Physics 111

Optional Extra Credit Assignment

Assigned: Wednesday, March 13 2013 at 8:00^{AM}

Due: Friday, March 15 2013 at 12:00^{PM}

Name___________________________________

Problem #1	/10
Problem #2	/10
Problem #3	/10
Problem #4	/10
Problem #5	/10
Problem #6	/10
Total	/60
Extra Credit	/5

I affirm that I have carried out my academic endeavors with full academic honesty.

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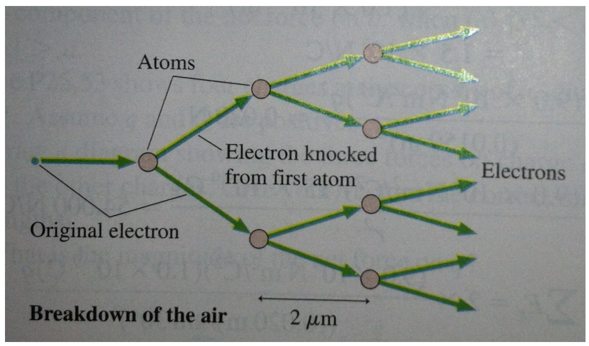
By submitting this packet in whole or in part, you agree to the following:

1. Late extra credit assignments will not be looked at or graded and no extra credit points will be assigned.
2. This optional extra credit assignment will award you up to 5 additional points on top of your final class score. You are under no obligation to do any of this assignment and you may do none, some or all of the assignment. Any amount you do will earn you some number of extra points on your final class score.
3. I will grade this optional assignment and to determine the number of extra credit points you will earn on your final score I will multiply the percent you earn on this assignment by five.
4. I will determine grades for the class as if this assignment was not present and after assigning grade ranges will then add your extra credit points onto your final score. This has the potential to raise your grade by one letter grade. For example, if your final grade were at the class average of a B-, these extra points could potentially bring your grade up to a B.
5. Since this assignment is optional, you do not have to do it and your final grade will be whatever you earn. This assignment can only help you, not hurt you.
6. You must work independently and consult no one on the preparation of this optional extra credit assignment.
7. You are not allowed to look up solutions on websites like Chegg.com or the like. This is absolutely cheating (and you shouldn't be using these for your homework either! That's cheating too! Remember the honor code at the beginning of each assignment? It's not there just for an extra homework point.) Copying my solutions to problems is also not allowed.
8. You may consult only your textbook, class notes, class website, or class homework assignments solutions for guidance.
9. You may not consult tutors of any kind, other faculty, other students, or the Internet for solutions – this is cheating.
10. *If I suspect anyone of not submitting their own work, or looking up solutions, or anything that doesn't demonstrate your own understanding of the material, I reserve the right to not allow any extra credit for any reason. There are no exceptions!*
11. I will treat this optional assignment as if it were an exam. I will answer questions, but will not help you with the solutions. The solutions are for you to determine.
12. Your work must be legible, and the organization clear and you must show all work, including correct vector notation.
13. You will not receive full credit for correct answers without adequate explanations, and you will not receive full credit if incorrect work or explanations are mixed in with correct work. So make sure your final version is what you want graded.
14. Make explanations complete but brief. Do not write a lot of prose.
15. Include diagrams and give standard SI units with your results unless otherwise specified.
16. Show what goes into a calculation, not just the final number. For example,

$$|\vec{p}| \approx m|\vec{v}| = (5kg)\times\left(2\,\tfrac{m}{s}\right) = 10\,\tfrac{kg\cdot m}{s}$$

1. Why is it that if you scuff your feet across a carpeted surface you get a shock when you reach out and touch, say a doorknob? Cosmic rays, steams of high-energy particles or photons usually from the sun incident in the upper atmosphere, if they have sufficient energy can ionize air molecules. If there is an external electric field present, a free electron may be accelerated until it collides with an air molecule. When this electron collides with an air molecule, it can give some of its kinetic energy to the molecule. If an electron's kinetic energy just before a collision is $2\times10^{-18}J$ or more, it can knock an electron out of the molecule it hits as is shown below in the diagram. If a strong external electric field is present, this causes a "chain reaction" of electron production. This is called a *dielectric breakdown of the air*. The chain reaction of electrons is what gives you the shock. The spark is generated by a burst of light energy when the electrons recombine with positive ions.



a. What strength electric field must act on the electron in order for the electron to gain $2\times10^{-18}J$ of kinetic energy in this distance? This is called the *breakdown field strength*.

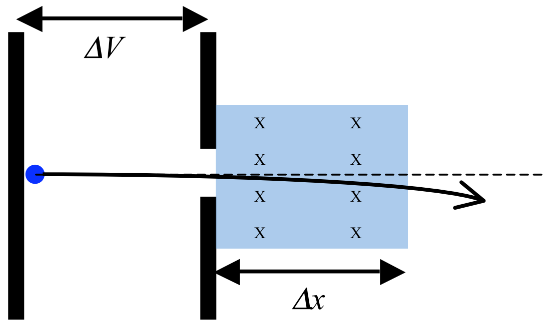
$$W = Fd = qEd = \tfrac{1}{2}mv^2 = 2\times10^{-18}J$$

$$E = \frac{W}{qd} = \frac{2\times10^{-18}J}{1.6\times10^{-19}C\times2\times10^{-6}m} = 6.25\times10^{6}\tfrac{N}{C}$$ in the direction opposite the motion of the electron.

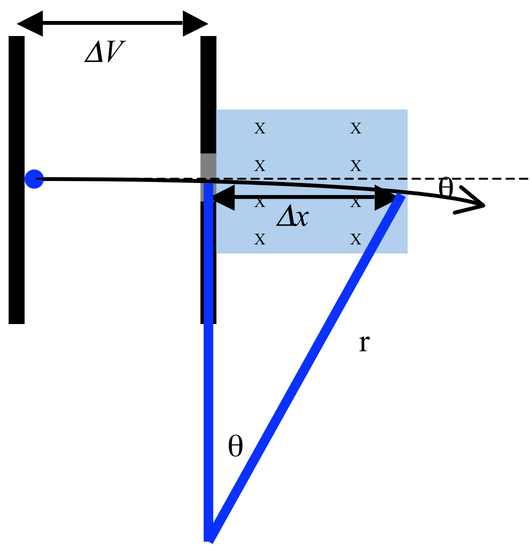
b. Suppose that a free electron in air is $1.0cm$ away from a point charge (say the doorknob). To create a spark, what is the smallest amount of charge $q_{\min}$ that this point charge must have?

$$E = \frac{kq_{\min}}{r^2} \rightarrow q_{\min} = \frac{Er^2}{k} = \frac{6.25\times10^{6}\frac{N}{C}\times(0.01m)^2}{9\times10^{9}\frac{Nm^2}{C^2}} = 6.9\times10^{-8}C = 69nC$$

2. An electron in a cathode-ray tube is accelerated through a potential difference of ΔV, and then passes through a region of space of width Δx and magnetic field strength B as shown in the figure below. Further, the electron is deflected downwards by an angle θ with respect to the horizontal.



a. Derive an expression for the deflection of the electron beam (measured from the horizontal) in terms of the given quantities, ΔV, B and Δx.



$$W = q\Delta V = \tfrac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2q\Delta V}{m}}$$

$$\sin\theta = \frac{\Delta x}{r} \rightarrow r = \frac{\Delta x}{\sin\theta}$$

$$F = qvB = \frac{mv^2}{r} \rightarrow qB = \frac{mv}{\Delta x}\sin\theta$$

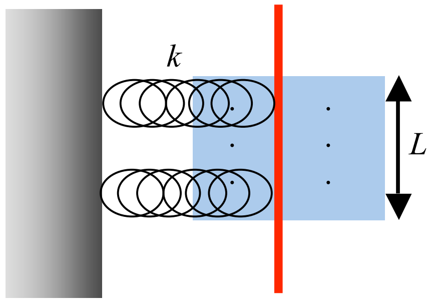
$$\Rightarrow \sin\theta = \frac{qB\Delta x}{mv} = \sqrt{\frac{q^2B^2\Delta x^2}{m^2v^2}} = \sqrt{\frac{q^2B^2\Delta x^2 m}{m^2 2q\Delta V}}$$

$$\therefore \sin\theta = \sqrt{\frac{qB^2\Delta x^2}{2m\Delta V}}$$

b. What magnetic field strength B, will deflect the electron by 10^0 if the potential difference is $\Delta V = 10.0kV$, and the magnetic field region is $\Delta x = 2.0cm$ wide?

$$B = \sqrt{\frac{2m\Delta V}{q\Delta x^2}}\sin\theta = \sqrt{\frac{2\times 9.11\times 10^{-31}kg\times 10000V}{1.6\times 10^{-19}C\times(0.04m)^2}}\sin 10 = 0.00144T = 1.47mT$$

3. Two springs shown in the figure below each have a spring constant k. They are stretched/compressed by a distance Δx when a current I is passed through the red wire (of length L) due to the current in the wire interacting with the external magnetic field B.



a. Derive an expression for the extension/compression of the springs in terms of the current I, magnetic field strength B, the spring constants k, the length of wire L, and the distance of extension/compression Δx.

$$\sum F_x: \quad -2k\Delta x + ILB = ma_x = 0 \rightarrow \Delta x = \frac{ILB}{2k}$$

b. If $k = 10\frac{N}{m}$ and the springs are stretched to the right by $\Delta x = 1.0cm$, how large is the current (magnitude and direction) in the wire if $L = 20cm$ and $B = 50mT$?

$$\Delta x = \frac{ILB}{2k} \rightarrow I = \frac{2k\Delta x}{LB} = \frac{2 \times 10\frac{N}{m} \times 0.01m}{0.2m \times 50 \times 10^{-3}T} = 20A$$ flowing up the wire.

4. Suppose that you have an incident photon of wavelength λ that makes a head-on collision with a free electron. The collision causes the electron to move in the direction of the incident photon's motion while the incident photon scatters in a direction exactly opposite its initial motion. ***In the questions that follow, you may not use the Compton shift in wavelength formula.*** You must find another way to do the problem to answer the questions.

   a. What is the kinetic energy of the electron?

   $KE = \frac{p_e^2}{2m_e} = \frac{hc}{\lambda} - \frac{hc}{\lambda'} = \frac{hc}{\lambda} - \frac{hc}{\lambda} + \frac{2m_e hc^2}{h} = 2m_e c^2$, where the results from *part b* are used. The full derivation is shown in *part b*.

   b. What is the wavelength of the recoiling photon?

   Conservation of momentum: $\frac{h}{\lambda} = \frac{h}{\lambda'} + p_e$ eqn. (1)

   Conservation of Energy: $\frac{h}{\lambda} + m_e c^2 = \frac{h}{\lambda'} + \left[m_e^2 c^4 + p_e^2 c^2\right] = \frac{h}{\lambda'} + m_e c^2 \left[1 + \frac{p_e^2}{m_e^2 c^2}\right]^{\frac{1}{2}}$

   Expanding the term in parenthesis in a power series ($(1+x)^n \approx 1 + nx$) for small $x = \frac{p_e^2}{m_e^2 c^2}$, we get $\frac{h}{\lambda} + m_e c^2 = \frac{h}{\lambda'} + \left[m_e^2 c^4 + p_e^2 c^2\right] \approx \frac{h}{\lambda'} + m_e c^2 \left[1 + \frac{p_e^2}{2m_e^2 c^2}\right]$.

   Therefore we have $\frac{hc}{\lambda} = \frac{hc}{\lambda'} + \frac{p_e^2}{2m_e}$ eqn. (2). Here we have two equations (1) and (2) with two unknowns. Solving (1) for the momentum of the electron, squaring the result and inserting that result into equation (2) produces: $\frac{1}{\lambda'^2} + \left(\frac{2m_e c}{h} - \frac{2}{\lambda}\right)\frac{1}{\lambda'} + \left(\frac{1}{\lambda^2} - \frac{2m_e c}{h\lambda}\right) = 0$. This is a quadratic in $\frac{1}{\lambda'^2}$ and when it's solved you get two solutions: $\frac{1}{\lambda'} = \begin{cases} \frac{1}{\lambda} \rightarrow \text{forward scattering} \\ \frac{1}{\lambda} + \frac{2m_e c}{h} \rightarrow \text{back scattering} \end{cases}$. So we choose the solution for backscattering since that's the condition in the problem.

   Therefore, $\frac{1}{\lambda'} = \frac{1}{\lambda} + \frac{2m_e c}{h}$. Using this in (2) we can solve for the kinetic energy and we get $\frac{h}{\lambda} = \frac{h}{\lambda'} + \frac{p_e^2}{2m_e} \rightarrow KE = \frac{p_e^2}{2m_e} = \frac{hc}{\lambda} - \frac{hc}{\lambda'} = 2m_e c^2$.

5. An object is placed on one side of a double convex lens of focal length f, and a screen placed on the opposite side of the lens as the object. The distance $d_T = d_o + d_i$ between the object and the screen is kept fixed, but the lens can be moved.

   a. If the total distance is greater than four times the focal length ($d_T > 4f$), there will be two locations where the lens can be placed such that a clear image will be produced on the screen. What are those two locations? Further, show that if the total distance is less than four times the focal length ($d_T < 4f$), that there will be no lens position where a clear image can be formed.

   From the thin lens equation we have $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} + \frac{1}{d_T - d_o}$. Get a common denominator and cross-multiply and you'll get a quadratic equation in the object distance $d_o^2 - d_o d_T + f d_T = 0$. The solutions depend on what's under the square root using the quadratic equation. We have 3 cases under the square root, $d_T^2 - 4 f d_T \to d_T - 4f$. The argument of the square root has to be greater than or equal to zero for real solutions to exist. Thus for each case we have $d_o = \begin{cases} d_T = 4f \to d_i \\ d_T < 4f \to imaginary \\ d_T < 4f \to \frac{d_T}{2} \pm \frac{\sqrt{d_T^2 - 4 f d_T}}{2} \end{cases}$

   b. Determine a formula for the distance between the two lens positions in *part (a)* and the ratio of the image sizes.

   For each object there is a corresponding image distance given by $d_i = d_T - d_o = \frac{d_T}{2} \mp \frac{\sqrt{d_T^2 - 4 f d_T}}{2}$.

   The image height is determined from the magnification and the object height. We have then $\frac{h_1}{h_0} = m_1 = \frac{d_{i1}}{d_{o1}} \to h_1 = m_1 h_0 = \left(\frac{d_{i1}}{d_{o1}}\right) h_0$ and $\frac{h_2}{h_0} = m_2 = \frac{d_{i2}}{d_{o2}} \to h_2 = m_2 h_0 = \left(\frac{d_{i2}}{d_{o2}}\right) h_0$. Dividing these two expressions gives the ratio of the image heights. We have $\frac{h_1}{h_2} = \frac{m_1 h_0}{m_2 h_0} = \frac{m_1}{m_2} = \left(\frac{d_{i1}}{d_{o1}}\right)\left(\frac{d_{o2}}{d_{i2}}\right)$. Inserting the expressions for the object and image distances we get

$$\frac{h_1}{h_2} = \left(\frac{d_{i1}}{d_{o1}}\right)\left(\frac{d_{o2}}{d_{i2}}\right) = \left[\frac{\frac{d_T}{2} - \frac{\sqrt{d_T^2 - 4 f d_T}}{2}}{\frac{d_T}{2} + \frac{\sqrt{d_T^2 - 4 f d_T}}{2}}\right]\left[\frac{\frac{d_T}{2} - \frac{\sqrt{d_T^2 - 4 f d_T}}{2}}{\frac{d_T}{2} + \frac{\sqrt{d_T^2 - 4 f d_T}}{2}}\right] = \left[\frac{d_T - \sqrt{d_T^2 - 4 f d_T}}{d_T + \sqrt{d_T^2 - 4 f d_T}}\right]^2$$

6. Radioactive elements have important applications in medicine, research or industry. A particular radioactive element is ${}^{60}_{27}Co$, which has a half-life of $5.27 yr$ and decays by emitting a beta particle ($0.311 MeV$) and two gamma rays ($1.17 MeV$ and $1.33 MeV$).

   a. Suppose that you want a sample of ${}^{60}_{27}Co$ with an activity of $10 Ci$, where $1 Ci = 3.7 \times 10^{10} \frac{decays}{s}$ after 30 months, what mass of ${}^{60}_{27}Co$ do you need?

$$\lambda_{Co} = \frac{\ln 2}{t_{½}} = \frac{\ln 2}{5.2 yr} \times \frac{1 yr}{31.5 \times 10^6 s} = 4.17 \times 10^{-9} s^{-1}$$

$$t = 30 mo \times \frac{1 yr}{12 mo} \times \frac{31.5 \times 10^6 s}{1 yr} = 7.89 \times 10^7 s$$

$$A(t) = A_0 e^{-\lambda_{Co} t} = (\lambda_{Co} N_{Co}) e^{-\lambda_{Co} t} \rightarrow N_{Co} = \frac{A(t)}{\lambda_{Co}} e^{\lambda_{Co} t}$$

$$N_{Co} = \left[\frac{10 Ci \times \frac{3.7 \times 10^{10} \frac{decays}{s}}{1 Ci}}{4.17 \times 10^{-9} s^{-1}} \right] e^{(4.17 \times 10^{-9} s^{-1}) \times (7.89 \times 10^7 s)} = 1.23 \times 10^{20}$$

$$\therefore M_{Co} = 1.23 \times 10^{20} \times \frac{1 mol}{6.02 \times 10^{23}} \times \frac{59.93 g}{1 mol} = 0.01226 g = 12.3 mg$$

   b. What rate will the source emit energy after 30 months? Assume that all three decay products are emitted at the same time.

$$P = \frac{E_{total}}{t} = \frac{E_{total}}{decay} \times \frac{decay}{time} = E_{total} \times A = \left[(0.310 + 1.17 + 1.33) \frac{MeV}{decay} \right] \times 37 \times 10^{10} \frac{decay}{s}$$

$$P = 1.04 \times 10^{12} \frac{MeV}{s}$$

$$= 1.04 \times 10^{18} \frac{eV}{s}$$

$$= 0.166 \frac{J}{s} = 0.166 W$$