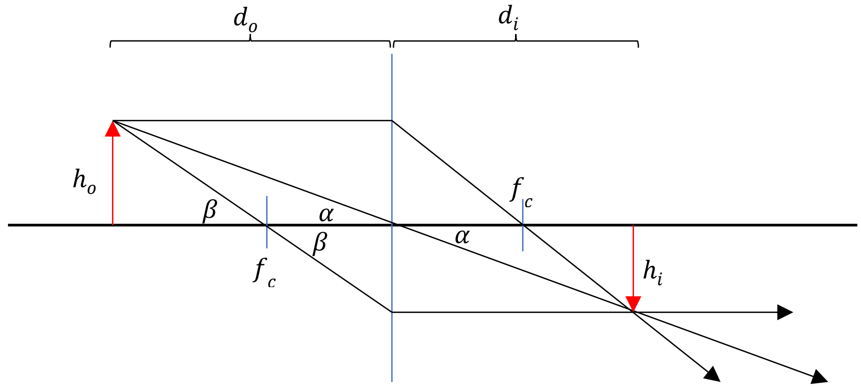

Figure 5.1: Ray diagram for a thin converging lens used to determine the formula for the magnification of the image by the lens and to determine the thin lens equation to calculate d_i.

As a second type of lens, consider the thin double-concave (diverging) lens of focal length f_d shown in Figure 5.2 below. An object with height h_o is placed a distance d_o (the object distance) to the left of the center of the diverging lens as shown in Figure 5.2. Rays of light are drawn to locate the image of the object using the refraction of light at the front and back surfaces of the lens. Again, if the ray diagram in Figure 5.2 is drawn to scale, the ray diagram could be used to measure, with respect to the lens, the location of the image d_i (the image distance) and the height of the image h_i. Using the geometry from the ray diagram in Figure 5.2, we see that the magnification M of the image (the ratio of the image height to the object height) can be determined from

$$\tan\alpha = \frac{h_o}{d_o} = \frac{h_i}{d_i} \rightarrow M = \frac{h_i}{h_o} = \frac{d_i}{d_o} \quad (5.5).$$

If the ray-diagram were drawn to scale, we could measure the image height h_i directly, or from equation 5.5, we could calculate the image height h_i if we knew where the image was located, d_i. From the scale drawing of the ray diagram, we could measure the image distance d_i. However, we'd like to be able to predict where the image should be for a given focal length lens and object distance d_o from the lens. Again, using the geometry from the ray diagram in Figure 5.2, we have

$$\tan\beta = \frac{h_i}{f_d - d_i} = \frac{h_o}{f_d} \quad (5.6).$$

Using equation 5.2 (or equivalently equation 5.5), we can determine an expression for the image distance d_i for the image of the object produced by the lens. This is shown in equation 5.7.

$$\frac{h_i}{h_o} = \frac{d_i}{d_o} = \frac{f_d - d_i}{f_d} \quad (5.7)$$

Equation 5.7 is the thin lens equation for a diverging lens. It is, however, not the most useful form of the thin lens equation. Cross-multiplying and dividing equation 5.7 by the product of $d_o d_i f_d$ will give the thin lens equation, equation 5.8.

$$\frac{1}{d_o} - \frac{1}{d_i} = -\frac{1}{f_d} \qquad (5.8)$$

Equations 5.4 and 5.8 share some a common form but have some differing signs. To combine equations 5.4 and 5.8 into a single thin lens equation for both converging and diverging lenses, we use some sign conventions. First, we assume that the objects are all real so that $d_o > 0$. Converging lenses have positive focal lengths $f_c > 0$, while diverging lenses are defined mathematically to have negative focal lengths $f_d < 0$. With this set of sign conventions, we also see that real images will have a positive image distance $d_i > 0$ and virtual images a negative image distance $d_i < 0$. Using these sign conventions, we have the thin lens equation given by equation 5.9 and the magnification of either lens is given by equation 5.2 or 5.5

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \qquad (5.9)$$

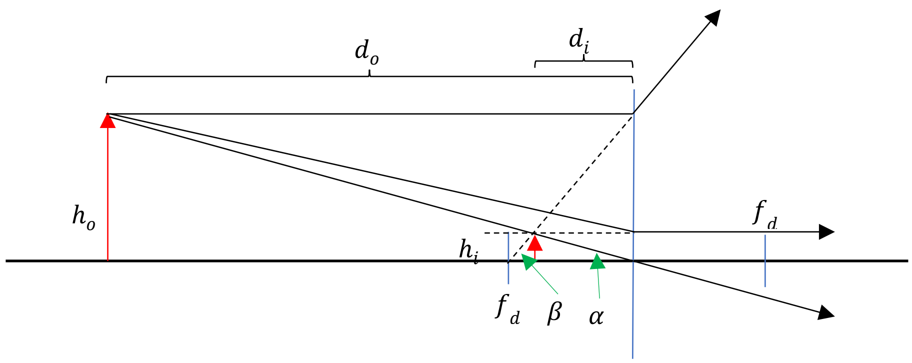

Figure 5.2: Ray diagram for a thin diverging lens used to determine the formula for the magnification of the image by the lens and to determine the thin lens equation to calculate d_i.

Experiment #5: Light as a Wave - Geometric Optics Pre-Lab Exercises

Read laboratory experiment #5 on geometric optics, then answer the following questions in complete sentences. Be sure to print out and hand in any data and graphs you made along with the answers to these questions. The pre-laboratory exercise is due at the beginning of the laboratory period and late submissions will not be accepted.

1. An object is placed at various distances from a thin lens and the corresponding real image distances are measured. Table PLE5.1 shows the results of the measurements. Plot the data according to equation 5.9 in such a way that the data are linear and that the slope of the line is minus one.

$d_o (mm)$	$d_i (mm)$
60	240
80	120
100	92
140	73
160	69
170	67
180	66
200	63
210	62

Table PLE5.1: Object and image distances used to determine the focal length of a lens.

2. What is the focal length of the lens and what type of lens (converging or diverging) was used in the experiment. Justify your answer of converging or diverging.

   Focal length of the lens: $f =$

   Lens type:

Experimental Procedure:

Activity 1. Determining the focal lengths of a converging lens

In this experiment we want to determine the focal length of a converging lens. We will do this by setting a fixed distance between the object (a light bulb filament) and a screen on which to see the real image produced by the lens. The setup of the experiment is shown in Figure 5.3.

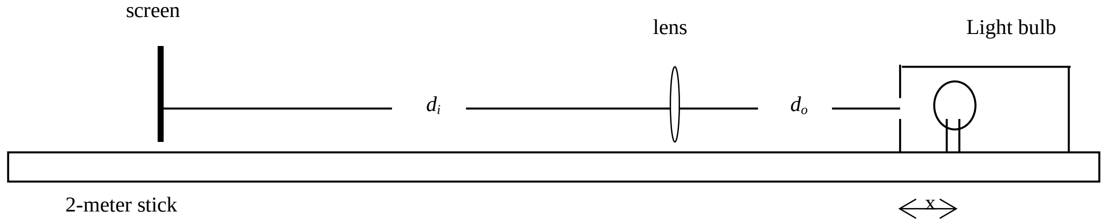

Figure 5.3: A schematic showing the basic setup for determining the focal length of a converging lens.

1. Starting with the light source and screen far apart along the scale and the converging lens labeled A find two different locations where the lens can be placed between the light source and screen, so that a focused image appears on the screen.
2. Record the location of all three, namely, the two locations that the lens can be placed and the location of the screen from the light source.
3. Now move the screen closer to the light source in increments of your choice. Repeat steps 1 & 2 for a total of five screen-to-light source locations. Each time you move the screen, again find two lens locations that produce a focused image. In the end you will have 10 object distances and 10 image distances.
4. From equation 5.9, determine a way to plot your data so that the data are linear with a slope of negative one ($slope = -1$).
5. Fit the data with a linear trendline and record the equation of the fit below.

   Equation of the fit to the data:

6. You may find that the slope of your graph is different from -1. The reason is that your measurement of d_o does not include the distance x that is seen Figure 5.3. We can correct for that. First make and record in your data table a rough estimate of the distance x. Modify your data table in your spreadsheet so that it adds the value of x to each value of d_o. Change the value of x until you get the slope of your plot as close as possible to -1.

   Value of x:

7. Determine the value of the focal length of the lens A.

Activity 2: Measuring the size of the light bulb filament.

How large is the light bulb filament in your light box?

1. Using one of your data points (that is pick a d_o and its correspoinding d_i) for the converging lens, determine the size of the curly part of the light bulb filament.
2. Measure the height of the curly part of the filament on the screen for the data point you chose.
3. From these measurements, determine the size of the light bulb filament.

Activity 3: Determining the focal length of a diverging lens

The method you used in activity 1 to determine the focal length of the converging lens will not work with the diverging lens, as it does not focus light. Mathematically, diverging lenses are treated as if they have a negative focal length and always produce virtual images. Since the light does not actually focus at the location of the virtual image, therefore its location cannot be measured.

1. With the same light source, put a diverging lens on the optical bench and verify that there is no real image produced on the screen.
2. To observe the virtual image, look through the lens at the light source. Don't do this for too long as the light source is very bright.
3. Place the converging lenses beyond the diverging lens and look for a real image produced on the screen from the converging lens. This may take some practice to find the real image produced from the converging lens.
4. When you have a real image produced, measure all relevant object, and image distances and record them in Table 5.1 below.

Focal length of the converging lens f_c	
Object distance from the diverging lens, $d_{o,d}$	
Distance between the diverging and converging lenses L	
Image distance from the converging lens $d_{i,c}$	

Table 5.1 Data table for determining the focal length of a diverging lens.

5. Compute the focal length of the diverging lens using equation 5.9 twice. You will need to do the calculations in steps, first computing the virtual image location, then the object distance for the second lens, and finally the final real image location.
6. The actual value of the focal length stamped on the diverging lens is $f_D = -2.2cm$. Using this value, the position of the diverging lens (from the light source $d_{o,d}$) and the distance between the diverging and converging lenes L, draw a complete ray diagram ***to scale***.
7. Measure the distance between the converging lens and the real image location on your scale drawing d_{ic}. You will need to select a convenient scale for the drawing both horizontally and vertically, and you will need to use ***at least a*** whole sheet of paper, a ruler, and a pencil to draw the ray diagram.

Activity 4: A second method to determine focal length of a converging lens

There is another way that we can use to determine the focal length of a converging lens. From your work in activity 1, you found that for each separation of the screen-to-light source, there are two different positions of the lens that will bring an image into focus.

1. Let D be the light bulb-to-screen distance, and for a given object distance d_o use equation 5.9 to relate d_o, D, and f_c.
2. Using equation 5.9 to relate d_o, D, and f_c will lead to a quadratic equation in d_o. Solving this quadratic will give you two positions that you can place the lens between the light source and the screen. Calling these two positions that you can put the lens from the light source $d_{o,1}$ and $d_{o,2}$, determine the expressions for $d_{o,1}$ and $d_{o,2}$.
3. Subtract $d_{o,1}$ from $d_{o,2}$ and define the result of the subtraction to be s. This is shown in Figure 5.4 below.
4. From your expression for s, solve for the expression for focal length of the converging lens f_c.
5. You already have the data you need to calculate f_c in this way. Modify your spreadsheet by adding new columns, one for D and one for s.
6. Using the expression, you derived in step 4, and the columns you created in step 5, calculate a value f_c.

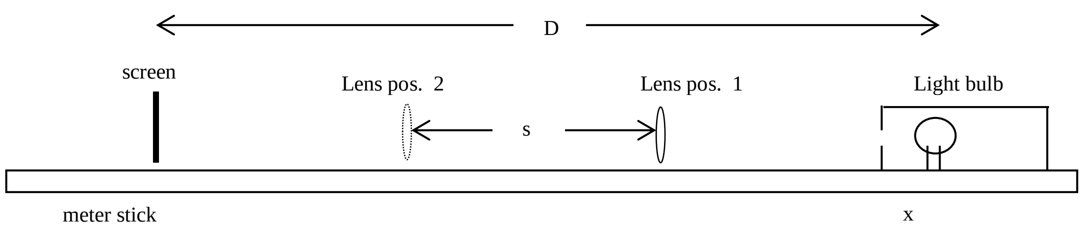

Figure 5.4: A schematic drawing showing the basic setup for determining the focal length of a converging lens using the two lens positions between a screen and light source.

Data Analysis & Post-Laboratory Exercises

Based on your data collected, graphs generated, and equations of fits to the data, answer the following questions. Be sure to print out and hand in your data and graphs along with the answers to these questions.

1. From the data you took on the object and image distances (d_o and d_i) for the converging lens, explain what you plotted and why so that the data were linear with a $slope = -1$.

2. What was the value of x that you needed to add to d_o to make the slope as close to negative one as possible? Does this value seem reasonable given how far the light bulb filament was behind the square opening on the light source? Explain.

3. From the plot you made for question 1, what does the y-intercept of the plot mean? Determine focal length $f_{c,expt}$ of the lens that you used from the y-intercept? Calculate a percent difference between $f_{c,expt}$ and the actual value $f_{c,theo} = 127mm$.

4. Describe your procedure that you used to determine the size of the light bulb filament. From this procedure, what is the size of the light bulb filament? Show all your calculations below. How reasonable is the value that you determined? Explain.

5. From Table 5.1, list your values for the distance the object was from the diverging lens $d_{o,d}$, the distance between your diverging and converging lenses L, and the distance from the converging lens to the screen on which the real image was formed $d_{i,c}$. From these data, calculate the value of the virtual object distance for the converging lens, $d_{o,c}$ from equation 5.9? Is it larger than the distance between the lenses? Should it be? Explain.

6 Subtracting the distance between the converging and diverging lenses from $d_{o,c}$, what is the location of the virtual image from the diverging lens $d_{i,d}$. Using this value and equation 5.9, calculate the focal length of the diverging lens f_d? How does your value for the focal length of the diverging lens compared to the actual value of $f_d = -22mm$? Calculate a percent difference.

7. From your scale drawing using the actual object to diverging lens distance $d_{o,d}$, the actual value of the focal length of the diverging lens f_d , the separation between the converging and diverging lenses L, measure (from the converging lens) the location of the real image that was produced, $d_{i,c}$. How does this value compare to what you measured in the experiment? Comment on your results? If the two values are not reasonably close to each other, explain why they are not. Attach your scale drawing to the lab report.

8. Derive the relationship that you used to determine the focal length f_c of the converging lenses that relates the variables D, s and f_c using Figure 5.4.

9. What is the average value of the focal length f_c for your converging lens using the expression you derived in question 8? Calculate a percent difference