Name____

Physics 111 Quiz #7, November 11, 2022

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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1. Oxygen-15 ($^{15}_{8}O,\ m_{^{15}_{8}O} = 15.003066u$) is a positron emitter. What is the reaction that characterizes the decay of $^{15}_{8}O$?

$$^{15}_{8}O \rightarrow {}^{0}_{+1}e + {}^{15}_{7}N + \upsilon_e$$

2. If the recoil of the decay product of $^{15}_{8}O$ can be ignored, what is the kinetic energy of the emitted positron in MeV? Some useful masses may be found in the table below.

$$Q = K_{e^+} + K_N \cong K_{e^+} = [M_O - M_N - 2M_{e^+}]c^2$$

$$Q \cong K_{e^+} = \left\{[15.003066u - 15.000109u - 2 \times 0.00054u] \times \frac{931.5\frac{MeV}{c^2}}{1u}\right\}c^2 = 1.748MeV$$

3. Suppose that the emitted positron collided with and annihilated with an orbital electron in the decay product. If we consider the orbital electron to be at rest at the time of annihilation, what is the energy of one of the gamma-ray photons produced?

$$m_{e^-}c^2 + m_{e^+}c^2 + K_{e^+} = 2E_\gamma \rightarrow E_\gamma = \frac{2m_{e^+}c^2 + K_{e^+}}{2}$$

$$E_\gamma = \frac{\left([2 \times 0.00054u] \times \frac{931.5\frac{MeV}{c^2}}{1u}\right)c^2 + 1.748MeV}{2}$$

$$E_\gamma = 1.377MeV$$

Table of Masses
$m_{^1_1p} = 1.007276u$
$m_{^2_1H} = 2.014102u$
$m_{^{0}_{-1}e} = m_{^{0}_{+1}e} = 0.00054u$
$m_{^4_2He} = 4.00260u$
$m_{^{15}_{9}F} = 15.017785u$
$m_{^{15}_{7}N} = 15.000109u$

4. Suppose some of the gamma-rays from the electron-positron annihilation were incident on stationary electrons in a block of insulating material. If the scattered gamma-ray photons are detected at an angle $\phi = 120^0$ measured with respect to the incident beam's direction, what is the energy of the scattered gamma-ray photons?

$$\frac{1}{E'} = \frac{1}{E} + \frac{1 - \cos\phi}{m_e c^2} = \frac{1}{1.377MeV} + \frac{1 - \cos 120}{0.511\frac{MeV}{c^2} \times c^2} = 3.662MeV^{-1} \rightarrow E' = 0.273MeV$$

5. What is the speed, expressed as a fraction of the speed of light, of the recoiling electron in the block of insulating material?

$$E = E' + K \rightarrow K = E - E' = 1.377MeV - 0.273MeV = 1.104MeV$$

$$K = (\gamma - 1)m_e c^2 \rightarrow \gamma = \frac{K}{m_e c^2} + 1 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \rightarrow v = \sqrt{1 - \frac{1}{\gamma^2}}\,c$$

$$\gamma = \frac{K}{m_e c^2} + 1 = \frac{1.104MeV}{0.511\frac{MeV}{c^2} \times c^2} + 1 = 3.1605 \rightarrow v = \sqrt{1 - \frac{1}{(3.1605)^2}}\,c = 0.949c$$