

Name _____

Physics 111 Quiz #3, January 24, 2025

Please show all work, thoughts and/or reasoning to receive partial credit. The quiz is worth 10 points total, and all parts may not be of equal weight.

I affirm that I have carried out my academic endeavors with full academic honesty.

An air-filled parallel-plate capacitor is constructed from two circular metal plates with radius 10cm separated by a distance of 1mm . The initially uncharged capacitor is connected to a $R = 100\text{M}\Omega$ resistor and a 50V battery.

1. What is the maximum amount of charge that can be stored on the capacitor when it is fully charged?

$$C = \frac{\kappa\epsilon_0 A}{d} = \frac{1 \times 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{Nm}^2} \times \pi(0.1\text{m})^2}{1 \times 10^{-3}\text{m}} = 2.8 \times 10^{-10}\text{F}$$

$$Q = CV = 2.8 \times 10^{-10}\text{F} \times 50\text{V} = 1.4 \times 10^{-8}\text{C}$$

2. What is the magnitude and direction of the electric field between the capacitor plates?

$$E = \frac{Q}{\epsilon_0 A} = \frac{1.4 \times 10^{-8}\text{C}}{8.85 \times 10^{-12} \frac{\text{C}^2}{\text{Nm}^2} \times \pi(0.1\text{m})^2} = 50000 \frac{\text{N}}{\text{C}} \text{ in magnitude.}$$

Or,

$$E = \left| -\frac{\Delta V}{\Delta x} \right| = \frac{50\text{V}}{1 \times 10^{-3}\text{m}} = 50000 \frac{\text{V}}{\text{m}} \text{ in magnitude.}$$

The direction of the electric field points from the positive to the negative plate.

3. Suppose that after the capacitor is fully charged the battery is removed. The capacitor is then connected to the $100M\Omega$ resistor and allowed to discharge. What is the time constant for the discharging circuit?

$$\tau = RC = 100 \times 10^6 \Omega \times 2.8 \times 10^{-10} F = 0.028s = 28ms$$

4. What is the voltage across the capacitor $4ms$ after it begins to discharge?

$$V = V_{max} e^{-\frac{t}{\tau}} = 50V e^{-\frac{4ms}{28ms}} = 43.3V$$

5. At a time $4ms$ after the capacitor begins to discharge, how much energy has been dissipated across the resistor?

The initial stored energy is: $U_i = \frac{1}{2}CV_i^2 = \frac{1}{2} \times 2.8 \times 10^{-10} F \times (50V)^2 = 3.5 \times 10^{-7} J$

$U = \frac{1}{2}CV_f^2 = \frac{1}{2} \times 2.8 \times 10^{-10} F \times (43.3V)^2 = 2.6 \times 10^{-7} J$ left to dissipate.

The amount of energy dissipated is therefore $3.5 \times 10^{-7} J - 2.6 \times 10^{-7} J = 9 \times 10^{-8} J$