Name____

Physics 111 Quiz #3, January 23, 2015

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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A paper-filled ($\kappa = 4$) parallel-plate capacitor has a length of $L = 60cm$ and a width of $W = 75cm$. The plates are separated by $d = 0.5mm$. The capacitor is connected to a battery $V = 24V$, a resistor $R = 20M\Omega$ and a switch S. At time $t = 0$ the switch is closed and the capacitor begins to charge.

1. What is the capacitance of the capacitor and what is the maximum charge that can be placed on the capacitor?

$$C = \frac{\kappa\varepsilon_0 A}{d} = \frac{4\times 8.85\times 10^{-12}\,\frac{C^2}{Nm^2}\times(0.6m\times 0.75m)}{0.5\times 10^{-3}m} = 3.2\times 10^{-8}F$$

$$Q_{\max} = CV_{\max} = 3.2\times 10^{-8}F\times 24V = 7.7\times 10^{-7}C$$

2. At what time $t > 0$ is the potential across the capacitor equal to that across the resistor?

Method 1:

$$V_C(t) = V_R(t) \to V_{\max}\left(1 - e^{-\frac{t}{RC}}\right) = V_{\max}e^{-\frac{t}{RC}} \to t = -RC\ln\left(\frac{1}{2}\right)$$

$$t = -\left(20\times 10^6\Omega\times 3.2\times 10^{-8}F\right)\ln\left(\tfrac{1}{2}\right) = 0.44s$$

Method 2: $V_R(t) = \dfrac{V_{\max}}{2} = V_{\max}e^{-\frac{t}{RC}} \to t = -RC\ln\left(\dfrac{1}{2}\right) = 0.44s$

Method 3: $V_C(t) = \dfrac{V_{\max}}{2} = V_{\max}\left(1 - e^{-\frac{t}{RC}}\right) \to t = -RC\ln\left(\dfrac{1}{2}\right) = 0.44s$

3. What is the energy stored in the capacitor when it is fully charged?

$$E_{\max} = \tfrac{1}{2}CV_{\max}^2 = \tfrac{1}{2}\times 3.8\times 10^{-8}F\times(24V)^2 = 9.2\times 10^{-6}J$$

4. Suppose that after a long time, the capacitor is fully charged to $Q_{\max}$. If the battery is removed from the circuit and the capacitor is connected to the resistor and allowed to discharge through the resistor, how long would it take for the energy stored in the capacitor to reach 1% of its maximum amount?

$$E(t) = \tfrac{1}{2}CV^2(t) = \tfrac{1}{2}C\left(V_{\max}e^{-\frac{t}{RC}}\right)^2 = \tfrac{1}{2}CV_{\max}^2 e^{-\frac{2t}{RC}} = E_{\max}e^{-\frac{2t}{RC}}$$

$$t = -\frac{RC}{2}\ln\left(\frac{E(t)}{E_{\max}}\right) = -\frac{20\times10^6\,\Omega\times3.2\times10^{-8}\,F}{2}\ln\left(\frac{0.01E_{\max}}{E_{\max}}\right) = 1.5s$$

5. The table below gives for sets of values for different resistor-capacitor circuits. Assume that at time $t = 0$ all capacitors are fully charged. Which circuit would take the least amount of time for the potential to decrease to one-half of its initial amount?

a. Circuit #1
b. Circuit #2
c. Circuit #3
d. Circuit #4
e. There is not enough information available to answer the question.

Circuit #	1	2	3	4
$V(V)$	12	12	10	10
$R(\Omega)$	2	3	10	5
$C(\mu F)$	3	2	0.5	2
$\tau = RC(\times10^{-6}s)$	6	6	5	10

Physics 111 Equation Sheet

Electric Forces, Fields and Potentials

$$\vec{F} = k\frac{Q_1 Q_2}{r^2}\hat{r}$$

$$\vec{E} = \frac{\vec{F}}{q}$$

$$\vec{E}_Q = k\frac{Q}{r^2}\hat{r}$$

$$PE = k\frac{Q_1 Q_2}{r}$$

$$V(r) = k\frac{Q}{r}$$

$$E_x = -\frac{\Delta V}{\Delta x}$$

$$W = -q\Delta V_{f,i}$$

Magnetic Forces and Fields

$$F = qvB\sin\theta$$

$$F = IlB\sin\theta$$

$$\tau = NIAB\sin\theta = \mu B\sin\theta$$

$$PE = -\mu B\cos\theta$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\varepsilon_{induced} = -N\frac{\Delta\phi_B}{\Delta t} = -N\frac{\Delta(BA\cos\theta)}{\Delta t}$$

Constants

$g = 9.8\,\frac{m}{s^2}$

$1e = 1.6\times10^{-19}C$

$k = \frac{1}{4\pi\varepsilon_o} = 9\times10^9\,\frac{C^2}{Nm^2}$

$\varepsilon_o = 8.85\times10^{-12}\,\frac{Nm^2}{C^2}$

$1eV = 1.6\times10^{-19}J$

$\mu_o = 4\pi\times10^{-7}\,\frac{Tm}{A}$

$c = 3\times10^8\,\frac{m}{s}$

$h = 6.63\times10^{-34}Js$

$m_e = 9.11\times10^{-31}kg = \frac{0.511MeV}{c^2}$

$m_p = 1.67\times10^{-27}kg = \frac{937.1MeV}{c^2}$

$m_n = 1.69\times10^{-27}kg = \frac{948.3MeV}{c^2}$

$1amu = 1.66\times10^{-27}kg = \frac{931.5MeV}{c^2}$

$N_A = 6.02\times10^{23}$

$Ax^2 + Bx + C = 0 \rightarrow x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$

Electric Circuits

$$I = \frac{\Delta Q}{\Delta t}$$

$$V = IR = I\left(\frac{\rho L}{A}\right)$$

$$R_{series} = \sum_{i=1}^{N} R_i$$

$$\frac{1}{R_{parallel}} = \sum_{i=1}^{N}\frac{1}{R_i}$$

$$P = IV = I^2R = \frac{V^2}{R}$$

$$Q = CV = \left(\frac{\kappa\varepsilon_0 A}{d}\right)V = (\kappa C_0)V$$

$$PE = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}$$

$$Q_{\text{charge}}(t) = Q_{\max}\left(1-e^{-\frac{t}{RC}}\right)$$

$$Q_{\text{discharge}}(t) = Q_{\max}e^{-\frac{t}{RC}}$$

$$C_{parallel} = \sum_{i=1}^{N} C_i$$

$$\frac{1}{C_{series}} = \sum_{i=1}^{N}\frac{1}{C_i}$$

Light as a Wave

$$c = f\lambda = \frac{1}{\sqrt{\varepsilon_o\mu_o}}$$

$$S(t) = \frac{energy}{time\times area} = c\varepsilon_o E^2(t) = c\frac{B^2(t)}{\mu_0}$$

$$I = S_{avg} = \tfrac{1}{2}c\varepsilon_o E_{\max}^2 = c\frac{B_{\max}^2}{2\mu_0}$$

$$P = \frac{S}{c} = \frac{Force}{Area}$$

$$S = S_o\cos^2\theta$$

$$v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{n}$$

$$\theta_{inc} = \theta_{refl}$$

$$n_1\sin\theta_1 = n_2\sin\theta_2$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$M_{total} = \prod_{i=1}^{N} M_i$$

$$S_{out} = S_{in}e^{-\sum_i \mu_i x_i}$$

$$HU = \frac{\mu_w - \mu_m}{\mu_w}$$

Light as a Particle & Relativity

$$E = hf = \frac{hc}{\lambda} = pc$$

$$KE_{\max} = hf - \phi = eV_{stop}$$

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\phi)$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$p = \gamma mv$$

$$E_{total} = KE + E_{rest} = \gamma mc^2$$

$$E_{total}^2 = p^2c^2 + m^2c^4$$

$$E_{rest} = mc^2$$

$$KE = (\gamma - 1)mc^2$$

Nuclear Physics

$$E_{binding} = (Zm_p + Nm_n - m_{rest})c^2$$

$$\frac{\Delta N}{\Delta t} = -\lambda N_o \rightarrow N(t) = N_o e^{-\lambda t}$$

$$A(t) = A_o e^{-\lambda t}$$

$$m(t) = m_o e^{-\lambda t}$$

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

Misc. Physics 110 Formulae

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\Delta(mv)}{\Delta t} = m\vec{a}$$

$$\vec{F} = -k\vec{y}$$

$$\vec{F}_C = m\frac{v^2}{R}\hat{r}$$

$$W = \Delta KE = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right) = -\Delta PE$$

$$PE_{gravity} = mgy$$

$$PE_{spring} = \tfrac{1}{2}ky^2$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

$$\phi = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

Geometry

$Circles: \quad C = 2\pi r = \pi D \qquad A = \pi r^2$

$Triangles: \quad A = \tfrac{1}{2}bh$

$Spheres: \quad A = 4\pi r^2 \qquad V = \tfrac{4}{3}\pi r^3$